

ARTS2570

⇒hYfa YX]UhY GdUb]g\ 5

HYfa 'H\fYY''##'&\$%-

7ci fgY⁻Cj Yfj]Yk

'GhJZZ'7cbhJWh'8YhJ]`g'

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
B Ul'U`]U'C fl·]n 7 YVYf]c	b"cft]nVWVYf]c4 i bgk "YXi "Ui		A cfj Yb 6fck b Yj Y``&`c <i>Z</i> JW &&,	fŽ*%:\$& -',):%\$*(

GWXcc`7cbHJWhi±bZcfa Uh]cb

GW.cc``cZ<iaUb]h]Yg'UbX'@Ub[iU[Yg

@cWUhjcb. GWkcc`CZZJWYž'AcfjYb'6fckb'6i]X]b[ž'@YjY``&ž'&),

CdYb]b['< ci fg. 'A cbXUm! ': f]XUmž'- Ua '!') da

D\cbY. 'Ž*%'&'-',)'%*,%

: UI . Ž*%'&'-',) ', +\$)

9a U]`. \<u>U`4 i bgk "YXi "Ui</u>

7ci fgY 8YHJ] g

7 fYX]hDc]bhg * *

Gi a a UfmcZh\Y 7ci fgY

 $GiVYWh5fYU.EbS`[eZS`V>Sf[`3_Wd[US`EfgV[We]]$

K]h\ a cfY h\ Ub'' ' a]``]cb`GdUb]g\`gdYU_Yfg']b'h\ Y`I G5`U`cbYž'<]gdUb]WW\`hi fY`]g`]bWfYUg]b[`m g][b]Z\WUbh'`H\]g`Wci fgY``Yb\ UbW'g`mci f`UV]`]mihc`UddfYV\JUhY`h\ Y`Wca d`YI]micZ<]gdUb]WW\`hi fY`UWfcgg hk c`Wcbh]bYbhg"'H\ Y`Wci fgY`]g`UVci h`YUfb]b[`hc`gdYU_ž'fYUX`UbX`k f]hY`GdUb]g\`Vmi g]b[`h\ Y`a YX]i a `cZ W\`hi fY"`J cWUVi `UfmUbX`[fUa a Uh]WU``ghfi Whi fYg`UfY`dfYgYbhYX`]b`h\ Y`WcbhYI hcZW\`hi fU``mfY`Yj Ubhhcd]Wg" Hcd]Wg`a Um]bWi XY.``Ub[i U[Y`UbX`Wca a i b]WUh]cbž'VY`]YZgmghYa gžUbX`h\ Y`Ybj]fcba Ybh'

H\Y``Ub[i U[Y`cZ]bglfi Wf]cb`]g`GdUb]g\"`K Y`UfY`h\Y`Z]fghi b]j Yfg]lmi]b`5i glfU`]U`i g]b[`h\Y`]bbcj Uf]j Y`cb`]bY WcUW\]b[`dfc[fUa`k\]W\`cZYfg`mci `U`i b]ei Y`cddcfh b]lmihc`k cf_`ci hg]XY`cZWUgg`k]h\`U`dYfgcbU`ž'bUf]j Y! gdYU_Yf``Ub[i U[Y`WcUW\`]b`U`fYU``h]a Yžcb`]bY``YUfb]b[`Ybj]fcba Ybf'`K Y`\cdY`h\Uhih\]g`i b]ei Y`dYfgcbU`YI dYf]YbWY`k]```VY`lfi`miYb^cmUV`Y`UbX`k]``a cfj UhY`mci `hc`Wcbf]b[`GdUb]g\"

7 ci fqY @YUfb]b['Ci hMca Yq

- &" DfcXi Wrg]a d`YžWc\YfYbh'hYl hg`fcfU``UbX'k f]hhYbŁcb`Wca a cb'hcd]Wg`cf`UfYUg`cZdYfgcbU``]bhYfYgh'
- ' "8YgWf]VY YI dYf]YbWg UbX Yj Ybhg UbX dfcj]XY fYUgcbg cf YI d`UbUh]cbg Zcf cd]b]cbg UbX d`Ubg
- ("8]gh]b[i]g\`g][b]Z]WUbhiX]ZZYfYbWYg`VYhk YYb`gca Y`W ghca gž`Uhh]hi XYg`cf`VY`]YZg`dfYj U`Ybh`]b`h\Y <]gdUb]WgdYU_]b[`Wca a i b]hmUbX`h\cgY`cZh\Y]f`ck b"
-)" I gY glfUhY[]Yg hc WccdYfUhY YZZYWhj Y mk]h\ ch\Yf a Ya VYfg cZh\Y [fcid"

