

# **ARTS1690**

H\Y'Ghfi WhifY'cZ@Ub[iU[Y

HYfa 'C bY''##'&\$%

## 7ci fgY 8YW] g

7 fYX]hDc]bhg\*\*

Gi a a UfmcZh\Y 7ci fgY

 $Gi V \Upsilon Wh 5 f Y U. >[ Yg[ef[Ue$ 

 $\label{thm:continuous} $$H\g^{\prime\prime}g^{\prime\prime}d$ 

#### 7 ci fgY @YUfb]b[ 'Ci hWca Yg

%" 5dd`mih\Y`hYW\b]ei Yg`cZd\cbYh]Wg#d\cbc`c[mihc`hfUbgWf]dh]cb`UbX`XUhU`UbU`mg]g" &" 5bU`mgY`h\Y`a cfd\c`c[]\WJ``ghfi Wfi fY`cZk cfXg`UbX`WUi gYg#gYbhYbWfg" \ " 91 d`U]b`gYa Ubh]\W\cbW\dhg`UbX`X]gh]b\W]cbg" \ (" 5bU`mgY``]b[i]gh]\WXUhU`UbX`Uf[i Y`h\Y`a Yf]hg`cZh\Y`UbU`mg]g"

#### HYUWX]b[ 'GhfUhY[]Yq'

5FHG%\*-\$`]g`cbY`cZh\Y`rkc`[UhYkUmWcifgYg`]b`@]b[i]gh]Wg"`=hdfcj]XYg`Ub`]brfcXiWf]cb`rc`[YbYfU``]b[i]gh]WgžUbX`h\Y`ZcibXUh]cb`cb`k\]W\mci~WUb`gY`YWf@YjY``&'UbX''`WcifgYg`fY`YjUbhrc'mcif']bhYfYghg"

H\Y`k\YY\_`m``YWi\fYg`UfY`X]j ]XYX`]bhc`hk\c`dUfhg"`\#\`h\Y`Z]fghdUfhž`VUg]WWcbW'dhg`UbX`h\YcfYh]WU``]ggi Yg`UfY\dfYgYbhYX`k]h\]b\h\Y`hTUX]hjcbU``Xca\U]bg`cZ``Ub[i\U[Y`ghfi\Wi\fY`]bWi\X]b[i\d\cbc`c[mž'ubX\gYa\Ubh]Wg"\#\h\Y`gYWcbX`dUfhžj\Uf]ci\g`dfcV`Ya\g`UbX`XUhU`gYhg`UfY`dfYgYbhYX`UbX`X]gWi\ggYX`]b`Ub\]bhYfUWhj\Ya\UbbYf`UbX`mci\UfY`YbWci\fU[YX`hc`Wcbff]Vi\hY`hc`h\Y`X]gWi\gg]cb"

H\Y'hi hcf]U'g'dfcj ]XY'mci 'k ]h\'h\Y'cddcfhi b]hm'Zcf'Ub']b!XYdh\'YI Ua ]bUh]cb'cZ`]b[i ]gh]WXUhU'fXfUk b'Zfca 'U k ]XY'fUb[ Y'cZ`Ub[ i U[ YgŁź'k ]h\'dUfh]w' `Uf'ZcW g'cb'WfYUh]j Y'dfcV'Ya !gc'j ]b[ 'g\_]``g']b'U'Wc``UVcfUh]j Y Ybj ]fcba Ybh'

## 5ggYgga Ybh

#### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hUg_	K Y][ \h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
7`Ugg'hYgh'=	' \$1	K YY_')	&ž(
7`Ugg'hYgh'=	' \$ı	K YY_',	% <u>*</u> (
HU_Y!\ca Y`dfcV`Ya 'gYh	(\$1	\$' #\$) #&\$% '\$(.\$\$`DA	%ž&Ž Ž(

#### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% 7`Ugg hYgh=

GhUfhXUhY. Bch5dd`]WUV`Y

5ggYgga Ybh& 7`Ugg hYgh=

GhUfhXUhY. Bch5dd`]WUV`Y

**8YHJ]`g**. `(\$`a]bi HY`HYgh`Wcj Yf]b[ `'h\Y`WcbHYbh`cZ`YWh fYg`UbX`h hcf]U`g"`Mci `k]```fYWY]j Y`]bX]j ]Xi U` ZYXVU<u>W</u>"

