# 7ci fgY'Cj Yfj ]Yk

### GhJZZ7cbhJWh8YhJ]`g`

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUhjcb	D\cbY
Gi Y`GhUfZJY`X	g"gfUfZJY`X4 i bgk "YXi "Ui	D`YUgY`WcbHJWfia Y VmYa U]`"	9	f\$&Ł'-',) &)&(

## GWXcc~7cbHJWh~abZcfa Uh]cb

GW.cc``cZ9Xi WUh]cb 5fhg`UbX`GcW]U``GW]YbWYg

@Y|Y`'%z'>c\b'; ccXgY``'6i]`X]b[ 'fl &\$Ł

H. 'Ž \* %'f&Ł'-',) '%-++

9. YXi WUhcb4 i bgk "YXi "Ui

K. YXi WUhjcb"Ufhg"i bgk "YXi "Ui

#### 5 h Y b X U b W F Y e i ] f Y a Y b h g

#### 5XX]h]cbU`GWXcc`cZ'9Xi WUh]cb'UhhYbXUbWY'fYei ]fYa Ybhq

#i]gYl dYWYX'h\UhUbmghi XYbhYbfc``YX']b'Ub'98GH'Wci fgY'k]h\Y'GW\cc``cZ9Xi WUhjcb'k]``a U\_Y'U Wca a ]ha Ybhhc'h\Y]f``YUfb]b['UbX'UhYbX'U```WUggYg']b'Z ```k\YfY'WcbhYbh]g'XY`]j YfYX']b'U'ZJ\W!hc!ZJ\W'a cXY'Yl Wrdh]b'WjfW a ghUbWrg'k\YfY'UVgYbW']g'Xi Y'hc']``bYggža ]gUXj Ybhi fY'cf'i bZcfYgYYb WjfW a ghUbWrg'VYmcbX'h\Y'ghi XYbhg'Wcbhfc`'''5'a ]b]a i a 'UhhYbXUbW'fYei ]fYa YbhcZ, \$i ']g'fYei ]fYX'Zcf YUW\'Wci fgY'''\#b'\WffU]b'\WjfW a ghUbWrgžU'ghi XYbha Uma ]gg'i d'hc'&\$i 'cZU'Wci fgY'k]h\ci h\Zcfa U` Udd']\WUhjcb'UbX'i d'hc''' i 'cZU'Wci fgY'k]h\'Zcfa U``XcW a YbhUhjcb'fgYbhj ]U'Ya U]`hc'\wci fgY'\wcbj Ybcft dfcj ]X]b['U``h\Y'fYei ]fYa Ybhg'XYhU]YX'VY`ck 'UfY'a Yh'''Ghi XYbhg'bcha YYhjb['h\Y'UhhYbXUbW\'fYei ]fYa Ybhg'ci h]bYX'UVcj Y'k]```VY'Uk UfXYX'Ub'I bgUhjgZU\wcfm: U]`fI : t'Zcf'h\Uh\\ci fgY'fY[UfX`Ygg'cZ h\Y]f'dYfZcfa Ub\W']b'h\Y'UggYgga YbhhUg\_g'cf'ch\Yf'fYei ]fYa Ybhg'Zcf'h\Y'\wci fgY''

: cfiZ fh\Yfi]bZcfa Uhjcbicbi9Xi WUhjcbiWci fgYiUhhYbXUbWYifYei ]fYa YbhgžidiYUgYifYZYfihcih\YifYgdYWhji Y 98GHiA ccXiYia cXi `Yfgt"

