

ARTS2485

9|d`cf]b[': fYbW\'@]b[i]gh]Wg

HYfa 'H\fYY''##'&\$%-

7 ci fgY Cj Yfj]Yk

'GhJZZ'7cbhJWh'8YhJ]`g'

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUhjcb	D\cbY
<i[iyg˙dyhyfg< td=""><td>\"dYhYfg4 i bgk "YXi "Ui</td><td>,</td><td>A 6 &+)</td><td>\$&</td></i[iyg˙dyhyfg<>	\"dYhYfg4 i bgk "YXi "Ui	,	A 6 &+)	\$&
		Vm'Uddc]bha Ybh		- ' ,) %((\$

GWXcc`7cbHJWhi±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b['<ci fg. 'A cbXUm'! : f]XUmž - Ua '! ') da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . 'Ž * %'&'-' ,) ', +\$)

9a U]`. '<u>\U`4 i bgk "YXi "Ui</u>

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y 7ci fgY

Gi VYWi5fYU.: fYbW.'Gh X]YgH\]g'Wci fgY'HUi [\h']b'9b[`]g\'cZYfg'U'gmghYa Uh]WWca dUf]gcb'cZ: fYbW.'UbX 9b[`]g\'Ug'ZJf'Ug'gci bXgž'j cWUVi `Ufmž'k cfX'Zcfa Uh]cbž'UbX'gYbhYbW'ghfi Wi fY'UfY'WcbWfbYX''=hk]``ZcW g cb'h\Y'hmd]WU``X]Z]W'`h]Yg'YbWci bhYfYX'Vm`YUfbYfg'cZ: fYbW.']b'dfcbi bW]Uh]cbž'cfh\c[fUd\mž'UbX'[fUa a Uf]b'cfXYf'hc'[]j Y'h\Ya 'h\Y'hcc`g'hc']a dfcj Y'h\Y]f'UWW fUWmUbX'VYWca Y']bXYdYbXYbh'YUfbYfg''

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 5dd`mU`Wf]h]WU`i bXYfgHUbX]b['cZh\Y'hcc`g`UbX'a Yh\cXg`cZ`]b[i]gh]W]bei]fm" &" 9I Ua]bY'h\Y'j Uf]ci g``]b[i]gh]WWUhY[cf]Yg`]b`: fYbW\. 'gci bXgž'a cfd\Ya Ygž'k cfXgž'd\fUgYg`UbX WUi gYg"
- ' " 7 ca dUfY'j Uf]ci g``]b[i]gh]WWUhY[cf]Yg`]b': fYbW\`UbX`9b[`]g\"
- ("⇒XYbh]Zmgca Y ZYUhi fYg cZ`]b[i]gh]WX]j Yfg]lm"

HYUWX]b['GhfUhY[]Yq'

FUhlcbU`Y.

H\]g`Wci fgY`dfcj]XYg`Ub`]blfcXi Wfjcb`hc`h\Y`ghfi Wi fY`cZh\Y`: fYbW\``Ub[i U[Y`UbX`k]```ZcW g`cb`h\Y`hmd]WU`X]ZZ]W``hJYg`YbWci bhYfYX`Vm9b[`]g\!gdYU_]b[```YUfbYfg`cZ: fYbW\"`5```cZci f`ghi XYbhg`\Uj Y`9b[`]g\`Ug`cbY cZh\Y]f``Ub[i U[Yg`Ug`UbUh]j Y`cf`U`gYWcbX``Ub[i U[Y"`Gc`WcbhfUgh]b[`h\Y`h\ c`a U_Yg`gYbgY"`K Y`k]```U`gc []j Y`h\Y`cddcfhi b]hmZcf`ghi XYbhg`k\cgY`Z]fgh``Ub[i U[Y`]g`bch'9b[`]g\`hc`a U_Y`di bWfi U``Wca dUf]gcb`k]h\ h\Y]f`Z]fgh``Ub[i U[Y`]b`Ub`cfU``[fUa a Uf`dfYgYbhUh]cb"

H\]g`WcbHYI fi U``ghi X]Yg`Wci fgY`]g`]bHY[fUHYX`k]h\]b`h\Y`: fYbWk`ghi X]Yg`dfc[fUa ž'Ug`]hiVi]`Xg`cb`h\Y`YUfb]b[`cZh\Y`ghfi Wfi fYg`X]gWcj YfYX`]b``Ub[i U[Y`Wci fgYgž'UbX`Wca d`Ya Ybhg`ch\Yf``]b[i]gh]WU``m cf]YbhUhYX`Wci fgYg.`H\Y``wci fgY.`H\Y`: fYbWk`GdYU_]b[`K cf`X`ffUi [\h]b`9b[`]g\Łž'UbX h\Y`DfcZYgg]cbU```Yj Y``Wci fgY.`5ddfcUWkYg`hc`Gdc_Yb`: fYbWk`ffUi [\h]b`: fYbWk\L''`H\Y`Wci fgY`\Y`dg ghi XYbhg`XYj Y`cd`h\Y]f`UWW`fUWmUbX`gcW]c`]b[i]gh]WWca dYhYbWf`]b`h\Y`: fYbWk```Ub[i U[Yž'UbX`[]j Yg`h\Ya h\Y`hcc`g`hc`VYWca Y`]bXYdYbXYbh`YUfbYfg"

HYUW]b[Ghf UhY[]Yg.

