

# **ARTS3375**

A cXYfb; Yfa Ub'D\]cgcd\m

HYfa 'C bY''##'&\$%-

# 7ci fgY<sup>·</sup>Cj Yfj ]Yk

## GHJZZ:7cbHJWh:8YHJ]`g`

·**7 c bjj Ylbooffo**j" c bj Ybcfg

## 5 h Yb X Ub WY F Yei ] f Ya Yb hg

5 hh/b x Ub W/ c Z `Y Whi f Y g `Ub X 'hi hc f] U`g`] g`a Ub X Uhc fmi]b 'h\]g `Wc i fg Y "`I b Y I W g Y X `UV g Y b W/ 'Z fca 'a c f Y 'h\ Ub & I c Z `Y Whi f Y g `c f 'hi hc f] U`g 'a Um f Y gi 'hi] b 'h\ Y `Uk Uf X `c Z U `Z U] `[ f U X Y "

 $\pm \text{ ``YWhi fYg'UbX'hi hcf]U'g'mci'k ]```UWhj Y'mYb[U[Y'k]h\ 'WcfY'Wci fgY'WcbhYbhžYbUV']b['mci'hc'UhhU]b'7@C'\% 7@C'&'UbX'7@C'' "$ 

## 7ci fgY GWX YXi `Y

#### J]Yk WUgg'ha YhUV'Y

### H]a YHJV`Y

Hmd Y	7 cbh/bh
@YWn fY	K Ugʻ< Y]XY[[YfʻUʻBUn]3ʻ=gʻBUn]gaʻ]a d`]YXʻVm\]g d\]`cgcd\m3 FYUX]b[ʻZcfʻh\]gʻk YY
	"Gd]Y[Y`=bhYfj]Yk k]h\ A Ufh]b < Y]XY[[Yf"žhfUbg" @gU' < Uff]Yg']b? Sd[`: W[VWYYWdS`V @Sf[a`S^ E aU[S/[e_, CgWef[a`e S`V 3`ei WežYXg"; Øbh\Yf B Yg_Y UbX 9a]`? YhhYf]b[ fB Yk Mcf DUfU[cb < ci gYž% - \$\frac{1}{2}(%d**" "H\Y GY`Z 5ggYfh]cb cZh\Y; Yfa Ub'l b]j Yfg]hm"ž hfUbg"K]`]Ua G"@Yk]g']b FZW: W[VWYYWd 5 a`fcahWekžYX"F]W\UfXK c`]b f7 Ua Vf]X[YžA 5. H\YA =HDfYggž% - '\frac{1}{2}&-!'-"
Hi kcf]U`	Hi hcf]U``X]gW gg]cb`cb`h\Y`h\Ya Y`cZh\Y``YWi fY`UbX h\Y`hYI hg"`K \Uhg\ci `X`k Y`a U_Y`cZ <y]xy[[yffig fY`Uh]cb`hc`B Un]ga 3`K \Uh`]g`h\Y`fY`Uh]cbg\]d VYhk YYb`d\]`cgcd\]WU``h\Ycf]Yg`UbX`dc`]h]WU` ]XYc`c[]Yg3</y]xy[[yffig 
@YWi fY	#orfcxi Which ho < Y]XY[[Yffig 4W] Y S V F L WUbX h\Y dfc YWhicZi bxUa YbhU cbhc c[m"  F YUX]b[."#orfcxi Which"]b 6Y]b[ 'Ubx H]a YžhfUbg" >c\b A Uwei Uff]Y 'Ubx 9xk Ufx F cV]bgcb ]b 6Ug]W K f]hjb[gžYX" 8Uj ]X: UffY``? fY``fC I ZcfX. 6'Uw_k Y``ž % *&&z'%!*("
Hi kcf]U`	Hi hcf]U``Zcf`ei Ygh]cbg`UbX`X]gW gg]cb"
	@YWifY  @YWifY

