

# **IEST5001**

: fUa Yk cf\_g'Zcf'9bj ]fcba YbhU``A UbU[ Ya Ybh

## 7ci fgY<sup>·</sup>Cj Yfj ]Yk

### `GHJZZ`7cbHJWh`8YHJ]`g`

#### . 7 cbj Ybcfg<sup>.</sup>

BUa Y	9a U]`	5j U]`UV]`]m	@cWUh]cb	D∖cbY
5#DfcZHYa U A ]`ghY]b		Uddc]bha Ybhij ]U	A cfj Yb 6fck b 6i ]X]b[" A 6' *(	

### °@Y₩nifYfg`

B Ua Y	9a U]`	5j U]`UV]`]m	@cWUh]cb	D∖cbY
8f`>c∖b`7Uff	-	Uddc]bha Ybhij ]U	A cfj Yb 6fck b 6i ]X]b[" A 6' *'	

GWXcc`7cbHJWhf=bZcfaUhjcb

### 7cifgY<sup>·</sup>8YHJ]`g

### 7 fYX]hDc]bhg<sup>·</sup>\*<sup>·</sup>

### Gi a a UfmcZh\Y'7ci fgY

H\]gʻWci fgYʻdfcj [XYgʻUbʻ]bHfcXi Wijcbʻtcʻh\YʻA UghYfʻcZ9bj ]fcba YbHUʻA UbU[ Ya Ybhdfc[ fUa "'Mci ʻk ]```[ U]b Ub'UddfYVijUijcbʻcZh\YʻWca dʻYl 'UbXʻHfUbg!X]gWjd`]bUfmbUhi fYʻcZYbj ]fcba YbHUʻ`a UbU[ Ya Ybhdfc[ fUa "'Mci ʻk ]```[ U]b WcbWfdh Uʻ`UbX`dfUWijWUʻZfUa Yk cf\_gʻUbX`[ `cVUʻgHfUhY[ ]YgʻZcfʻUXUdhj YʻYbj ]fcba YbHUʻa UbU[ Ya Ybh' GHUft]b[ 'Zfca ʻh\YʻdfYa ]gYʻcZʻ´´gi gHU]bUVʻYʻXYj Yʻcda Ybh[ cUʻgʻ´´UQ'UʻW ffYbhVfcUXmiYbXcfgYXʻZfUa Yk cf\_ ZcfʻYbj ]fcba YbHUʻa UbU[ Ya Ybhźmci fʻYl d`cfUh]cbʻk ]```]bWi XYʻh\YʻXYj Yʻcda YbhcZh\YʻWcbWfdhgʻcZ gi gHU]bUVʻYʻXYj Yʻcda YbhUbXʻgi gHU]bUVʻYʻXYj UbhdYfgdYWij Ygʻ]bWi X]b[ 'ZcfʻYl Ua dʻYa Ybhb[ gi gHU]bUVʻYʻXYj Yʻcda YbhUbXʻgi gHU]bUV`]`]mUbXʻdfcVʻYa gʻ]b'dfUWijWUʻ`m]bH'fdfYhb[ 'UbX`]a dʻYa Ybhb[ gi gHU]bUVʻYʻXYj Yʻcda YbhUbXʻgi gHU]bUV]`]mUbXʻdfcVʻYa gʻ]b'dfUWijWUʻ`m]bH'fdfYhb[ 'UbX`]a dʻYa Ybhb[ gi gHU]bUV`]`]mi`Mci `k ]``YbVci bH'f`UʻfUb[ YʻcZff`Yj UbhdYfgdYWij Ygʻ]bWi X]b[ 'Zcf`Yl Ua dʻYʻ+bX][ Ybci g k cfʻX'j ]Yk gžd\]`cgcd\mUbX`Yh,]VjbʻzgcVjUʻ`A ghJWfžd`Ubb]b[ žYVcbca ]Wjz`\YUʻh,`gWjYbWfgžYV¢`c[ mža YX]Už UbX`W``hi fU``UddfcUWXYg''Mci `k ]``Yl d`cfY`h\Y`df]bVjd`YgʻcZgi gHJ]bUV`Y`XYj Y`cda Ybhi]b'hY]f`Udd`]WUhjcbž ]bWi X]b[ '[ cj YfbUbWi`fYgdcbgYgʻtc`h\Y`´´gi gHU]bUV`Y`XYj Y`cda Ybh[ cU`gʻ´´`UhX]ZYfYbh`Yj Y`gʻcZ [ cj Yfba YbhžVmbcb`[ cj Yfba Ybhcf[ Ub]nUhjcbgžVmV¢fdcfUh]cbgžUbX`VmdfcZygg]cbU`cf[ Ub]nUhjcbg''Mci k ]``YbVci bhYf`Wf]hei YgʻcZgi gHU]bUV]`]mUgʻUʻzfUa Yk cf\_ 'Zcf`Ybj ]fcba YbhU`a UbU[ Ya YbhžUbX`YI d`cfY U'hYfbUhj] Y`a cXY`gžUbX'dcgg]VY`gʻci hjcbg''