HYUWX]b['GhfUhY[]Yq'

FUNCOUY Zcfh\Y]bWiq]cbcZWcbNYbhUbXnYUWX]b[UddfcUWX

@Ub[i U[Yg'UfY'VYgh`YUfbYX'h\fci [\`i gY'cZh\Y``Ub[i U[Y"'K Y'dfcj]XYžh\YfYZcfYžcddcfhi b]h]Yg'Zcf ghi XYbhg'hc'i gY'h\Y``Ub[i U[Y'UbX'k Y'Xc'gc']b'U'a YUb]b[Z\``k Um''H\Uh]gžk Y'i gY'U'h\Ya Uh]WUddfcUW\'hc h\Y'cf[Ub]nUh]cb'cZWci fgY'a UhYf]U'g"'H\Y'hcd]Wg'Wcj YfYX'fYdfYgYbh'W ffYbh']ggi Yg']b'GdUb]g\!gdYU_]b[Wci bhf]Yg"

 $\label{thm:condition} $$H^g'' a d`cmg'ghi XYbhi WbhYfYX``YUfb]b['Ug'h\Y'VUg]g'Zcf']hg']bghfi Wh]cbU``XYg][b'UbX'Ya d\Ug]nYg h\Y']a dcfhUbW'cZUWhj Y``YUfb]b["$

H\Y'hYUWX]b[']b'h\]g'Wci fgY']g'V'YbXYX.'cb`]bY'UbX']b'WUgg'UWh]j]h]Yg''''

 $HY^{"}Ub[iU[Y^{"}cZ]bghfiWh]cb^{"}g^{"}GdUb]gY^{"}$

: fca 'h\Y'; i]XY`]bYg`cb`@YUfb]b['h\Uh\\$Zcfa 'HYUW\]b['ghi XYbhg`Wca a i b]\WhY`]bZcfa Uh]cbž]XYUg`UbX Uf[i a Ybhg`Vch\`cfU``mUbX`]b`k f]h]b["'H\Ym[Uh\Yf`UbX`dfc\Wgg`]bZcfa Uh]cb`Zfca 'U'j Uf]YhmcZdUdYfž Ui X]cj]gi U``UbX`Y`Y\Wfcb]\Wgci f\Wg''H\Ymi gY`\H\Ymi gY`\H`YZZY\Wh]j Y`mVch\`Ug`U'a YUbg`cZ\Wca a i b]\WUh]cb`UbX`Ug`Ub U]X`hc``YUfb]b['Ug`K Y```Ug`XYa cbghfUhY`gca Y`UV]`]hmUg`Ub`]bXYdYbXYbh`YUfbYf"

5ggYgga Ybh

Ghi XYbhgʻUfYʻYl dYWYX'hcʻWca Yʻhcʻ: UWYʻhcʻ: UWYʻ`YWhi fYgʻUbX'hi hcf]UʻgʻdfYdUfYX"ʻD`YUgYʻWca d`YhYh\Y`k YY_`mUWhji]YgʻVYZcfYʻ`YWhi fYʻUbXʻh\Y`UWNji]hJYgʻVYZcfYʻhi hcf]UʻgʻflGYYʻA ccXYʻZcfi'Z fh\Yf`XYhU]`YgŁ"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[CihWcaYgʻ5ggYggYX
9!Dcfh7c`]c'	&\$I	cb^[c]b[%ž&ž(
&:±b!7:`Ugg:'HYghg:	*\$1	k YY_^) ^UbX^%\$	%ž&Ž Ž(
C fU [*]	&\$ı	k YY_',	

‰

5XX]h]cbU`XYhJ]`g.

