7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g`

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D\cbY
F YVYVWU 7 c ``]Y	fYVYVWW"Wc``]Y4 i bgk "YXi "Ui	-	Fa'% %'>c\b ; ccXgY`` 6i]`X]b[\$&`-' ,) -' %+

GWkcc~7cbhJWh~abZcfa Uhlcb

GWXcc``cZ9Xi WUhjcb 5fhg`UbX`GcWjU``GWjYbWYg

@Y|Y``%ž'>c\b'; ccXgY```6|]`X]b['f! &\$Ł

H. Ž*%f&Ł'-',) '%++

9. YXi WUhcb4 i bgk "YXi "Ui

K . YXi WUNcb"Ufhg"i bak "YXi "Ui

5 hryb XUb WY F Yei] fYa Ybhg

5XX]h]cbU``GWXcc``cZ'9Xi WUh]cb'UhhYbXUbWY`fYei]fYa Ybhq

#i]gYl dYWYX'h\UhUbmghi XYbhYbfc``YX']b'Ub'98GH'Wci fgY'k]h\'h\Y'GW\cc``cZ9Xi WUhjcb'k]``a U_Y'U Wca a]ha Ybhhc'h\Y]f``YUfb]b['UbX'UhYbX'U```WUggYg']b'Z ```k\YfY'WcbhYbh]g'XY`]j YfYX']b'U'ZJ\W!hc!ZJ\W'a cXY'Yl Wrdh]b'WfW]a ghUbWrg'k\YfY'UVgYbW']g'Xi Y'hc']``bYggža]gUXj Ybhi fY'cf'i bZcfYgYYb WfW a ghUbWrg'VYmcbX'h\Y'ghi XYbhg'Wcbhfc`"'5'a]b]a i a 'UhhYbXUbW'fYei]fYa YbhcZ, \$i ']g'fYei]fYX'Zcf YUW. 'Wci fgY''' \delta \windfallo WfWa a ghUbWrgžU'ghi XYbha Uma]gg'i d'hc'&\$i 'cZU'Wci fgY'k]h\ci h\Zcfa U` Udd`]\WUhjcb'UbX'i d'hc'' 'i 'cZU'Wci fgY'k]h\'Zcfa U``XcW a YbhUhjcb'fgYbhj]U'Ya Uj`'hc'Wci fgY'Wcbj Ybcfb dfcj]X]b['U``h\Y'fYei]fYa Ybhg'XYhU]'YX'VY`ck 'UfY'a Yh'''Ghi XYbhg'bcha YYhjb['h\Y'UhhYbXUbW'fYei]fYa Ybhg'ci h]bYX'UVcj Y'k]``VY'Uk UfXYX'Ub'I bgUhjgZJ\Wcfm: U]`fI : b'Zcf'h\UhWci fgY'fY[UfX`Ygg'cZh\Y]f'dYfZcfa UbW']b'h\Y'UgqYgga YbhhUg_q'cf'ch\Yf'fYei]fYa Ybhg'Zcf'h\Y'Wci fqY''

: cfiZ fh\Yfi]bZcfa Uh]cbicbi9Xi WUh]cbiWci fgYiUhhYbXUbWifYei]fYa YbhgžidiYUgYifYZYfihcih\YifYgdYWf]j Y 98GHiA ccXiYia cXi `YfgŁ"

5 WUXYa]W = 1 Zcfa Uh]cb

: cf`YggYbh]U``ghi XYbh`]bZcfa Uh]cb`fY`Uh]b[`hc.`fYei Yghg`Zcf`Yl hYbg]cb/fYj]Yk `cZa Uf_g/cWW dUh]cbU``\YU`h\UbX`gUZ/hm'Yl Ua]bUh]cb`dfcWXi fYg/gdYW]U``Wcbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/ghi XYbhYei]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbh]U``UWUXYa]W]bZcfa Uh]cbž qYY`\hdg.#k k k "Ufhq"i bak "YXi "Ui #W ffYbhqhi XYbhq#UWUXYa]W]bZcfa Uh]cb#dfchcWc`a![i]XY`]bYq#