5ggYgga Ybh' HU\_Y!\ca Y dfcV Ya gYh

GHUfhXUhY. Bch5dd`]WUV`Y

Gi Va ]qq]cb'bchYq.H\Y'hU\_Y!\ca Y'dfcV'Ya 'qYh]q'\UbXYX'ci hcbY'k YY\_'df]cf'hc']hq'Xi Y'XUhY''

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhf]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\"Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$ 

## 5 h Yb X Ub WY F Yei ] f Ya Yb hg

 $5\,hhybXUbWh^*cZ^*Hi^*hcf]U^*g^*]g^*a^*UbXUhcfm^*]b^*hh^*]g^*Wc^*i^*fgY^*'I^*bYI^*W^*gYX^*UVgYbWh^*Z^*fca^*a^*cfY^*hh^*Ub^*&^*i^*cZ^*Hi^*hcf]U^*g^*k^*]^*^*fYgi^*h^*]b^*hh^*Y^*Uk^*UfX^*cZ^*ZU]^*^[^*fUXY^*'$ 

 $\pm 0.16 \ \text{hc} \ \text{fgY.Wc} \ \text{bhYbhz.YbUV.} \ \text{b[...inc...inc...uhtu]} \ \text{b...} \ \text{7 @C...} \ \text{5...inc.} \ \text{bh...} \ \text{5...} \ \text{5...inc.} \ \text{5...} \ \text{5...}$ 

## 7ci fgY GWX YXi `Y

#### J]Yk 'WUgg'ha YhUV'Y

### H]a YtUV`Y

@YV <b>Ki</b> fY Hihcf]U`	#blfcXi Whcb UbX A cfd\c`c[mfle #blfcXi Whcb hc h\Y gh XmcZ`Ub[i U[Y/gca Y `]b[i ]gh]Wdi nn`Yg"  8YhJ]`YX`]bghfi Whcb Zcf`YUWX k YY_fij h hcf]U` hJg_gžUWhj jhYg`UbX`YI dYWhhcb k]``		
Hi læfju`	`]b[i]gh]Wdinn`Yg" 8YhJ]`YX`]bghfiWh]cb`Zcf`YUW\`kYY_fig`hihcf]U`		
	VY'Uj U]`UV`Y'cb'A ccX`Y"'Mci 'a i gh'Wcbgi `h A ccX`Y'fY[ i `Uf`m'Zcf'i dXUhYg"		
@YWhifY	A cfd\c`c[ mif⊨Ł		
Hi hcf]U`	A cfd\c`c[ miYl YfW]gYg"		
C b`]bY'5Wf]j ]hm	A cfd\c`c[]\WJ``hmdc`c[m'		
@YWhifY	GmbHJI 'flŁ		
Hi hcf]U`	A cfd\c`c[mUbX'GmbHJl 'Yl YfW <u>l</u> gYg"		
@YWhifY	GmbHJI 'f⊨Ł'UbX'GYa UbhJVg'flŁ		
Hi hcf]U`	GmbHJI "UbX"GYa UbHJWg"YI YfWJgYg"		
C b`]bY'5Wfjj]fm	GmbHUWjWmdc`c[m"		
@YWhifY	GYa UbhjWg`flet`UbX`D\cbYhjWg`flet		
Hi hcf]U`	GYa Ubhj\\g`UbX`D\cbYhj\\g`Yl Yf\\jgYg"		
C b`]bY 5 Whjj ]hm	GYa UbhjWhmdc`c[m'`D\cbYhjWYI YfWjgYg"		
@YWhifY	D\cbYhJWg`fl=t`UbX`D\cbc`c[mfl±		
Hi hcf]U`	D\cbYhJWg`Yl YfWJgYg"		
@YWhifY	D\cbc`c[mf\text{\tin}\text{\tex{\tex		
Hi hcf]U`	D\cbYhJWg`UbX`D\cbc`c[mYlYfWJgYg"		
C b`]bY'5Whjj]hm	D\cbYhJWg`YIYfWJgYg`UbX`D\cbc`c[]WU``hmdc`c[m"		
@YWhifY	D\cbc`c[mf⊨Ł		
Hi hcf]U`	D\cbc`c[mYlYfW]gYg"		
C b`]bY 5 Whjj ]hm	D\cbc`c[mYlYfWjgYg'UbX'dfcgcXjWd\cbc`c[m'		
@YWnifY	D\cbc`c[mUbX`[fUa a Uf`]bHYfUWjcb"		
Hi hcf]U`	91 Yf]WgYg`cb`d\cbc`c[ m#( fUa a Uf`]bhYfUWf)cb"		
C b`]bY 5 Whjj ]hm	91 Yf]WYg'cb'd\cbc`c[ m#[ fUa a Uf']bhYfUWf]cb"		
@YWnifY	Gi a a UfmUbX fYj ]g]cb cZVUg]WWcbWdhg"		
Hi hcf]U`	; YbYfU``X]gW gg]cb'ei Ygh]cbg'cb'h\Y``Ub[i U[Y		
	Hi hcf]U` C b`]bY 5 Whj ]hm  @Y Whi fY  Hi hcf]U` @Y Whi fY  Hi hcf]U` C b`]bY 5 Whj ]hm  @Y Whi fY  Hi hcf]U` C b`]bY 5 Whj ]hm  @Y Whi fY  Hi hcf]U` C b`]bY 5 Whj ]hm  @Y Whi fY  Hi hcf]U` C b`]bY 5 Whj ]hm  @Y Whi fY  Hi hcf]U` C b`]bY 5 Whj ]hm  @Y Whi fY  Hi hcf]U` C b`]bY 5 Whj ]hm  @Y Whi fY  Hi hcf]U` C b`]bY 5 Whj ]hm  @Y Whi fY		

