

# **MODL5108**

HfUbg`Uhcb'7Yfh]ZWUhcb'DfYdUfUhcb

HYfa 'Hk c''##'&\$%

## 7ci fgY<sup>\*</sup>Cj Yfj ]Yk

## GHUZZ:7cbHUWh8YHJ]`g`

## '7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUŊcb D\cbY
: Y`]W]/miA i Y``Yf	Za i Y``Yf4 i bgk "YXi "Ui	j ]UʻYa U]`	A cfj Yb 6fck b
		Uddc]bha Ybh	&* (

#### 'Hi hcfg'

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
5`]gU'H]Ub	U`]gU"hJUb4 i bgk "YXi "Ui			
@cb[ '@]	`cb[ "`]4 i bgk "YXi "Ui			
5`UbU'G]a dgcb	U`UbU"g]a dgcb4 i bgk "YXi "Ui			
A UgU_c C [ Uk U	a "c[UkU4 i bgk "YXi "Ui			
9`YbU'A ]_\U]`]_	Y"a ]_\U]`]_4 i bgk "YXi "Ui			
Gi nUb'D]dYf	g"d]dYf4 i bgk "YXi "Ui			
5XY`U'9nW ffU	U"YnW ffU4 i bgk "YXi "Ui			

## GWkcc`7cbhUWh ≠bZcfa Uh]cb

GWkcc``cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

### 7ci fgY 8YHJ] g

7 fYX]hDc]bhg \*

#### Gi a a UfmcZh\Y 7ci fgY

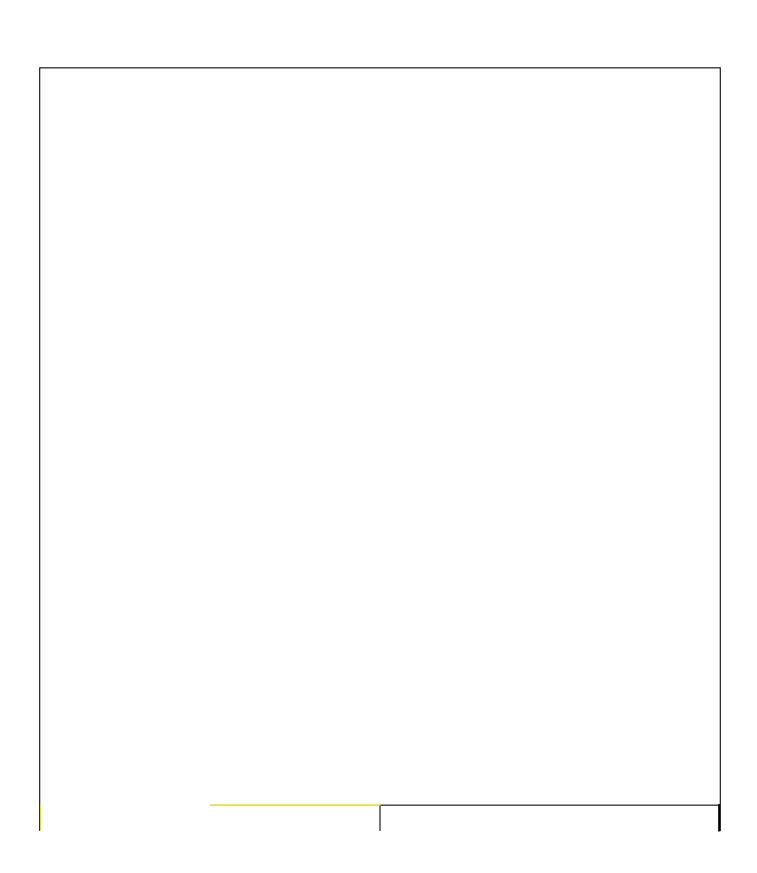
 $H\g"WcifgY'k] \label{eq:hybourded} H\g"wcifgY'k] \label{eq:hybourded}$ 

#### 7ci fgY @YUfb]b[ 'Ci hWca Yg

- %" XYa cbglfUhY`dfcZYgg]cbU``lfUbg`Uh]cb`Wca dYhYbV]Yg`]b`UfYUg`fYei ]fYX`Vmih\Y`BUh]cbU` 5\WhYX]liUh]cb`5i h\cf]lmiZcf`HfUbg`Uhcfg`UbX`\#\YfdfYhYfg`fB55H\\*\DfcZYgg]cbU``HfUbg`Uhcf YI Ua ]bUhlcb
- &" WilhWU``mUbU`mgY' IfUbg`Uhlcbg'cZh\Y]f'ck b'UbX'dYYfg'VUgYX'cb'h\Ycfm'UbX'fYgYUfW\
- " a U\_Y ]bZcfa YX 'ffUbg`Uhcb 'Wkc]WYg 'h\UhUfY 'Uddfcdf]UhY ]b [ ]j Yb 'WcbhYl hg

#### HYUWX]b[ GlfUhY[]Yg

$$\label{thm:converse} \begin{split} &\text{H}^{\text{T}}\text{fUhjcbUY}^{\text{T}}\text{VY}^{\text{D}}\text{X}^{\text{T}}\text{H}^{\text{T}}\text{UW}^{\text{D}}\text{b}^{\text{C}}\text{UW}^{\text{D}}\text{U}^{\text{D}}\text{C}}\text{UW}^{\text{D}}\text{J}^{\text{D}}\text{J}^{\text{D}}\text{C}}\text{J}^{\text{D}}\text{J}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{D}}\text{L}^{\text{$$



## FYgci fW/g

#### DfYgWf]VYX F Ygci fWYg

D`YUgY`fYUX`h\Y`a UhYf]U``cb`h\Y`7Yfh]Z]YX`HfUbg`Uhcf`hYghcb`h\Y`B55H=kYVg]hY`UhibUUh]"Wca "Ui 'j Yfm WUfYZ ``m''`=h'WcbhU]bg`]bZcfa Uh]cb`UVci hih\Y`WYfh]Z]WUh]cb`hYghz`h\Y`fYj ]g]cb'hYghg'UbX`\Ug'dfUWf)WY'a UhYf]U`"

#### FYWca a YbXYX FYgci fWYg

BchUj U]`UV`Y

#### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhJb[ 'hc'gi Va ]hmci f'Ugg][ ba Ybht\fci [ \ 'Hi fb]hJbž'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U] 'h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'\\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUfŁ" Image:

UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]hJb'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'hJW\_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUh]cb" Image: Ya U]''9! hYfbU''Gi ddcfhmci 'k ]''

Ui hca UhJWJ''mfYW'] Y'U'hJW\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]'''bYYX'hc'gdYWJZWJ''mUg\_'Zcf'cbY"

Hi fb]hJb'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk ]hYf"

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mj ]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\ U`hYfbUhj Y'gi Va ]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va ]hUgq][ ba Ybha ccXiY

#### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'=hWJb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhU`mWcdm]b[ 'Zfca `U`gci fWY'k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a ]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k ]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb.'k cf\_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b['mci f'k cf\_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa ]WhUg\_ž'ghYU']b['cf'UWei]f]b[ Ubch\Yf'dYfgcb'g'UWUXYa ]Wk cf\_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf\_'cf'gYY\_]b[ dUma YbhZcf'Wca d'Yh]b['UWUXYa ]Wk cf\_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fWYg'k\]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW\_bck`YX[]b[`h\Y`″gYWcbXUfm″gci fWY`Zfca 'k\]WX`\_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł gi Va]\h]b[`mci f`ck b`k cf\_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb[ 'dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b[ 'UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WcbWYdhg"

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY