

ARTS2455

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7ci fgY'Cj Yfj]Yk

'GhJZZ'7cbhJWh'8YhJ]`g'

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
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"@YWhi fYfg"

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
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H\]g'Wci fgY'Yl d`cfYg'h\Y'XfUa UhjWW\Ub[Yg'hc'[YbXYf'UbX'gYl i U`]mbcfa g'Yl dYf]YbWYX'VmdYcd'Y'`]j]b[
]b'7\]bU'cf'cdYfUhjb[']b'h\Y'G]bcd\cbY'k cf'X"'H\Y'Wci fgY'Yl Ua]bYg'h\Y'HfUbgZcfa Uhjcb']b'h\YgY'fYU'a g
Zca ']a dYf]U'`ha Yg'h\fci [\'hc'h\Y'W' ffYbhYfU"'+jg\ck g'h\Y'cb[c]b['hYbg]cb'VYlk YYb'dcdi 'Uf']XYUg'UVci h
fffUX]hjcbfiUbX'fa cXYfb]mfiUg'[YbXYf'UbX'gYl i U`]mbcfa g'UbX'dfUWfjW'g'\Uj Y'V\Ub[YX'cj Yf'ha Y"'+h
Yl Ua]bYg']bhYfj Ybhjcbg'Vm7\]bUfig'j Uf]ci g'[cj Yfba Ybhg'UbX'Y[U'`gmghYa g'UbX'WJa dU][bg'VmUWfj]]ghg
UbX'fYZcfa Yfg'hc'W\Ub[Y'h\Y'ghUhi g'ei c'Ug'k Y``'Ug'h\Y'i gY'cZgYl 'UbX'[YbXYf'VmWfYUhj] Y'Ufhjghgžk f]hYfg
UbX'UXj YfhjgYfg"'Mci 'k]```YUfb'UVci hd\Ybca YbU'gi W\'Ug. dfc[fYgg'hck UfXg'gYl 'Yei U']lmž'hYbg]cbg
VYlk YYb']bX]j]Xi U'`XYg]fYg'UbX'ZJa]mdfYggi fYž'@6; H=W\'h fY'UbX'f][\hgž'gcW]U']ghW\U``Yb[Yg'hc
ffUX]hjcbž'7\]bYgY'a UgW']b]hjYg'UbX'ZYa]b]b]hjYg']b'`cWU'`UbX'[`cVU'`WcbhYl hgž'X]j Yfg]lm]b'[YbXYf'UbX
gYl i U']mbcfa g'Ua cb['7\]bUfig'bcb! < Ub'Yh\b]MjhYg'UbX'X]ZYfYbhgcWJU``WUggYgž'W' ffYbhW\U'`Yb[Yg']b
[YbXYf'UbX'gYl i U`]mbcfa g']b'fY`Uhjcb'hc'd\Ybca Ybcb'`]_Y'ZJa]mZcfa UhjcbžYa d`cma Ybhž'dc`]hjWU`
dUfhjWjdUhjcbžgdcffjb['UbX'W'`h fU`Yl dfYgg]cb"

H\Y`Wci fgY`XfUk g`i dcb`U`X]j YfgY`fUb[Y`cZhYl hg`!`Ui X]c!j]gi U`ž`]bhYfbYhž`Z]`a ž``]hYfUhi fYž`cZZ]WJU` XcW a Ybhgž`bYk gdUdYfg`UbX`a U[Un]bYgž'dcghYfgž'UfhUbX`dYfZcfa UbW\"`\#i]g`[fci bXYX`]b`W`ffYbhh\Ycf]Yg cZ[YbXYf`UbX`gYl i U`]hmcdYfUh]b[`]bhYfbUh]cbU``mUbX`]b`7\]bU"`DfcZ]WJYbWm]b`7\]bYgY``Ub[i U[Y`]g`bch fYei]fYX"