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

K cf_`cUX

%) \$`\ci fg`]bWi X]b['WUgg'WcbHUWi\ci fgž'fYUX]b[gž'WUgg'dfYdUfUh]cbž'UggYgga Ybhž'Zc``ck 'i d'UWhj]h]Ygž YhW

Giaa UfmcZh\Y 7ci fgY

#b 'h]g'Wti fgYž'mci 'k]````YUfb'UVci hih Y]a dcfHubW' cZfYg]`]YbW' UbX'k Y``VY]b['Ua cb['YXi WJhcfg"'K]h Zci bXUh]cbg']b 'h YcfmUbX'Ya d]f]WJ 'fYgYUfW Z'h]g'Wti fgY']g'U'gc'\][\`mdfUW]WJ'"'Mci 'k]``'UWei]fY _bck `YX[Y'cZYj]XYbW!VUgYX'gHUhY[]Yg'Zcf'a U]bHJb]b['UbX'dfca ch]b['mci f'ck b'k Y``VY]b[ž'Ug'k Y``'Ug fYgci fW'g'UbX'gHfUhY[]Yg'fY'Yj Ubh'hc gW.cc```YUXYfg'fY"["'df]bW]dU'gž'\YUX'hYUW.YfgŁ'U]a]b['hc'dfca ch'V VfcUXYf'W.Ub[Y"'Mci 'k]```YUfb'UVci hh\Y']a dcfhUbW'cZYXi WJhcf'k Y``VY]b['UbX'fYg]`]YbW'Zcf'h\f]j]b[ghi XYbhgž'hYUW.YfgŁ'UbX'gW.cc`gž'Ug'k Y```Ug']ggi Yg'gdYW]ZJWhc'W'fhJ]b 'hmdYg'cZYXi WJhcfg'fY"["ž'df]bW]dU`gž YUf'mWJfYYf'hYUW.YfgŁ"``

5i ghfu`]Ub'DfcZYgg]cbU`; fUXi UhY'HYUWX]b['GhJbXUfXg

GMbXUfX	8YgWf]dh]cb
* "%"	5bU`mgY`h\Y`5i ghfU`]Ub`DfcZYgg]cbU``GhUbXUfXg`Zcf`HYUW\Yfg`hc`d`Ub`dYfgcbU` dfcZYgg]cbU``XYjY`cda Ybh[cU`gž'gi ddcfhWc``YU[i Yg`hc`]XYbh]ZmUbX`UW\]YjY`dYfgcbU` XYjY`cda Ybh[cU`g`UbX`dfY!gYfj]WY`hYUW\Yfg`hc`]a dfcjY`WUggfcca `dfUWf)WY"
*"("(5Xj cWUHYždUfhjWjdUHY`]b`UbX``YUX`gHfUHY[]Yg`hc`gi ddcfh\][\!ei U`]hmdfcZYgg]cbU` `YUfb]b[`cddcfhi b]hjYg`Zcf`Wc``YU[i Yg`h\Uh'ZcW g`cb`]a dfcj YX`ghi XYbh`YUfb]b["

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'HJg_	K Y][\h	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX	5 +HG@'DfcZYgg]cbU`; fUXi UhY HYUWX]b['GHJbXUfXg'5ggYggYX	8iY 8UhY
K Y``VY]b['UbX fYg]`]YbWY'dYfgcbU` fYZYW¶cb'	&\$I	%	*"%"	&+#\$, #& \$% \$).\$\$ DA
K_Y``VY]b[_'UbX fYg]`]YbWY'd`Ub'	(\$1	&Ž	*"("(&&#%\$#& \$% \$).\$\$ DA</td></tr><tr><td>9ggUm</td><td>(\$1</td><td>%ž&</td><td>* "%"</td><td>\$) #%#& \$% \$) .\$\$ DA</td></tr></tbody></table>

 $\label{eq:continuous} D`YUgY`fYZYf`hc`3gefcS`fS`BcaXWee[a`S^EfS`VSoVe~XadFWSUZWcb`cb`h\Y`dfYj~]ci~g`dU[~Y].$

5ggYgga Ybh'8YHJ]`g

 $5\,ggYgga\ Ybh\%\ 'K\ Y``VY]b[\ 'UbX'fYg]`]YbWY'dYfgcbU``fYZYW¶cb$

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g. 'HUg_'%! '9ggUm!'% \$\$`k cfXgGhi XYbhg`k]```fYWY]j Y`k f]hhYb`ZYYXVUW_'k]h\]b`%\$`Vi g]bYgg`XUmg`cZgi Va]gg]cb

5XX]h]cbU`XYhU]`g.

BSdf#, 6WeUd[bf[a`,

- H\]b_'UVci hUb'YI dYf]YbW'mci '\Uj Y'\UX']b'hYUW\]b['k\YfY'mci f'k Y``VY]b[#Yg]']YbW'k Ug gi ddcfhYX'UbX'Ubch\Yf'k\\YfY'mci f'k Y``VY]b[#Yg]']YbW'k Ug'bchgi ddcfhYX''
- 8 YgWf]VY 'h\YgY 'lk c 'YI dYf]YbWrg 'fUfci bX'&\$\$ 'k cfXg 'YUW\Ł" 'H\YgY 'WUb 'VY 'cbY!cZYI dYf]YbWrg 'cf cb[c]b['cbYg"

BSdf \$, >[] e i [fZ >[fWdSfgdW]

7 cbg]XYf'h\Y'Zc``ck]b['ei Ygh]cbg'UbX'a U_Y'Ui h\Ybh]W']b_g'hc'h\Y'_bck `YX[Y'mci '\Uj Y'[U]bYX Zfca 'h\Y'Ugg][bYX'fYUX]b[g"

 $\bullet \ \, \text{Hc'k \setminus Uh'YI hYbh'\setminus Uj Y'mci f'dYfWYdh]cbg'cZh\Y'YI dYf]YbWYg'W\Ub[YX'UZhYf'h\Y'fYUX]b[g3] }$

BSdf% 5a_bSd(`Y7jbWd(W)UWe

BSdf\$z7\ccgY`cbY`UfYU`]b`k\]W\`mci f`kY``VY]b[#Yg]`]YbW\`Wci `X`VY`VYfhYf`gi ddcfhYX`fY"["žkcf_`cUXž]bhYfUW]cbg`k]h\ gh XYbhg`cf`h\Y]f`dUfYbhgŁ"`\\Ybh]ZmgdYW]ZWdfUW]W\g\#JfUhY[]Yg`mci `Wci `X`di h]b`d`UW\`hcVYfhYf`gi ddcfhmci f`kY``VY]b[`UbX`fYg]`]YbW\"`AU_Y``]b_g`hc`h\Ycf]Yg\#\cbg\fi Wg\#JfUW]W\g`kY`\Uj Y\\circ \fi\Ycf]Yg\#\cbg\fi Wg\#JfUW]\Wg`kY`\Uj Y\\circ \fi\Yof]Yg\fi \fi\Y`Zc``ck]b[`ei Ygf]cbg.

- %" K \UhiUfY'h\Y'[cU`g`cZh\Y'd`Ub3'K \m]g'h\]g']a dcffUbh'hc'UXXfYgg3
- &" K \UhWcbglfi Wfg#\Ycf]Yg`Wcj YfYX`]b`h\Y`Wci fgY`UfY`fY`Yj UbhifUfci bX`&!' /a U_Y`gi fY`mci `XYZ]bY _YmihYfa q`Vf]YZmL3`K \m3
- ' "K\UngdYWJZWUWJcbg'WJb'mci i bXYfhU_Y'fYei]fYX'hc'a YYhih\Y'[cU'g'ffY`Yj Ubh'hc'fYgci fW/gž'h]a Yž UWJj]hJYgžYhWL3
- ("5fY'h\YfY'Ubmgiddcfhg'bYYXYX'hc'\Y'd'mcigiWWyggZ\`mibXYfhU_Y'h\YgY'UWnjcbg'fZfca'mcif kcf_d`UWYžZJa]`mžZf]YbXg'YhWt3
-) " K \UhVXU``Yb[Yg'UfY'h\YfY'hc']a d`Ya Ybh]b['h\YgY'UWn]cbg3'<ck'\WUb'h\YgY'VY'UXXfYggYX3
- *" K \UhYj]XYbWY!VUgYX`ci hWca Yg`UfY`UggcW]UhYX`k]h\`h\YgY`hmdYg`cZUWf]cbg`fY"["ž´cV`gUh]gZUWf]cbž gh`XYbh`UW\]Yj Ya Ybhž`ck Yf`UVgYbhYY]ga Ł3
- +" < ck 'Wci 'X'mci 'ZYUg]V'miUbX'dfUWfWU'`miUggYgg'th\Y'YZZYWfj YbYgg'cZth\Y'd'Ub3'K \Uhk ci 'X'mci 'VY