### 7cifgY`@YUfb]b[`CihWcaYg

- %" 8]gW ggʻh\Y`W\Ub[]b[`gHUHY`cZh\Y`Ybj]fcba YbhUbX`dYfgdYWjj Ygʻcb`W'ffYbh\_YmYbj]fcba YbHU` ]ggi YgʻUgʻ]bZi YbhJU`ZUWcfgʻ]b`ZfUa Yk cf\_gʻ[cjYfb]b[`Ybj]fcba YbHU`a UbU[ Ya Ybh
- &" =XYbhJZmhXY`\_YmghU\_Y\c`XYfg`UbX`fY`Yj UbhX]gWJd`]bYg`Zcf`UXXfYgg]b[ `gdYWJZJWYbj ]fcba YbhU` a UbU[ Ya Ybh]ggi Yg`UbX`YI d`U]b`hXY]f`fY`UhJj Y`fc`Yg`UbX`a cXYg`cZdUfhJWJdUhJcb
- ' "HU\_Y`U`\c`]ghJWž]bhYfX]gWJd`]bUfmUddfcUWX`hc`UbU`mg]b[`Ybj]fcbaYbhU``aUbU[YaYbh]ggiYg
- ("91 d`U]b`UbX`Wf]h]ei Y`h\Y`j Ufm]b[`WcbWfdhg`cZgi ghU]bUV]`]hmUbX`gi ghU]bUV`Y`XYj Y`cda Ybhz`[]j ]b[ Xi Y`fYWc[b]h]cb`hc`h\Y]f`a Ubm]bh/fdfYhUh]cbgz`UbX`U`h/fbUh]j Y`WcbWfdhg`UbX`ZfUa Yk cf\_g
- ) " =XYbhJZmVUff[Yfg`hc`gi ghU]bUV]`]hmXYf]j ]b[ `Zfca `U`VfcUX`fUb[ Y`cZd\]`cgcd\]WU``UbX`Yh\]WU`ž UXa ]b]ghfUh]j Yž`Y[ U`ž'dc`]hJWU`žgcW]U``UbX`YWcbca ]WZJWfcfg
- \*" 5dd`mh`Y`df]bW]d`Yg`cZgi`gHJ]bUV]`]mUbX`X]gW gg`\ck [ cj Yfba YbhgžVi g]bYgg`UbX`Wca a i b]m cf[ Ub]gUh]cbg`WUb`]a d`Ya Ybhgi gHJ]bUV`Y`XYj Y`cda YbhdfcWYggYg`]b[]j Yb`WUgY`gh`Xmg]h Uh]cbg