K YY_'(.''‱'A UfWX'!'%+ A UfWX	@YWhifY	< Y]XY[ [ Yf'WcblfU'8YgWUfhYg/VY]b[ !k ]h\ 'ch\Yfg'UbX h\Y'fbchcf]ci gŁ'"h\Ym""
		F YUX]b[ . '6Y]b[ 'UbX'H]a Yž'‱&!*, "
	Hi hcf]U`	E i Yghcbg'UbX'X]gW gg]cb"
	Cb`]bY`5WNjj]hm	Cb`]bY`UWfj ]maa Uf_]b[`cZUbcbma]nYX`gh XYbh YggUmg`UbX`X]gW gg]cb`cZk \Uh`]g`[ccX`cf`VUX`UVci h h\Ya"`8YhJ]`g`UbX`hjaY`hc`VY`Ubbci bWXX`]b`WUgg UbX`cb`AccX`Y"
K YY_').'% 'A UfWX'!'&( A UfWX	@YWn fY	"7 UfY 'Ug'th Y '6Y]b[ 'cZ8UgY]b""' <y]xy[ "'k="" "<="" 'cf'_bck="" 'ubx'h]a="" .'6y]b[="" [="" \uh="" ]b[="" a="" cfx']g'bchu="" ci="" f="" f'df]a="" hczwuf]b[="" td="" ufhyf'czvy`]j="" ufmfy`uf]cb'tc'th="" xcyg'th]g'a="" y'k="" yffg'fux]wu`]xyu.="" yub3'="" yux]b[="" yž'&&)!+'="" ž'vi=""></y]xy[>
	Hi lcf]U`	EiYghlobg'UbX'X]gWigg]cb'cb'h\Y'hYIh'UbX'h\Y 'YWfifYg"'8cYg'h\Y'fUX]WU`]XYU'aU_Y'gYbgY3
K YY_'*.'&) 'A UfWX'!'' % A UfWX	@YWn fY	"H\Y`HYa dcfU`]lmicZ8UgY]b.`6Y]b[!hck UfXg!XYUh\"" A U_]b[ 'gYbgY'cZUbch\Yf`fUX]WU`]XYU"
		F YUX]b[ . '6Y]b[ 'UbX'H]a Yž'&+(!' %%"
	Hi hcf]U`	K \UhXcYg' <y]xy[[yf'a ufxg!<br="" yub'vm'"vy]b[!hck="">XYUh\"'UbX'XcYg'h\Uha U_Y'gYbgY3</y]xy[[yf'a>
K YY_'+.'%'5df]`'!'+'5df]`	@YWn fY	5i h\YbhJ\Wfm'UbX`fYgc`i hYbYgg"`6Ug]\W\cb\W\dhg`cZ <y]xy[[yf]ub`yl]ghybhju`]ga "<="" td=""></y]xy[[yf]ub`yl]ghybhju`]ga>
	Hi hcf]U`	FYUX]b[.:6Y]b[:'UbX:H]a Yž:' :
K YY_', .`, '5df]`'!'%( '5df]`	@YWn fY	"HYa dcfU`]hmUbX`h\Y`cfX]bUfmWcbW\dh]cb`cZh]a Y"" < Y]XY[ [ Yffg`Wf]h]ei Y`cZWcbW\dh]cbg`cZh]a Y ]b`dfYj ]ci g`d\]`cgcd\mUbX`\]g`WcbZcbhUh]cb`k ]h\ < Y[ Y`"  F YUX]b[ . '6Y]b[ 'UbX`H]a Yž'() *!, , "
	Hi lcf]U`	E i YghcbgʻUbXʻX]gW gg]cb"ʻ=g` <y]xy[[yf`f][\huvci h<br=""><y[y`3`fĭbc`k fgy″ʻgum<br="" umrʻgumgca="" yž″czwci="">ch\Yfg"Ł</y[y`3`fĭbc`k></y]xy[[yf`f][\huvci>
K_YY_''%)	@YWnifY	K \Uh]gʻa YhUd\mg]Mgžk\Uh]gʻk fcb[ ˈk ]h\ ]hžʻUbX k\YfYʻX]Xʻ]hghUfhZfca 3
		<pre>&lt; Y]XY[[Yffgi]bU[ifU`UXXfYgg'Uh: fY]Vif['Ib]jYfg]hm ]b'% &amp;-žhkc'mYUfg'UZhYf'h\Y'diV`]WUhjcb'cZ4W['Y S`VF[_W'FiXc`Z7UfbUdfgijYfmX]ZYfYbhhU_Y'cb a YHUd\mg]Wg'UbX'\]g'Wf]h]eiY'cZ<y]xy[[yffgi'hyihug "bcbgybg]wu`""="" .<="" pre=""></y]xy[[yffgi'hyihug></pre>

	F YUX]b[.	
		< Y]XY[ [ Yf. "K \Uh]g A YHJd\mg]Vg3"ž"HJbg" 8 Uj ]X

### FYgci fW/g

#### DfYgWf]VYX F Ygci fWYg

Mci 'bYYX'hc'cVhU]b'U'WcdmcZA Uff]b'<Y]XY[[Yf.'4W[YS`VF[\_W'HfUbg`UhYX'VmA UWei Uff]Y'UbX FcV]bgcb"fK]``VY'Uj U]`UV`Y'Uhi b]j Yfg]hm'Vcc\_g\cd"'₹mci 'di fWkUgY']hÆca 'Y`gYk\YfYža U\_Y'gi fY'hc'[Yh h\Y'A UWei Uff]Y'/ 'FcV]bgcb!hfUbg`Uh]cb°Ł

Ch\Yf'hYI hg'k ]``'VY'Uj U]`UV'Y'cb'A ccX'Y"

#### FYWca a YbXYX FYgci fWyg

5a cb['h\Y'a Ubmj c`i a Yg'cb'<Y]XY[[Yf']b'h\Y'l BGK ``]VfUfmih\Y'Zc``ck ]b['UfY'YgdYW]U``mifYWca a YbXYX Zcf'Z fh\Yf'fYUX]b[.