 $\label{thm:linear} $$ Mci_k = Mci_k$

% 7 fYUHY 'Ub 'UbbcHJHYX 'V]V`]c[fUd\m

I BGK 'G7<CC@'C: '98I 75H€B'FI 6F = 7 # 998657? 'G<99H' 98GH)' %(

 $5\,ggYgga\ Ybh'HUg_.\ 'K\ Y``VY]b[\ 'UbX'fYg]`]YbWY'dYfgcbU``fYZYWI]cb$

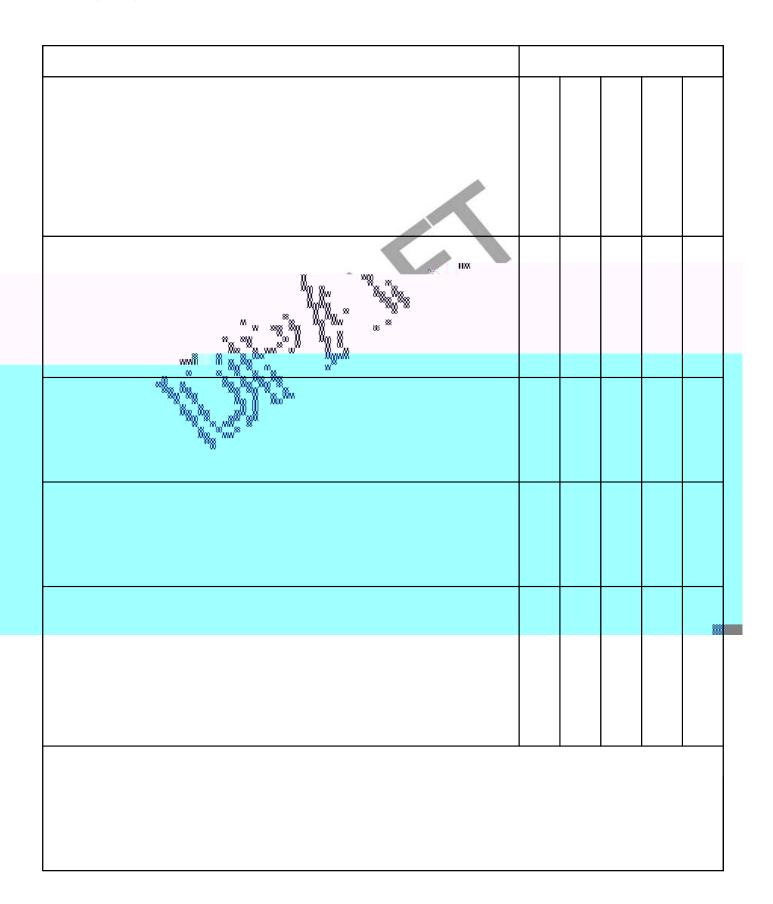
I bXYfghUbX'cZh\Y'el Yghjcb'cf']ggl Y'UbX'h\Y'_YmWcbWdfg]bj c'j YX • F YZYWjcb'cb itk c'df]cfYl dYf]YbW'g cZk Y"YY]b['UbX'fYg]]YbW' • 7 'YUF']b_g itc inci f'df]cfYl dYf]YbW'g jb DUftg %l 'cZh\Y 5ggYgga YbhHUg_ 8 Ydh\'cZUbUmg]g'UbX#cf'W]lei Y']b'fYgdcbgY'ltc'h\Y'HUg_ • Gcd\]gh\Wlh\X'UbX']biXYdh\'fYZYWjcb'cb'df]cfYl dYf]YbW'g • -XYbh]\Wligb'cb'cZgi ddcftg h\Uhk YfY' dfYgYbhcf'UW_b[']b'h\Y'hc' df]cfYl dYf]YbW'g ***********************************
 7 'YUf' ']b_g'hc'mci f'df]cf'Yl dYf]YbWrg']b'DUffg'%l' 'cZh\Y 5ggYgga YbhHUg_ 8Ydh\ 'cZUbUmg]g'UbX#cf'Wf]hei Y']b'fYgdcbgY'hc'h\Y'HUg_ Gcd\]gh]WJhYX'UbX']b!XYdh\ 'fYZYWf]cb'cb'df]cf'Yl dYf]YbWrg ¥Ybh]Z]WJh]cb'cZgi ddcffg'h\Jhk YfY'dfYgYbhicf''UW_]b[']b'h\Y'hk c df]cf'Yl dYf]YbWrg
Gcd\]gh]WhYX`UbX`]b!XYdh\`fYZYW]cb`cb`df]cf`Yl dYf]YbWfg →XYbh]ZJWJh]cb`cZgi ddcffg`h\Uhik YfY`dfYgYbhcf``UW_]b[`]b`h\Y`hk c df]cf`Yl dYf]YbWfg
→ XYbh]Z]WUh]cb cZgi ddcfhg h\ Uhik YfY dfYgYbhcf \ UW_]b[']b h\ Y hk c df]cf YI dYf]YbWg → W W W W W W W W W W W W W W W W