 $\pm \text{ 'h'gh'\%' Ub X' h'gh'\&' h' Y' $Z\!c``ck]b['k]```VY' UggYgggYX{\tt Z}'' = \text{ `k'l} + \text{ `k'l} +$

- DUffn%; fUa a Ufžij cWUVžfYUX]b['Wca dfY\Ybq]cbžYHW'Cb`]bY'Ei]n
- DUfhi& K f]h]b["

HYgh%DUfh%UbX^&'tU_Yg'd`UWY']b'k YY_') ž'fl \$1 \ \ \text{E'".7 cbhYbhWcj YfYX\]b'k YY_g'%(

 $HYgh\&^{\circ}DUfh\&^{\circ}ggUmK\ f]h]b[\ \check{z}^{\circ}hU_{Y}g^{\circ}d^{\circ}UWY^{\circ}]b^{\circ}k\ YY_{-}^{-}\ Xi\ f]b[\ `^{\circ}YWi\ fY^{\circ}h]a\ Y^{\circ}f\%) + \&^{\circ}CbhYbhWcbj\ YfYX^{\circ}k\ YY_{g}^{\circ})!\%$

.

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh' . 'CfU'

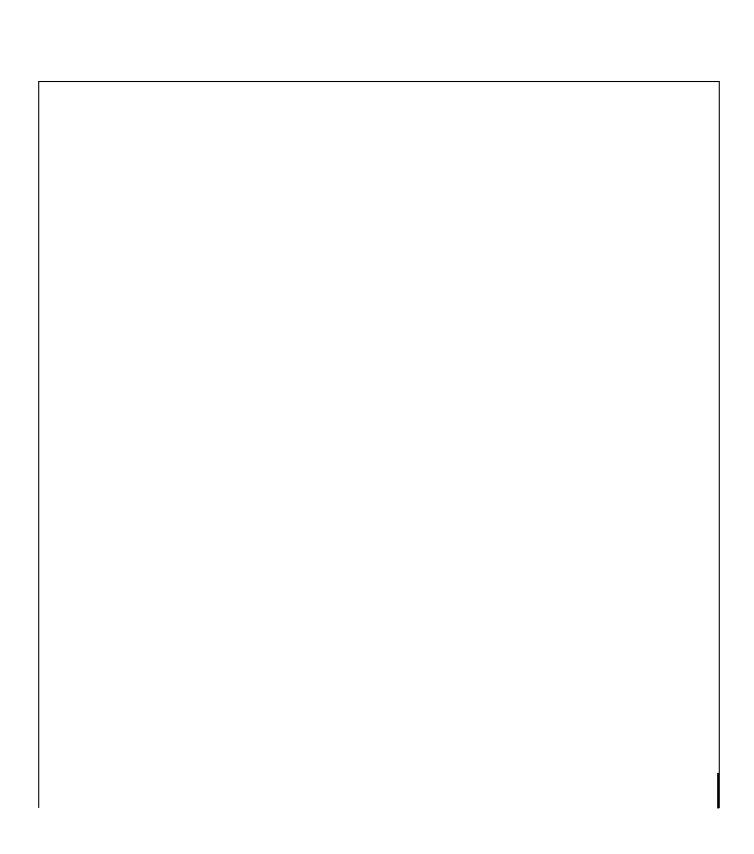
GhUfhiXUhY. cb [c]b[

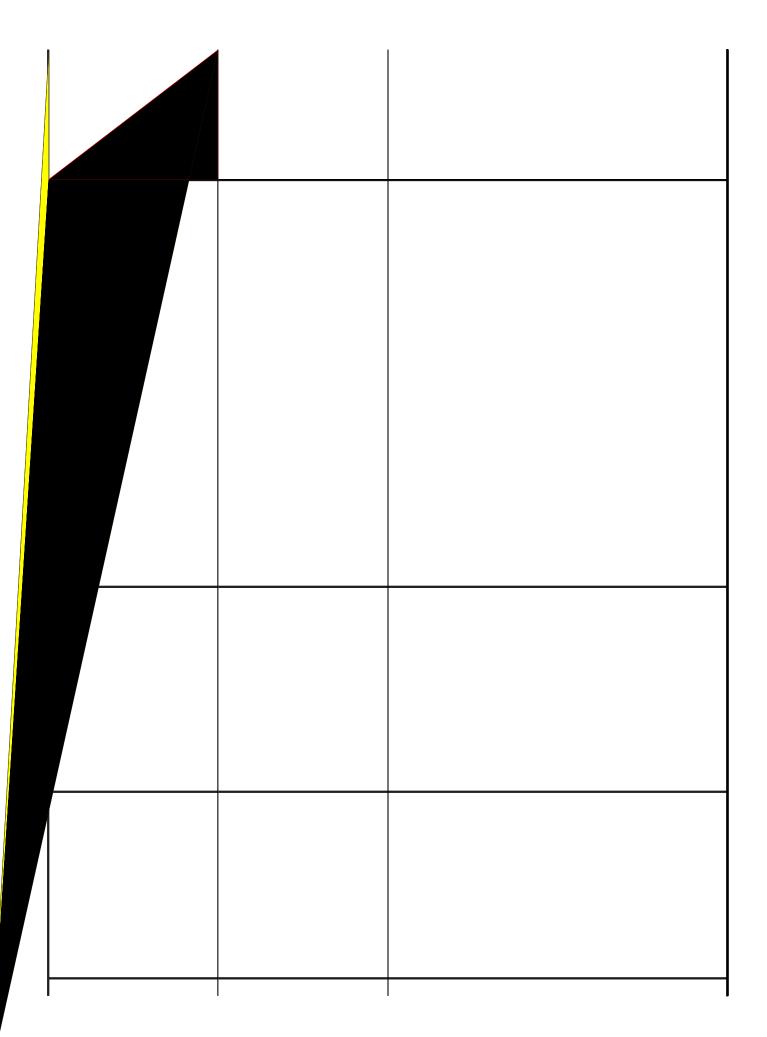
@Yb[h\. j Uf]Yg

 $8YHJ]^*g. "Cb[c]b["UbX"]^bU``flish" a]bi hYgL"VmUddc]bha Ybh"CfU``ZYYXVUW_UbX`a Uf_g'dfcj]XYX"$

5XX]h]cbU`XYhU]`g.

K YY_`m' \$ a]bi hYg'cb`]bY'cfU`'dfUWnJgY'Ž