7ci fgY @YUfb]b['Ci hWca Yg

- %" Uff]W `UhY`U`gdYW]Z]WVcXmcZ_bck `YX[Y`UVci hWcfY`UgdYWfg`cZ7\]bYgY`gcV]YhmUg`dYfhU]b]b[`hc Yj c`j]b[`[YbXYf`UbX`gYI i U`]hmbcfa g
- &" YI d`U]b`h\Y`j Uf]YX`bUh fY`cZG]bcd\cbY`YI dfYgg]cbg`cZ[YbXYf`UbX`gYI i U`]hm'U\fcgg`h]a Yž VYhk YYb``c\Wh]cbažYh\b]\WhYa`UbX`gi V!\W`hi fYa
- ' " Uff]W `UhY`_bck `YX[Y`UVci hh\Y`X]ZZYfYbhk Umg`[YbXYf`UbX`gYl i U`]hm\Uj Y`VYYb`a cV]`]nYX`]b X]ZZYfYbh[YbfYg`cZdi V`]WmW]fW `Uh]b[`hYl hg
- ("Yl Ua]bY`UbX`Wca dfY\YbX`h\Y`WcfY`X]ZZYfYbWYg`VYłk YYb`7\]bU'g`W\Ub[]b[`[YbXYf`UbX`gYl i U`]lm bcfa g`]b`h\Y`]bHYfbUh]cbU``UbX`Wfcgg!W`hi fU``WcbHYl h
-) "XYa cbghfUhY`Ub`i bXYfghUbX]b[`cZ_Ymh\Ycf]Yg`Zcf`i bXYfghUbX]b[`[YbXYf`UbX`gYliU`]hmiUg cdYfUh|b[`]b`W ffYbh`UWUXYa]WX]qWci fgY

HYUWX]b['GhfUhY[]Yg'

5ggYgga Ybh

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5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
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±b!7`Ugg'HYgh'	' \$ı	\$) #%#&\$%-	%ž&ž ž(ž)
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@Yb[h\.*&\$*a]bihYg*f]bWiX]b[*) *a]bihY*E / *5Ł

8YHJ]`g. `Ghi XYbhg`k]```dfYgYbhh\Y]f`i bXYfgHJbX]b[`UVci hU`gY`ZgY`YWYX`WJgY`fY`UhYX`hc`h\Y`k YY_fg X]gW gg]cb`hcd]W`H\Ymk]```VY`Ug_YX`hc`dfca dhX]gW gg]cb`Zcf`h\Y]f`dYYfg"`H\Y`ZYYXVUW_k]```VY`dfcj]XYX j]U`U`F i Vf]WZcf`DfYgYbHJhjcbg`W ffYbhm]b`i gY`]b`5g]Ub`Ghi X]Yg`Wci fgYg"@Yb[h\.`&\$`a]bi hYg`f]bWi X]b[`) a]bi hY`E / '5ŁC fU``ZYYXVUW_'`

Gi Va]gg]cb bch/g.ghi XYbhg hc gi Va]hh\Y]f dfYgYbhUh]cb Z]Yg UbX hfUbgWf]dhg hc Hi fb]h]b j]U A ccXY

 $\label{thm:conditional} \begin{tabular}{ll} Hi & fb \end{tabular} $$ h' \end{tabular} $$ fb \end{tabular} $$ fo \end{tabular} $$ h' \end{tabular} $$ fo \end{tabular$

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GhUfh'XUhY. \$) #%#&\$%-

@Yb[h\.') \$ a]bi h/g

8 YHJ]`g. `Ghi XYbhg`k]```VY`ei]nnYX`cb`h\Y]f`XYa cbghfUhYX`i bXYfgHUbX]b[`cZh\Y`Wci fgY`WcbhYbh`UbX fYUX]b[g`UbX`h\Y]f`UV]`]hmhc`Uff]Wi`UhY`Uf[i a Ybh`UbX`a Ufg\U``Yj]XYbWY`]b`fY`Uh]cb`hc`h\Uh`Uf[i a Ybh`]b`U gYf]Yg`cZg\cfh`Ubgk Yf`ei Ygh]cbg`UbX`cbY``cb[`Ubgk Yf`ei Ygh]cb"@Yb[h\.`) \$`a]bi hYgDUdYf`WcffYW]cb

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh' . F YgYUfW 9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g. 'Ghi XYbhg`k]```VY`Ug_YX`hc`gY`YWh'U`fYgYUfW\`ei Ygh]cb`Zfca 'U`bca]bUhYX``]ghl`H\Y]f`HJg_`]g`hc [Uh\Yf`fYgYUfW\`a UhYf]U`gž'Wcbghfi Wh'U`WYUf`UbX``c[]\W``Uf[i a Ybh'Ufci bX`h\Y`fYgYUfW\`ei Ygh]cbž'Vi htfYgg h\]g`Uf[i a Ybh'k]h\`Yj]XYbW'`UbX`\Wca a i b]\WUhY`]b`U`WYUf`k f]hhYb`ZJg\]cb"'H\Y`Yj]XYbW'`k]```VY`dfYgYbhYX]b`Zcfa U``U\WJXYa]\WbchJh]cb'UbX`XcW`a YbhUh]cb"K f]hhYb`ZYYXVU\W_`k]```VY`dfcj]XYX`cb'h\Y`Fi Vf]\WZcf 9ggUmg`W`ffYbhm]b`i gY`]b`5g]Ub`Ghi X]Yg`Wci fgYgžj]g`Hi fb]h]b"'H\]g`]g`h\Y`Z]bU``UggYgga Ybh'Zcf