GdYWJZJW7 f]hYf]U	··flt2fŽt
GdYWJZJW7fJhYfJU	"flt2flt

@YWhi fYf

``B6.`h\]g`h]W_g`]b`h\Y`j Uf]ci g`Vcl Yg`UbX`XYg][bYX`hc`dfcj]XY`ZYYXVUW_`hc`gh XYbhg/h\YmUfY`bch`[]j YbYei U``k Y][\h]b`XYhYfa]b]b[`h\Y`fYWca a YbXYX`[fUXY"`8YdYbX]b[`cb`h\Y`bUh fY`cZh\Y`UggYgga Ybh'hUg_ž

I BGK 'G7<CC@'C: '98I 75H€B'FI 6F = 7 # 998657? 'G<99H' 98GH)' %(

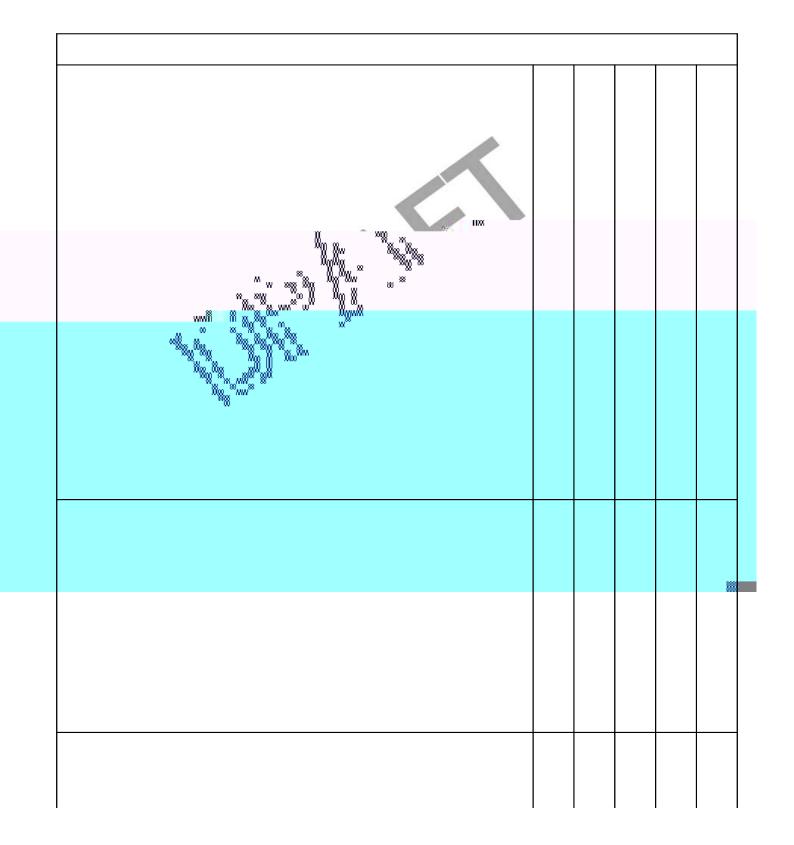


GdYW <mark>]Z</mark>]W7 f]hYf]U	"flt2flt

@YWhifYf

 $FYWca \ a \ YbXYX. \qquad ```#&\$fl @`DG`7F`8B`<8Ł` \qquad \qquad k \ Y][\ \ h]b[\ \ . ``&\i

"B6."h\]g'h\W_g']b'h\Y'j Uf]ci g'Vcl Yg'UbX'XYg][bYX'hc'dfcj]XY'ZYYXVUW_'hc'ghi XYbhg/h\YmUfY'bch[]j Yb
Yei W"k\Y][\\h\h]b\X\\h\fatab]b[bh\Y'\\tau\]a YbXYXb[f\tau\X''\8YdYbX]b['cb'h\Y'bUhi fY'cZh\Y'UggYgga YbhhUg_ž b]b['h\
'Y\Wi fYfg'a UmU'gc'\\cbh\I hi U`]nY'UbX\\chi Tua YbX'h\YgY'gdY\\J]\\Wf]l\Yf]U"'\\h\Y'fY\\can a YbXYX\[f\UXY']g
h\Ybh\Jh]j Y'cb`mžgi Va \(Y\Whic'gh\DX\UfX]gUh]cb'dfc\W'ggYg'UbX'Uddfcj U`'Vmh\Y'G\X\cc`'cZ'9Xi \WJh]cb



GdYWJZJW7 f]HYf]U	¨flŁ—		 2fŽŁ

lfb]b['UbX'HYUV\\]b	<u>['7 ca a]hhYY"</u>			