#### 5 WUXYa 1W = b Zcfa Uhlcb

cf'YggYbhjU`ghi XYbhjbzcfa Uhjcb'fY`Uhjb['hc.'fYei Yghg'zcf'Yl hYbg]cb/fYj ]Yk 'cZa Uf\_g/cWW dUhjcbU``\YU'h\
UbX'gUZYhm'Yl Ua ]bUhjcb'dfcWYXi fYg/gdYWjU``Wcbg]XYfUhjcb']b'h\Y'Yj YbhcZ]``bYgg'cf'a ]gUXj Ybhi fY/
ghi XYbhYei ]hm'UbX'X]gUV]`]hm'UbX'ch\Yf'YggYbhjU``UWUXYa ]W]bZcfa Uhjcbž
aYY`\hdg.##k k k "Ufhg"i bak "YXi "Ui #W ffYbhjahi XYbhg#JWUXYa ]W]bZcfa Uhjcb#dfchcWc`a![i ]XY`]bYg#

UWX]Yj YX:h\Y]f'di fdcgY""Mci 'UfY YbWci fU[ YX:hc 'h\]b\_'UVci h\ck 'mci 'k ci 'X:`]\_Y'hc ''gci bX']b 'mci f'h\Yg]g UbX:h\Y'\_]bX:cZ'UWUXYa ]W]XYbh]m'mci 'hU\_Y:cb""H\]g'gYW]cb'WcbWi XYg'k ]h\'U'X]gW gg]cb'cZh\Y'fYgYUfWX cb'Yl Ua ]bYfg'Yl dYWUh]cbg'cZfYgYUfWX:h\YgYg"H\Y'Yl Ua ]bYf']gž'UVcj Y'U``žU'fYUXYf"K Y'k ]``h\Yb X]gW gg'lk c'ch\Yf'g][ b]Z]WUbhfYgYUfWX:[ YbfYg. 'h\Y'WcbZYfYbWY'dUdYf'UbX'h\Y'ci fbU``Ufh]WY'UbX'dfcj ]XY Ub'cddcfhi b]mmZcfmci 'hc Yl Ua ]bY'U'ci fbU``Ufh]WY'Zfca 'mci f'Z]Y'X""A cghX]gW]d`]bYg'bck 'Yl dYWhfYgYUfWX ghi XYbhg'hc'di V`]g\'dudYfg'Zfca 'h\Y]f'fYgYUfWX:]b'h\Y'Wci fgY'cZk f]h]b[ 'h\Y]f'h\Yg]g'UbX'h\]g'Wb'VYbYZ]hh\Y ghi XYbh]b'Ubi a VYf'cZk Umg""K \]Y'h\YfY'k ]``VY'a Ubm]bZcfa U``cddcfhi b]h]Yg'Zcf'X]gW gg]cb'UbX dUfh]W]dUh]cb'Xi f]b[ 'h\Y'Wci fgYž'cbY'gYgg]cb'`cc\_g'gdYW]ZWU``mUh[ ]j ]b[ 'cfU'`dfYgYbhUh]cbg""5```ghi XYbhg k ]```VY'Ug\_YX'hc'[ ]j Y'U'g\cfhdfYgYbhUh]cb'hc'h\Y]f'dYYfg'cb'U'fYgYUfWX!fY'UhYX'hcd]WUbX'[ ]j Y'UbX'fYWY]j Y ZYYXVUW\_"'H\Y'U]a 'cZh\]g'Wca dcbYbhcZh\Y'Wci fgY']g'hc'\Y'd'ghi XYbhg'dfYdUfY'Zcf'h\Y'j Uf]YX'WcbHYl hg']b k \]W\'h\Ymk ]``bYYX'hc'Wa a i b]WbHY'-gYa ]bUfg/fYj ]Yk g/WcbZyfybWg"

 $\label{thm:condition} $$HY'W' ifgY']bjc'jYg'U'XYHU]YX'ghXmcZhY'hYcfmz'fYgYUfWX'UbX'dfUWJWY'UggcWJUhYX'k]hX'hX]g'hcd]WUgkY''`Ug'hXY'cddcfhib]mhc'Wcad'YhY'Ub'UWJcb'fYgYUfWX'dfcYW''$ 