H\Y`Wci fgY`]g`HJi [\h]b`9b[`]g\`Ug`]h]g`cbY`cZh\Y`WcbHYI hi U``ghi X]Yg`Wci fgYg`]b`h\Y`a U'cf`]b`: fYbW\ Ghi X]Yg"

H\Y'HYUW\]b['UddfcUW\']bjc`jYg'WUgg'X]gW gg]cbg'Ug'kY``'Ug'ga U``'[fcid'X]gW gg]cbg'Xif]b['h\Y h hcf]U`gž'UbX'Xif]b['h\Y'`YWhfYg"'H\]g']g'UW\]YjYX'VmaYUbg'cZU'jUf]YhmcZhUg_g'acghcZk\]W\'fYei]fY gh XYbhg'hc'dfYdUfY']b'UXjUbW'''

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
HYghi) \$1	8 i f]b[``YWni fYgžf]k:k YY_''. \$&#\$-#&\$%-/f]]k:k YY_'+. '\$#%\$#&\$%-	%
; fci d'cfU`dfYgYblUljcb'	&\$i DfYgYbrUhjcb Xi fjb['hi hcf]U`g. k YY_g`, 'hc'%\$/K fjhhYb gi a a Ufm Xi Y Vm') da 'cb'h\Y XUmcZh\Y'hi hcf]U` dfYgYbrUhjcb		' <u>ž</u> (
:]bU`5gg][ba Ybh	' \$ı	&-#%#&\$%-`\$).\$\$`DA	&

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% HYgh

GHJfhXUhY. Bch5dd`]WUV`Y

@Yb[h\."WU""() a]b""YUW

 $8YHJ)^*g. \text{ "Hk c "]b! WUgg "hyghg 'Wcj Yf]b[\text{ 'h\Y 'WcbhybhcZh\Y 'Wci fgYžWU'' () 'a]bi hyg 'YUW\": YYXVUW_j]U']b! WUgg 'X]gW gg]cb 'UbX 'WcffYWf]cb"$

5ggYgga Ybh&; fci d cfU dfYgYbhUh]cb

GHJfh'XUhY.

@Yb[h\.'WU"'&\$'a]b"'dYf'[fcid

8 YHU]`g. '\"U`[fci d'dfYgYbHUh]cb'cZWU"'&\$ 'a]bi hYgžgh XYbhg'k]``Wca dUfY'[]j Yb'[fUa a Uh]WU``ZYUhi fYg'cZ : fYbWX'UbX'9b[`]g\"'H\Ymk]``gi Va]hU'k f]hhYb'gi a a UfmcZh\Y]f'[fci d'dfYgYbHUh]cb'cb'Hi fb]hJb'Zcf Yj U'i Uh]cb": YYXVUW_.']b!WUgg'X]gW gg]cb'UbX'ei Ygh]cbg'UZhYf'dfYgYbHUh]cbž'Ug'k Y```Ug'fi Vf]Wg'UbX'k f]hhYb Wca a Ybhg"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$

5ggYgga Ybh' . :]bU` 5gg][ba Ybh

GHJfh:XUhY.

@Yb[h. % \$\$\$ k cfXg

8YHJ]`g. `DfcV`Ya `gYh'UbX`g\cfhYggUmei Ygh]cbg`VUgYX`cb`h\Y`WcbhYbhWcj YfYX`Xi f]b[`h\Y`gYa YghYf`h\Uh fYei]fY`h\Y`gh XYbhg`hc`UbU`mgY``]b[i]gh]WXUhU`]b`: fYbW\"`5gg][ba YbhcZ\W\"`\%\$\$\$`k cfXg`gi Va]hhYX`cb

Hi fb]hjb": YYXVUW_. fi Vf]Wg"UbX'k f]hhYb"Wca a Ybhg"

 $\label{thm:condition} \mbox{Hi fb]h]b"gYh]b" Label Label$

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

- @cX[Y'5"'WfS'''f&\$\$' \L'''7j b'ad[YfZW8dWUZ >SYgSYW''@cbXcbz'5fbc'X"
- A UhYf]U`gʻdfcj]XYX`j]U`A ccX`Y

FYWca a YbXYX FYgci fWYg

Mci 'g\ci `X'h\]b_'UVci hUWei]f]b['U'[ccX'X]Wf]cbUfm''=\mci ']bhYbX'hc'Xc'U'a U'cf']b': fYbW\žkY'fYWca a YbX h\Uhmci 'Vi m'U`'Uf[Y'V]`]b[i U`'fi fYbW\!9b[`]g\#9b[`]g\!: fYbW\!X]Wf]cbUfm''\Mci 'g\ci `X'Vi m'h\Y'V][[Ygh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybht\fci [\ 'Hi fb]hJbž'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-',) '' ' ' %cf'Ya U] 'h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh

\ci fg'UfY', .\$\$Ua '-'\\$.\$\$Ua '-'\\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ" =Zmci

UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]hJb'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbž'Vi hmci

a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc

]bWi XY'Ug'Yj]XYbWY'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUhJcb" =Zmci 'Ya U]''9! hYfbU''Gi ddcfhmci 'k]'`

Ui hca UhJWJ'`mfYWY]j Y'U'hJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'``bYYX'hc'gdYWJZWJ``mUg_'Zcf'cbY"

Hi fb]hJb'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUlfYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hzil BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUv`Y`I gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi VYWfi]XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/ fYj]Yk ʻcZa Uf_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/