### HYUWX]b[ GhfUhY[]Yg

H\fci [\`f\]g'Wci fgY'mci `k]```[U]b'Ub'i bXYfgHUbX]b['cZh\Y'ZUa Yk cf\_g'k]h]b'k \]W.'UXUdHj Y Ybj ]fcba YbHU`'a UbU[ Ya YbhHU\_Yg'd'UW'''K Y'k]```gHUfhVmYa d\Ug]g]b['UbX`YI d`cf]b['h\Y`Wca d`YI ]fmUbX bUh fY'cZYbj ]fcba YbHU`'a UbU[ Ya Ybh]ggi Yg'UbX'h\Y']b\YfYbhWU`Yb[ Yg']b']bHYfX]gW]d`]bUfm[ fci d UddfcUWXYg'hc Ybj ]fcba YbHU`'a UbU[ Ya YbhUbX'gi gHU]bUV]`]m''Mci f'[ fci d`Vf]b[ g'U'k]XY'j Uf]YfmcZ X]gW]d`]bUfmVUW\_[ fci bXg'UbX'dfcZyg]cbU``UbX``]ZY YI dYf]YbWYg'hc 'h\YgY YbXYUj ci fgž'Ug'k Y```Ug'U'k]XY gdfYUX'cZ[ Yc[ fUd\]WVUW\_[ fci bXg'UbX'YI dYf]YbW'Vch\'Zfca 'k]h]b'5i gffU`]U`UbX`]bHYfbUh]cbU``m'`: cf'cb! WJa di g'gh XYbhg`X]gW gg]cbg'k]```VY`]bHY[ fUHYX`]bhc'h\Y' !\ci f'WUgg'h]a Y'YUW 'k YY\_ž'k \]Y'Zcf'X]gHUbW' gh XYbhg`X]gW gg]cbg'k]```HJ\_Y'd'UW'cb`]bY''Hc'dfcj ]XY'VUW\_[ fci bX`Zcf'h\]g'Wci fgY'k Y'k]```Vf]YZmfYj ]Yk W ffYbh\_YmYbj ]fcba YbHU``]ggi Yg'cZWcbWfb']bWi X]b['W]a UhY'WLUb[ Yž'U]f'UbX'k UhYf'dc``i h]cbž`cgg'cZ V]cX]j Yfg]mUbX'YWc'c[]WU``]bHY[ f]mz'Ug'ZUWcfg']bZi YbV]b['H\Y'ZUa Yk cf\_g'Zcf'Ybj ]fcba YbHU` a UbU[ Ya Ybhz'UbX'Wcbg]XYf'HfYbXg']b'h\YgY''K Y'k]``YI d`cfY'h\Y'WUi gUh] Y'UWjj [h]Yg'UggcW]UhYX'k ]h`h\YgY ]ggi Yg'UbX'h\Y'fYgi `h]b['Ybj ]fcba YbHU`a UbU[ Ya YbhV\U`Yb[ Yg''H\Y'a U]b'Ya d\Ug]g']b`: fUa Yk cf\_g'Zcf 9bj ]fcba YbHU`A UbU[ Ya Ybh]g'cb'YI d`cf]b['WcbWfdh'U`Yb[Yg'''H\Y'a U]bYa d\Ug]g']b`: fUa Yk cf\_g'Zcf a UbU[Ya YbH'5`\_Yma YggU[Y`]g`h\UhH\YgY`ZfUa Yk cf\_g`UfY`bchghUh]WVi hWcbh]bi U``mYj c`j ]b[žk]h\'h\Y dUWY`cZW\Ub[Y`dUfh]W``Uf`mfUd]X`UhdfYgYbH'

### 5ggYgga Ybh

### : i ```XYHU]`g`Uj U]`UV`Y`j ]U`mci f`A ccX`Y`g]HY

### 5ggYgga YbhHUg\_g

5ggYgga YbhHJg_	K Y][ \h	8iY`8UhY	GhiXYbhi@YUfb]b[ CihWcaYgʻ5ggYggYX
5bbcHJHYX~6]V~]c[fUd\m	' *	8 UHY 1465	%ž&ž ž(ž)
; fcid`DfYgYbHJh]cbg`	%&I	€ mcif"Ugg][bYX'kYY_	%ž&ž ž( ž*
: ]Y`X`Hf]d`9I \WfW]gY`	%&I	Gi Va ]ha ]X HYfa " 8 UHY H65	%ž&ž ž( ž*
F YgYUfVX '9ggUmi	(\$1	5hih\Y`YbX`cZHYfa "`8UhY H65	%ž&ž ž( ž) ž*