# 5ggYgga Ybh

Q: YfY 'mci 'WUb'ci h]bY 'UbmfY`Yj Ubh]bZcfa Uh]cb'h\Uhk Ug'bch]bWi XYX`]b'5  $\Rightarrow$  G'Vi ha Umdfcj Y'\Y`dZ\`Zcf mci f'ghi XYbhg": cf'Yl Ua d`Yžmci 'a ][\hdfcj ]XY'XYhU]`g'cb'h\Y'fYZYfYbW]b[ 'gmghYa ž``]b\_g'hc'dfYj ]ci g ghi XYbhYl Ya d`Ufg'cf'h\Y'XYg][ bUhYX'k YY\_']b'h\Y'Wci fgY'h\Uhmci 'k ]```X]gWi gg'h\Y'UggYgga Ybh'Uh`Yb[ h\"  $\Rightarrow$  dcfhUbhmž'h\]g'gYWfcb']g'Ub'UfYU'Zcf'mci 'hc'dfcj ]XY']bZcfa Uh]cb'h\UhXcYg'bch[ c'h\fci [\'h\Y'Uddfcj YX [cj YfbUbW'ghfi Whi fY''Q

# 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'HJg_	K Y][ \h		

#### fYdcfhg"

5ggYgga Ybh& 5ggYgga Ybh' fZJbU'Ł

#### GhUfh XUhY.

8 YHJ]`g.: ]bU``5ggYgga Ybhfl Ł`5bU`mg]g`cZU`X]gW gg]cb`W.UdhYf``GY`YWhU`X]gW gg]cb`W.UdhYf`Zfca `U fYWYbhfdcgh&\$%\$£'D\8 `cf`A UghYfg'h\Yg]g`]b`h\Y`GcWJU``GWJYbW'g`cf`<iaUb]hJYg`gi Va ]hhYX`Uh'Ub 5i ghfU`]Ub`i b]j Yfg]hm'`I g]b[ 'h\Y`A cj Y`ZfUa Yk cf\_`ZcfX]gW gg]cbg`dfYgYbh'X`]b`WUgg`X]gW gg`h\Y`YI hYbhhc k\]W. 'h\Y`Ui h\cf`i gYg`h\YgY`a cj Yg`]b`ghfi Wfi f]b[ 'h\Y`W.UdhYf''``Mci 'g\ci `X`W&bg]XYf`bchcb`mk\]W. a cj Yg`UfY`dfYgYbhUbX`Wa a Ybhcb`k\mmci 'h\]b\_'h\Y`Ui h\cf`\Ug`i gYX`h\Ya 'Vi h'U`gc`cb`h\cgY`bch UXcdhYX`Vmh\Y`Ui h\cf`UbX`gi [ [ Yghk\mh\]g`a ][ \hVY`gc''`: ]bU``mz`mci 'g\ci `X`Wa a Ybhcb`k\Yh\Yf`h\Y Ui h\cf`YI d`]WJhm]XYbhJZJYg`h\Y`g][ b]ZJWUbhWbbfJVi hJcb`cZh\Y`h\Yg]g`hc`h\Y`ZJY`X`cZghi XmUbX`h\Y`YI hYbhhc k\]W.``Ub[i U[ Y`ZYUhi fYg`gi W.`Ug`\YX[]b[ 'UfY`i gYX''K cfX``Yb[h\.`&\$\$\$K f]hhYb`ZYYXVUW\_k ]```VY`[ ]j Yb k]h\]b'' kYY\_g`cZgi Va ]gg]cb

