

ARTS2908

DfYa cXYfb >UdUb. GHJh gž GYI 'UbX Dck Yf

HYfa 'C bY''##'&\$%

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y 7ci fgY

 $\label{eq:GivYWh5fYU.'I a_W ie S`V 9 W VWdEfgV[We H\]g'Wci fgY WUb'U`gc'VY`ghi X]YX`]b'h\Y`Zc``ck]b['gdYW]U`]gUh]cbg. `3e[S` EfgV[Wel : [efackl <SbS` WeW EfgV[We$

 $H\g`Wci fgY`cZYfg`U`h\Ya UhjWcj Yfj]Yk `cZ>UdUbYgY`\]ghcfmiZca `dfY\]ghcfmiCh\Y`A Y]'J\=a dYf]U`F YghcfUhjcb`cZ% *, ž]hg`WbhfU`h\Ya Y`VY]b[`h\Y`\]ghcfmiZca `dfY\]ghcfmiCk ca Ybž'[YbXYf`UbX`gYI i U`]h]Yg"`5g`h\Y`Wci fgY]g`YggYbhJU``mU`W`hi fU``UbX`gcWJU``\]ghcfmžmci `k]```bchcb`mX]gW gg`h\Y`]XYUg`UbX`gcWJU``dfUWJ\Wg`cZ fi `]b[`WUggYg`fWJj]``Uf]ghcWfUhg`UbX`h\Yb`Vi g\]# Uff]cfgŁžVi hk]```U`gc`YI Ua]bY`dcdi `Uf`6i XX\]ga ž`Zc`_fY`][]cbž'dYUgUbhfYVY``]cbž'Wca a cbYf`fYg]ghUbW'žUbX`UgdYWg`cZdcdi `Uf`W`hi fY"`$

7 ci fgY @YUfb]b['Ci hWca Yg

%" 8 Ya cbglfUhY _bck `YX[Y cZh\Y \]ghcfmcZdfYa cXYfb '>UdUb"

- &" 5bU`mgY`dfYacXYfb`>UdUbYgY`<]qhcfmih\fci[\`h\Y``Ybg`cZ[YbXYf`UbX`ZYa]b]qhqV\\c`Ufg\]d"
- ' " 7 cbg/fi Whuf[i a Yb/g UVci hdfYa cXYfb >UdUbYgY <]g/cfm]b k f]/ffyb UbX#cfi YfVu Zcfa '
- ("→XYbhJZm_YmXYVUhYgʻUbX`WcbWYdhgʻ]bʻdfYacXYfbʻ>UdUbYgY`\]ghcfm"
-) " @cWUHY 'UbX 'UbU'mgY 'df]a UfmUbX#cf'gYWcbXUfma UHYf]U``fY`Yj Ubhhc 'dfYa cXYfb'>UdUbYgY'\]ghcfm"

HYUWX]b[GhfUhY[]Yg

H\]g`Wci fgY`]bHfcXi WYg`ghi XYbhg`hc`_YmXYVUhYg`UVci hih\Y`YUf`ma cXYfb`\]ghcfmcZ>UdUbžk]h\`Ub Ya d\Ug]g`cb`k ca Ybfg`UbX`[YbXYf`\]ghcfm' \di gYg``YWi fYg`hc`hYUW.`WcbhYbhž'UbX`hi hcf]U`g`hc`U``ck ghi XYbhg`hc`X]gW gg`h\Y]f`fYUX]b["`Hi hcf]U``UggYgga Ybhg`UfY`XYg][bYX`Vch\`hc`YbWci fU[Y`ghYUXm Yb[U[Ya YbhfdUfh]W]dUh]cb&UbX`hc`UXX`XYdh\`hc`hi hcf]U``X]gW gg]cb`f&l `fYgdcbgYgž'd`i g`X]gW gg]cb ei Ygh]cbg&"'H\Y`fYgYUfW\`dfc`YWi\Y`dg`ghi XYbhg`hc`i bXYfhU_Y`ghfi Wi fYXž]bXYdYbXYbhfYgYUfW\"

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[CihWcaYgʻ5ggYggYX
±b!WUgg`HYgh'%	&\$ı	% #\$' #&\$% `%%\$\$`5A	%ž&ž(
±b!WUgg"HYgh'&"	&\$ı	\$'#\$(#&\$%-'%%\$\$'5A	%ž&ž(
±b!WUgg"HYgh"	&\$ı	%+#\$(#&\$%-`%%\$\$`5A	%ž&ž(
7 f]h]WU`6]V`]c[fUd\m	(\$1	&, #\$(#&\$% `%%) - `DA	%ž&ž ž(ž)