UhrybXUbW di fdcgYg"

5XX]h]cbU`XYhU]`g.

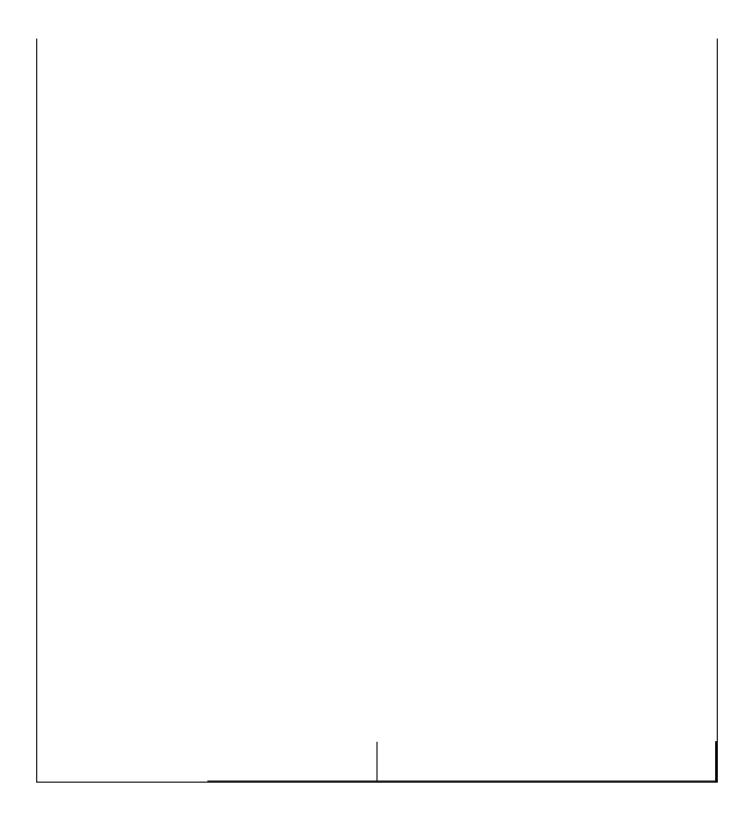
.

Gi Va]gg]cb'bchYg.Gh XYbhg'hc'gi Va]hh\Y'fYgYUfW\'dUdYf'Vmi d`cUX]b['Ub'Y`YWfcb]WWcdmhc'Hi fb]h]b j]U'A ccX`Y"'5'GW\cc``5gg][ba Ybh7cj Yfg\YYha i ghVY'UhUW\YXž'k]h\'mci f'WcbhUWhXYhU]`g'UbX'g][bUhi fY _Z``YX']b"'@UhY'gi Va]gg]cb'k]```UhffUWhdYbU'h]Yg"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tab$

5 http://bxubww.F.Yei]fYa Ybhg

5 hhy b X U b W 'c Z @Y Whi f Yg 'U b X 'Hi hcf] U 'g 'U f Y 'a U b X U hcfm] b 'h\]g 'W ci fg Y "'I b Y I W g Y X 'U V g Y b W 'Z f ca 'a cf Y h\ U b '&\$ i 'c Z `Y Whi f Yg 'U b X 'hi hcf] U 'g 'k] `` f Y g i `h] b 'f\ Y 'U k U f X 'U 'Z U] ``[f U X Y "



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\ci fg'UfY', .\$\$Ua '-'\\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ" =Zmci
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]hJb'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'YY] UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUhJcb" =Zmci 'Ya U]''9! hYfbU''Gi ddcfhmci 'k]'`
Ui hca UhJWJ'`mfYW']j Y'U'hJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'``bYYX'hc'gdYWJZWJ'`mUg_'Zcf'cbY"
Hi fb]hJb'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW_bck iYX[Ya Ybh'i

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7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

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- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]lmUbX X]gUV]`]lm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

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