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM &$ 

5ggYgga Ybh' . 5ggYgga Ybh&

#### GHJfhXUhY.

8 YHJ]`g. '5 ggYgga Ybh'& '5 bU`mg]b[ 'U'h\Yg]g`]bhfcXi Whjcb'GY`YWhh\Y`±bhfcXi Whjcb'W\UdhYf`Zfca 'U'fYWbh fdcgh'&\$\%\$\LD\8 'h\Yg]g`]bh\Y`GcW]U`GW]YbWrg`cf'<ia Ub]h]Yg`gi Va ]hhY`UhUb'5i ghfU`]Ub'i b]j Yfg]hm''l g]b[ h\Y`7 fYUh]b[ 'U'F YgYUfW\GdUWY`fl7 5 F GŁ'ZfUa Yk cf\_'UbU`mgY'h\Y`W\UdhYf`]b'hYfa g`cZh\Y`Ui h\cf'g`i gY`cZh\Y`Ui h\cf'g`i gY`cZh\Y`Ui h\cf'g'i gY`cZh\Y`Ui h\cf'\Ug'W\cgYb'hc`cf[ Ub]gY'h\Y]f`W\UdhYf`]b'h\]g k UmUbX`\ck 'mci 'Ug'U'fYUXYf`fYUWhc'h\]g`cf[ Ub]gUh]cb'' =\mathred{Zmci 'k YfY`UXj ]g]b[ 'h\Y`Ui h\cf'cb'\ck 'hc ]a dfcj Y`h\]g`W\UdhYf`k\Uhk ci 'X`mci 'gi [ [ Ygh3'K cfX``Yb[ h\. '&\$\$\$K f]hhYb'ZYYXVUW\_k ]``VY`[ ]j Yb'k ]h\]b'' k YY\_g`cZgi Va ]gg]cb

# I BGK 'G7<CC@'C: '98I 75H€B'FI 6F = 7 # 998657? 'G<99H' 98GH) %(+

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GdYWJZJW7 fJhYf]U	``f!Ł—-	——2fŽŁ
I bXYfghUbX`cZh\Y`ei Ygh]cb`cf`]ggi Y`UbX`h\Y`_YmWcbWYdhg ]bj c`j YX		
8Ydh\cZUbUmg]g`UbX#cf`Wf]heiY`]b`fYgdcbgY`hc`h\Y`HJg_		
: Ua ]`]Uf]hmk ]h\ 'UbX'fY`Yj bWY'cZ'dfcZYgg]cbU`'UbX#cf'fYgYUfVX `]hYfUh fY'i gYX'hc'gi ddcfh'fYgdcbgY		
GHfi Whi fY'UbX'cf[Ub]nUh]cb'cZfYgdcb		
DfYgYbhUhjcbicZifYggcbgy www.fxjb[ihciuddfcdf]UhYiUWUXYa]WUbX `]b[i]ghjWWcbjYbhjoog		
; YbYfU`Wca a Ybhg#YWca a YbXUh]cbgʻZcfʻbYl hh]a Y.ʻ		

@YWhi fYf

FYWcaa YbXYX. ' "#&\$"fl@"DG"7F"8B"<8E" k Y][\h]b[ '. "&\$i

"B6. h\]g'h\Wg']b'h\Y'j Uf]ci g'Vcl Yg'UbX'XYg][bYX'hc'dfcj]XY'ZYYXVUW\_hc'ghi XYbhg/h\YmUfY'bch[]j Yb
Yei U`k Y][\h]b'XYhYfa ]b]b['h\Y'fYWca a YbXYX'[fUXY"8YdYbX]b['cb'h\Y'bUhi fY'cZh\Y'UggYgga YbhHUg\_ž
'YWhi fYfg'a Um'U`gc'WcbhYl hi U`]nY'UbX#cf'Ua YbX'h\YgY'gdYV]ZJWWf]hYf]U"''H\Y'fYWca a YbXYX'[fUXY']g
hYbhUh]j Y'cb`mžgi Va 'YWh'nc'ghUbXUfX]gUh]cb'dfcWyggYg'UbX'Uddfcj U`'Vmh\Y'GW\cc`'cZ'9Xi WUh]cb
@YUfb]b['UbX'HYUW\]b['7ca a ]hhYY"