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gh XYbhgʻUfYʻYl dYWYXʻhcʻdi hh\Y]fʻbUa YgʻUbXʻgh XYbhbi a VYfgʻcbʻYj YfmdU[YʻcZh\Y]fʻUgg][ba Ybhgʻ Hi fb]h]bʻGi Va]gg]cb 7j S_ b^W`5 'ghi XYbhigi Va]hg`Ub`YggUm(, `\ci fg`UbX`%\$`a]bi hYg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY"'H\Y`YggUm]g`a Uf_YX`ci hcZ%\$\$i "'H\Y`YggUmfYWY]j Yg`U`a Uf_`cZ*, "'H\Y`ghi XYbhg`a Uf_`]g`h\YfYZcfY`*, '-'%) `1`)'

- K cf_'gi Va]hh/X'%\hc'% 'XUmg'UZh/f'h\Y'gh]di `Uh/X'XYUX`]bY'k]``'VY'UggYggYX'UbX'ZYYXVUW_dfcj]XYX'Vi hU'a Uf_'cZnYfc'k]``'VY'fYWcfXYX"'=Zh\Y'k cf_'k ci `X'\Uj Y'fYWf]j YX'U'dUgg'a Uf_'Vi hZcf h\Y'`UhYbYgg'UbX'h\Y'k cf_']g'U'Wca di `gcfm'Wci fgY'Wca dcbYbhf\i fX`Y'fYei]fYa Ybh\ZU'gh XYbhk]``VY'XYYa YX'hc'\Uj Y'a Yhh\UhfYei]fYa Ybh/
- K cf_'gi Va]httX:&\$'cf'a cfY'XUmg'UZhYf'h\Y'gh]di `UhYX'XYUX`]bY'k]``bchVY'UWWdhYX'Zcf UggYgga YbhUbX'k]``fYWY]j Y'bc'ZYYXVUW_Z'a Uf_'cf'[fUXY"'=Zh\Y'UggYgga YbhHUg_']g'U'Wca di `gcfm Wca dcbYbhcZh\Y'Wci fgY'U'gh XYbhk]``fYWY]j Y'Ub'l bgUh]gZJWcfm: U]`fl : Ł'[fUXY'Ug'U'fYgi `hcZ i bgUh]qZJWcfmdYfZcfa UbWY']b'Ub'YggYbh]U`'Wca dcbYbhcZh\Y'Wci fgY"