>YUbʻ6YUi ZYYRʻ6[S'aYgWi [fZ: W[VWYYWd, 9 dWW] BZ['aeabZkž'ffUbg"'A Uf\_'G]bWU]fʻfb`cca ]b[ hcb.ˈ\days]UbU I b]j Yfg]hmDfYggž&\$\$\*t"

GWchiA "'7 Ua dVY``žFZW7Sdk: WVWYWde BZ['àeabZk aX>[XW 8SUf[U[fkł 4W]' Ył S` V >S` YgSYWfBYk 'Mcf\_. : cfX\Ua 'I b]| Yfq]\mDfYqqž&\$\&\!"

>c\b'7Udi hcž: WVWYYWdS`V3cg[Se, 3`7eeSk[AhWdUa\_[Y?WfSbZke[Ue'fBYk'Mcf\_:cfX\Uallb]jYfg]hmDfYggž%, &t"

 $> UWei Yg'8Yff]XUžA XEb[d[f,: W[VWYYWdS] V fZWCgWef[a] ž'lfUbg''; YcZfYm6Ybb]b[hcb'UbX'FUWXY] 6ck `Vmf7 \]WU[c.H\Y'I b]j Yfg]hmcZ7 \]WU[c'DfYggž'%, - Ł''$ 

5`YI UbXYf`G"`8`i ZZ`: W[VWYYWdS` V Ba`[f[Ue, FZWA` fa'aYk aXDSV[US^6 [eUa` fW`f`fBYk `Mcf\_. `7 Ua Vf]X[ Y I b]j Yfg]miDfYggž&\$%) Ł"

<Ubg!; Ycf['; UXUa Yfž: W[VWYYWde | Skež'hfUbg"'>c\b'K "'GhUb`Ymfb`VUbm'GhUhY'| b]j Yfg]micZBYk 'Mcf\_ DfYqqž'% - ( t"

: fUbÃc]g`>UfUb`UbX`7\f]ghcd\Y`DYff]bž`FZW: W[VWYYWd5a` UadVS` UWf@cbXcb.`7cbh]biia ž`&\$% Ł" 7f]gh]bU`@UZcbhž: W[VWYYWd >S` YgSYW S` V I adVŽ6 [eU'aegdWz'hfUbg"; fU\Ua '< Ufa Ub`fl7 Ua Vf]X[Y.7 Ua Vf]X[Y`I b]j Yfq]hmDfYqqz`&\$\$\$Ł"

K ]``]Ua '@Uf[ Yž': W[VWYYWde 6Y]b[ 'UbX'H]a Y'f9X]bVi f[ \ .' 9X]bVi f[ \ 'I b]j Yfg]lmiDfYggž&\$\$, Ł" ? Uf''@Òk ]l\\ž'? Sdf[` : W[VWYYWdS` V 7gdabWS` @[Z[{e\_ ž'lfUbg"'; UfmGlY]bYf'fBYk 'Mcf\_. '7 c`i a V]U I b]j Yfq]lmiDfYggž'% - ) Ł"

K YfbYf'A Ufl ž: WWYYWdS`V fZWFcSV[f[a`ž'ffUbg"'H\YcXcfY'?]g]Y``UbX'A i ffUm; fYYbY'f9j Ubghcbž'-@. Bcfh\k YghYfb'I b]j Yfg]hmDfYggž% +%"

<i [ c C Hž? Sdf) : WVWYYVd 3 Ba ff[US^>[XWf@cbXcb. < UfdYf'7 c``]baž%--' Ł"

: fUbÃc]g`F UZCi ``UbX'8Uj ]X`DYh][ fYk `fYXg"Łž': WVWYYWdS` V BcSUf[US^BZ[?aeabZk`f5`VUbm`GhJhY I b]j Yfg]hmcZBYk `Mcf\_DfYggž&\$\$&L"

H\Y`Zc``ck]b[`fYgdYWYX`UWUXYa]W'cifbU`g`fY[i`Uf`mZYUhifY`dYYf!fYj]Yk YX`Ufh]WYg`cb`: W\VWYYWd BZ['aeabZk FaVSkžDWeWSdJZ [`BZW`a\_W`a'aYkž5a`f[`W`fS^BZ['aeabZk DWh[W`ž<agd`S^aXfZW4d[f[eZ EaU[Wfk XadBZW`a\_W`a'aYkž;`fWd`Sf[a`S^<agd`S^aXBZ['aeabZk]CS^EfgV[Wež7baUZW]3 <agd`S^XadfZW : [efack aXBZ['aeabZkžBZ['aeabZk S`V EaU[S^5d[f[U[e\_ž'9cbVgSfW8SUg\*fk BZ['aeabZk <agd`S^UbX DWh[W aX? WfSbZke[Ue"