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% ₺!WUgg HYgh%

GhUfhiXUhY. K YY_'(

8 YHU]`g. '7 cj Yfg'WcbHYbhZfca 'k YY_g'&! ("'H\Y'HYghk]``'Wcbg]ghcZ&\$'a i `h]d`Y!W.c]\W'ei Ygh]cbg'\VugYX'cb h\Y'fYUX]b[gž``Y\Wi fY'UbX'hi hcf]U``WcbHYbhfY`Yj Ubhhc'h\Y'k YY_g'\Y]b['UggYggYX'''\Mci f'a Uf_'k]``\VY fY`YUgYX'Uddfcl]a UhY`mcbY'k YY_'UZHYf'cb'A ccX`YžUbX'Z fh\Yf'ZYYXVU\W_'k]``\VY'[]j Yb']b'U'gi VgYei Ybh hi hcf]U``j]U'U'[YbYfU``X]g\W gg]cb''

5ggYgga Ybh& '\Digg HYgh&

GhUfhXUhY. K YY_+

8 YHU]`g. '7 cj Yfg'WcbHYbh'Zfca 'k YY_g')!+"'H\Y'HYghk]``Wcbg]gh'cZ&\$'a i `h]d`Y!W\c]WY'ei Ygh]cbg'VUgYX'cb h\Y'fYUX]b[gž``YWh fY'UbX'h hcf]U``WcbHYbh'fY`Yj Ubhhc'h\Y'k YY_g'VY]b['UggYggYX''Mci f'a Uf_'k]``VY fY`YUgYX'Uddfcl]a UhY`mcbY'k YY_'UZhYf'cb'A ccX`Yž'UbX'Z fh\Yf'ZYYXVUW_'k]``VY'[]j Yb']b'U'gi VgYei Ybh hi hcf]U`'j]U'U'[YbYfU``X]gW gg]cb''

5ggYgga Ybh' . ≠b!WUgg HYgh'

GhUfhXUhY. K YY_'-

8 YHU]`g. '7 cj Yfg'WcbhYbhZfca 'k YY_g'&! ("'H\Y'hYghk]``Wcbg]ghcZ&\$`a i `h]d`Y!W\c]W'ei Ygh]cbg'VUgYX'cb h\Y'fYUX]b[gž`YWh fY'UbX'h hcf]U``WcbhYbhfY`Yj Ubhhc'h\Y'k YY_g'VY]b['UggYggYX''Mci f'a Uf_'k]``VY fY`YUgYX'Uddfcl]a UhY`mcbY'k YY_'UZhYf'cb'A ccX`Yž'UbX'Z fh\Yf'ZYYXVUW_'k]``VY'[]j Yb']b'U'gi VgYei Ybh h hcf]U`'j]U'U'[YbYfU``X]gW gg]cb"H\]g']g'h\Y'Z|bU``UggYgga YbhZcf'UhYbXUbW''di fdcgYg"

 $5ggYgga Ybh(.7f]h]WU`6]V`]c[fUd\m]$

GhUfh'XUhY. K YY_'%

8 YHU]`g. Mci 'k]``XYj Y`cd'mci f`ck b'fYgYUfW\'ei Ygh]cb'hc'Ubgk Yfž'cb'U'hcd]WfY`UhYX'hc'cbY'cZh\Y'k YY_g]b'h\Y'Wci fgY"'Mci f`Wf]h]WU`V]V`]c[fUd\mg\ci `X`VY'ghfi Wfi fYX'i g]b['h\Y'hYa d`UhY'h\Uhk]``VY'dfcj]XYX'Zcf mci "'H\Y'hchU```Yb[h\']g'%(\$\$'k cfXg'!'mci 'g\ci `X'i gY'*'gci fWfg'UbX'k f]hY'&\$\$'k cfXg'Zcf'YUW\'gci fWfž U`cb['k]h\'U'%\$\$!k cfX']blfcXi Wfjcb'fk \YfY'mci 'k]``dfcj]XY'U'Vf]YZVUW_[fci bX'hc'h\Y'hcd]WUbX'ghUhY'mci f

fYgYUfW. 'ei YghJcbŁ'UbX'U'%\$\$!k cfX'WcbWi g]cb'fk \YfY'mci 'k]```fY]hYfUhY'h\Y'Ubgk Yf'hc'mci f'ei YghJcb'UbX dfcj]XY'U'Vf]YZgi a a UfmcZmci f'fYgYUfW\Ł"5```gci fWg'i gYX'a i ghVY'Zca 'Y]h\Yf'dYYf!fYj]Yk YX'ci fbU`g cf'W\UdhYfg'Zca 'YX]hYX'Vcc_'Wc``YWJcbg": YYXVUW_k]```VY'[]j Yb'hc'mci 'k]h\']bX]j]Xi U``Wca a Ybhg'j]U; fUXYa Uf_'cb'Hi fb]hJbž|b'UXX]hJcb'hc'U'ei U`]hUhJj Y'[fUX]b['fi Vf]W

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$

5 https://bxubww.Fyei]fyaybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$

7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YHJV`Y

8 UHY	HmdY	7 cbhYbh
K YY_'% '% ': YVfi Ufm! '&(
: Yldfi Ufm %	9	00

MY (UbX⁻ † bX⁻ b YY H (%

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUlfYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5WUXYa]W±bZcfaUhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/`UhY gi Va]gg]cbg [i]XY`]bYg/fYj]Yk 'cZa Uf_g/