H\]q^]bZcfa Uhcb^]q^U`qc^Uj U]`UV`Y^Uh

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWUXYa \\W\]bZcfa Uhcb#dfchcWc`g![i \\XY`]bYg#

GdYWJU`7cbg]XYfUhJcb5dd`]WUhJcbg

 $\label{eq:linear_continuity} $$Mc_i = W_0 - W_$

G | W b Y g g ž'a | g U X j Y b h f Y 'c f 'c h \ Y f ' W f W a g h U b W g 'V Y m c b X 'm c i f 'W c b h f c ``a U m

ł DfYj Ybhmci Zfca Wca d`Yhjb['U'Wci fgY'fYei]fYa Ybhž

ł? YYd mci Zfca 'UthYbX]b['Ub 'UggYggUV'Y 'UW] j]tmž

ł Ghod moi gi Va]hh]b['UggYggUV'Y'k cf_ Zcf'U'Wci fqYž

ł G][b]Zwubhmiuzzywhrci fidyfzcfa ubwhi]biuggygguviyik cf_žvyi]hiuizcfa uniybx!czlgya yghyfiyl ua]buh]cbžu wuggihyghžuniuvcfuhcfmhyghžuigya]bufidfygybhuh]cbicfiubmich\yfizcfa iczluggygga Ybh'

F Ygci fWyg

DfYgWf]VYX F Ygci fWYg

GYY'AccX`Y'g]hY'Zcf'fYei]fYX'fYUX]b[g"

FYWca a YbXYX FYgci fW/g

 $GYY^*A\ ccX^*Y^*g]hY^*Zcf^*fYgci\ fWYg^*UbX^*Z\ ffh Yf^*fYUX]b[\ g'''$

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

: cfa U`ghi XYbhZYXVUW_`]g`cVhU]bYX`Uhh\Y`YbX`cZh\Y`Wci fgY`j]U`A m9I dYf]YbWY`UbX`h\]g`]g`i gYX`hc a U_Y`W\Ub[Yg`hc`h\Y`Wci fgY`]b`h\Y`Zc``ck]b[`mYUf"`+b`UXX]h]cbž]bZcfa U``ghi XYbhZYXVUW_`]g`Wc``YWMYX a]Xk Umh\fci [\`h\Y`Wci fgY": cf`h\]gžghi XYbhg`UfY`Ug_YX`hc`]bX]WUhY`W\Ub[Yg`h\Uhh\Ymk ci `X``]_Y`hc`gYY]a d`Ya YbhYX`]b`h\Y`Wci fgY`f][\h'Uk Umž`Ug`k Y```Ug`W\Ub[Yg`hc`VY`]a d`Ya YbhYX`h\Y`bYI hh]a Y`h\Y`Wci fgY`]g fi b"`H\YgY`Wca a Ybhg`UfY`Wc``UhYX`UbX`a U^cf`h\Ya Yg`UfY`]XYbh]Z]YX"`H\YgY`W\Ub[Yg`UfY`h\Yb`g\UfYX`k]h\ghi XYbhg`]b`WUgg`UbXžk \YfY`dcgg]V`Yž`]a d`Ya YbhYX`]a a YX]UhY`m'`: cf`YI Ua d`Yždf]cf`]bZcfa U``ZYYXVUW_