### 5ggYgga Ybh8YHJ]`g

#### 5ggYgga Ybh% 5bbcHJHX 6]V ]c[fUd\m

 $\label{eq:constraint} GhufhXUhY.~~H\Y~V]V~]c[~fUd\mXYj~Y~cdg~UWfcgg~h\Y~Wci~fgY$ 

**8YHJ]`g**. '6f]YZgi a a Uf]Yg'Zcf'\_YmdUdYfgžVcc\_'W\UdhYfg'UbX'ch\Yf'fYZYfYbWyg''5ddfcl '%) \$\$`k cfXg'' K f]hYb'ZYYXVUW\_'dfcj ]XYX'UWVcfX]b[ 'hc'U'fi Vf]W

#### 5XX]h]cbU``XYHJ]`g.

: i ```XYHU]`g`Uj U]`UV`Y`j ]U`mci f`A ccX`Y`g]HY

5ggYgga Ybh&; fci d DfYgYbhJh]cbg

GHJfhXUhY. 'Mci 'k ]``'VY'Ugg][ bYX'U'k YY\_']b'k \]W. 'nc'dfYgYbh'

8YHJ]`g. '5 'WJgY'ghi XmidfYgYbHJh]cb']b'WJgg'cf'Ug'Ub'cb!`]bY'Zcfi a 'dcgh''K f]HYb'ZYYXVUW\_'dfcj ]XYX"

5XX]h]cbU``XYHJ]`g.

: i ```XYHU]`g`Uj U]`UV`Y`j ]U`mci f`A ccX`Y`g]HY

5ggYgga Ybh' .: ]Y`X Hf]d 9I WffW]gY

 $\label{eq:charged_constraint} Ghufh`XUhY.``H \ Y`Z]Y`X`hf]d`]g`YUf`m`]b`h \ Y`HYfa$ 

8YHJ]`g. `Ghi XYbhg`k]```VY`fYei]fYX`hc`gi Va]hU`kf]hYb`fYdcfhiVUgYX`cb`U`Z]Y`X`hf]d"

5XX]h]cbU``XYHJ]`g.

: i ```XYHU]`g`Uj U]`UV`Y`j ]U`mci f`A ccX`Y`g]HY

5ggYgga Ybh(. FYgYUfW 9ggUm

#### GHUfh'XUHY.

**8YhU]`g**. DUdYf`]b`k \]W\`mci `Udd`mU`g]| `ghYd`gi ghU]bUV`Y`XYj Y`cda YbhUbU`mg]g"`: ]bU``Ugg][ ba YbhZcf`h\Y Wci fgY"`UddfcI '' ) \$\$`k cfXg"`K f]hYb`ZYYXVUW\_`dfcj ]XYX`UWVcfX]b[ `hc`U`fi Vf]W

#### 5XX]h]cbU``XYhJ]`g.

: i ```XYHU]`g`Uj U]`UV`Y`j ]U`mci f`A ccX`Y`g]HY

### 5WUXYa ]W<cbYghmUbX^D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[`f\Y`k cfXg`cf`]XYUg`cZcf\Yfg`UbX`dfYgYbh]b[`f\Ya `Ug`mci f`ck b"`=fiWUb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUHY`W\YUh]b[`fc`UWV]XYbhU``mWcdm]b[`Zfca `U`gci fWFk]h\ci hUW\_bck `YX[Ya Ybh'

I BGK '[fci dg'd`U[]Uf]ga ']bhc'h\Y'Zc``ck ]b[ 'WUhY[cf]Yg.

7 cdm]b[. `i g]b[ `h\Y`gUa Y`cf`j Yfmig]a ]`Uf`k cfXg`hc`h\Y`cf][ ]bU``hYI h`cf`]XYU`k ]h\ci h`UW\_bck`YX[ ]b[ `h\Y

### 5WUXYa]W=bZcfaUh]cb<sup>.</sup>

: cf<sup>·</sup>YggYbhJU<sup>··</sup>gh XYbhJbZcfa UhJcb<sup>·</sup>fY<sup>·</sup>UhJb[<sup>··</sup>hc.

• fYei YghgʻZcf`Yl hYbg]cb/