## FYgci fW/g

#### DfYgWf]VYX F Ygci fWYg

F Yei ]fYX'hYl hcc\_.: fca \_]bž'J "'Yh'U`" &\$% "'5b` +bhfcXi Wf]cb'hc'@Ub[i U[Yž'-h\'5i ghfU`]Ub#BYk 'NYU`UbX YX]h]cbž7Yb[U[Y"

K YVg]hYg. 9hhbccc[iY/I7@5dhcbYh]Wg:`UV

5 b Yl hYbg]j Y``]ghcZUXX]hJcbU``Y`YWfcb]WfYgci fWYg']g'dfcj ]XYX`]b`A ccX`Y"`Mci 'a i ghWcbgi `h'A ccX`Y fY[i`Uf`mZcfi dXUhYq"

#### FYWca a YbXYX FYgci fWyg

6cc\_g. H\Y`gh ZcZh\ci [\h`@Ub[i U[Y`Ug`U`k]bXck ]bhc`\i a Ub`bUh fY"

5 b YI hYbg]j Y``]ghcZUXX]hJcbU``fYgci fWYg`]g`dfcj ]XYX`]b`A ccX`Y'''Mci `a i ghWcbgi `h'A ccX`Y`fY[ i `Uf`m'Zcf i dXUhYg''

#### 7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

7 ci fgYgʻUfYʻdYf]cX]\W\``mfYj ]Yk YXʻUbXʻgh XYbhgʻʻZYYXVU\W\_`]gʻi gYXʻhc`]a dfcj Yʻh\Ya "

: YYXVUW\_`]g`[ Uh\YfYX`Zfca `ghi XYbhg`i g]b[ `a m9I dYf]YbWY"`±n]g`YbWci fU[ YX`ghi XYbhg`Wca d`YhY`h\Y]f gi fj Ymg`VmUVWYgg]b[ `h\Y`dYfgcbU`]gYX`k YV``]b\_`j ]U`h\Y`A ccX`Y`Wci fgY`g]hY"

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

\_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb[ 'hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZUi 'hk ]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW\_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k ]``

#### 5 WUXYa | W < cbYghmUbX D`U[ | Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU\_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hUW\_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth Yfg'Vi hidUgg]b[ 'cZZth Y 'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUlfYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY'qYfj |WYq'UbX']bZcfa UHcb'UVci hi BGK 'UbX'l BGK '@|VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

## 5 WUXYa ]W = DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb[ 'hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va ]gg]cbgʻ[ i ]XY`]bYg/ fYj ]Yk ʻcZa Uf\_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/