 $\label{thm:condition} $$HY'XUHUVUgY'FZWBZ[^aeabZWde;^VW]']g'Ub'YIW'`Ybh'fYgcifW'Zcf``cWUh]b['Ufh]WYg'Ug'kY```Ug'Vcc\_g'cb'UgdYW]ZWhcd]WUbX'Uih\cf''$ 

#### 7 ci fgY '9j U'i Uhjcb 'UbX '8 Yj Y'cda Ybh

 $Ghi \ XYbhg`\ Uj Y'h Y'cddcfhi b]hmhc'dfcj \ ]XY'ZYXVUW_cb'h Y'Wci fgY'Zcfa \ U`mUhh Y'YbX'cZhYfa'j \ ]U'h YA m9I dYf]YbWY'gi fj YmUbX']bZcfa U`mj \ ]U'ZJWY!hc!ZJWY'Wca a i b]WUhjcb'cf'Y!a U]``k ]h 'h Y'Wci fgY Wcbj Ybcfg''' 5 g'h ]g']g'h Y'Zfghiha Y'h Uhk Y'\Uj Y'hUi [\hU'Wci fgY'hc[Yh\Yf'cb'<Y]XY[[Yfzk Y'UfY YgdYW]U`mcdYb'hc'gi [[Ygh]cbg'UbX'Wcbghfi Whj Y'Wf]h]W]ga "''H\]g'g\ci `X'VY'U'Wci fgY'UZhYf'k \]W 'ghi XYbhg \VY']Yj Y'h Ym\Uj Y'U'VYHHYf'i bXYfghUbX]b['cZcbY'cZh\Y'a U'cf'Z][i fYg'cZ; Yfa Ub'd\]`cgcd\m'''\=Zh\YfY'UfY fcUXV'cW_gz'hY``'i g'k \Uhh\YmUfY'''\=Zk Y'UfY'cb'h\Y'f][\hiffUW_z'`Yhi g'_bck 'gc'h\Uh'Z hi fY'ghi XYbhg`\Uj Yh\YvYbYZh'$ 

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

Image: Ybwb: bh/f'u'dfcv'ya k \yb'uhh/a dh/b[ 'hc'gi va ]hmci f'ugg][ ba Ybhh\fci [ \ 'Hi fb]h/bž'd'YugY

HY'Yd\cbY'9| h/fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U] 'h\Ya 'cb'Yl h/fbu'h/hgi ddcfh4 i bgk "YXi "Ui "'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUfŁ" Image: UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h/b'mci 'a UmUdd'mZcf'Ub'Yl h/bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h/w\_Yhbi a VYf'Zfca '9| h/fbu''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'YY] UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Yl h/bg]cb'Udd']WJh/cb" Image: Ya U]''9| h/fbu''Gi ddcfhmci 'k ]'`
Ui hca Uh/WJ'mfYW'] Y'U'h/w\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]'`'bYYX'hc'gdYW/ZWJ'`mUg\_'Zcf'cbY''
Hi fb]h/b'U'gc'dfcj ]XYg'i dXUh/g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk ]hYf"

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mj ]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\ U`hYfbUhj Y'gi Va ]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va ]hUgq][ ba Ybha ccXiY

#### 5 WUXYa | W < cbYghmUbX D`U[ | Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU\_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW\_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth Yfg'Vi hidUgg]b[ 'cZZth Y 'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wcaaib]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@G9'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.#\gi \YY\\f\i \\XY\g''\\f\f\f\i \bgk\\\YX\i \"\J\\Y\\\f\\\]aY

#### 5 WUXYa ]W = b Zcfa Uh cb.

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb[ hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va ]gg]cbg '[ i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK '< YU`h\ 'UbX'GUZYhmdc`]\\\Yg/
- YI Ua ]bUh]cb dfcWYXi fYg/
- gdYWJU``Wcbg]XYfUhjcb`]b`h\Y`Yj YbhcZ]``bYgg'cf'a ]gUXj Ybhi fY/
- gh XYbhYei ]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWUXYa ]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

.

#### =a U[ Y 7 fYX]h

A Ufh]b < Y]XY[ [ Yfž % &( "5 fVX)j A Ygg\_]fVX "

#### 7F = 7CG

7F = 7 C G 'Dfcj ]XYf'7cXY. '\$\$\$-,;