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

 $Gh\ XYbhg\ 'UfY\ 'YI\ dYWhYX\ 'hc\ 'di\ hh\ Y]f\ 'bUa\ Yg\ 'UbX\ 'gh\ XYbhbi\ a\ VYfg\ 'cb\ 'Yj\ YfmdU[\ Y\ 'cZh\ Y]f\ 'Ugg][\ ba\ Ybhg\ ''$ 

Hi fb]h]b Gi Va ]gg]cb

 $\bullet \ \ \, \text{K cf\_'gi Va ]} \\ \text{lh} \\ \text{X '\%' hc '\% 'XUmg 'UZhYf' h\ Y 'gh]} \\ \text{di ``UhYX 'XYUX`]} \\ \text{bY } \\$ 

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWfg'cZ9@ ${\mathfrak G}$ 9'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "f\hd.#gi V\YWfi \\X\yg''\YfUfm'i bgk\\"YXi\\"Ui\#\`\]qY\#JVci \\Y\\]qY\\

# <mark>77ċċġ</mark>yf<mark>ċŢĸċĠġŶĸĠ₩₩</mark>ŶXi`Y

J]Yk 'WUgg'ha YhUV'Y

ZcW gʻcbʻk f]njb[ʻWcbZYfYbWYʻUVghfUWngʻUbXʻci fbUʻ UfnjWYʻ]bhfcXi Wnjcbg"ʻ=k]```g\UfY`k]n\`mci a m YI dYf]YbWYʻcZVY]b[ʻUʻci fbU``YX]ncfʻUbXʻfYj]Yk Yf UbXʻdfcj]XYʻUXj]WYʻcbʻ[Yhnjb[ʻdi V`]g\YX´]b UWUXYa]W`ci fbU`g"ʻ5Wnjj]njYgʻk]``fY`UhY`nc`n\]gʻZcW g"
D`YUgY`Vf]b[ 'U''ci fbU``Ufh]WY'Zfca 'U''ci fbU``mci k ci 'X'`]_Y'hc'di V`]g\']b'hc'WUgg"

# FYgcifWFgYgcifWYg

# DfYgWf]VYX F Ygci fWYg

## FYWca a YbXYX FYgci fWfg

9j Ubgž'8 "ž; fi VUž'D "/ 'NcVY`ž >" 'f&\$\%' | fX 'YX]hjcbt" : ai fa i dfWS TWffWdfZWe[e" 'A Y`Vci fbYž'5 i ghfU`]U. A Y`Vci fbY`I b]j Yfg]hmDfYgg" ! h\YfY`UfY`gYj YfU``YX]hjcbg`cZh\]g`Vcc\_

 $A i ffUm \ref{thmoments} F "`f \& \$\%+ \& "``: ai fa i d[fWS fZWe[e"``f(h\ `YX]h]cb\&"`A U]XYb \ YUX \ref{thmoments} 9b[ `UbX.`CdYb`I b]j Yfg]hmDfYgg"``! h\ YfY`UfY`gYj YfU``YUf`]Yf`YX]h]cbg`cZh\]g`Vcc_'k \ ]W\ `UfY`j Yfm\Y`dZ\ ``hcc''$ 

DU'ff]X[Yž6"/GHUfZ]Y`XžG"'f&\$\$+Ł"``FZWe[eS`VV[eeWdfSf[a`idff[Y[SeWUa`V'S`YgSYW''@cbXcb.FcihYX[Y"]

## 7 ci fgY 9j U'i Uh]cb UbX 8 Yj Y`cda Ybh

 $: YYXVUW\underline{\ \ } cb 'h \underline{\ \ } g'Wc i fgY 'k \underline{\ \ \ } ``VY'gc i [\ hj ]U'A m9I dYf]YbW' gi fj Ymg'UbX'U'gc 'Xi f]b[ 'h\Y'Wc i fgY']hgY`Z' = k Y`Wc a Y'gi [[Ygh]cbg'UbX'ZYYXVUW\underline{\ \ \ } ca 'dUfh]W]dUbhg'Uh'Ubmh]a Y'j ]U'Ya U]`cf'h\fc i [\G''']$