K YY_', .'('Bcj Ya VYf'!', Bcj Ya VYf	Hcd]W	7 Ud]hi `c '8 Y 'j]U Y "`: SUWd Ub X ']b 'h]a Y 'Wobghfi Wh]cbg" F Yj]g]cb c Z E 7 'Z 'Yb X c Z 7 \UdhYf F Yj]g]cb
		9bX'cZ'7\UdhYf'ei]n
		7 ca d`YhY'cb`]bY``YWhi fY'ei]n'h\Y'XUmVYZcfY mcif': &: ``YWhi fY"'H\Y'ei]n'WcgYg'h\Y'XUm VYZcfY'mcif'`YWhi fY'Uh'&'") - "
K YY_''%%Bcj Ya VYf'! %) 'Bcj Ya VYf	Hcd]W	7 Ud]hi `c'%("'9``a i bXc'a cXYfbcž;bW&f[hWeźh\Y Zcfa 'Bcgchfcg'f@Yh%LWca a UbXg"'H\Y'EgT`g` Uf[hW k]h\'UXj YfV]U`'YI dfYgg]cbg'cZWcbX]hJcb'cf'di fdcgY"'
		9bX°cZ7\UdhYf°%(°ei]n
		HYgh&'DUfh&z'9ggUmK f]h]b[z'Xi f]b[''YWn fY'h]a Y' f%)ı Ł'
		7 ca d`YhY'cb`]bY``YWhi fY'ei]n'h\Y'XUm'VYZcfY mcif': &: ``YWhi fY"'H\Y'ei]n'WcgYg'h\Y'XUm VYZcfY'mcif'`YWhi fY'Uh'&'")-"
		7ca d`YhY'k YY_`mi: CFI A
K YY_^%\$.^% ^Bcj Ya VYf^!	Hcd]W	
&& Bcj Ya VYf		; YbYfU``fYj]g]cb`cZWci fgY`Wcbh/bh`]b`dfYdUfUh]cb Zcf`k f]hh/b`UbX`CfU``Yl Ua g"
		HYgh&`DUfh%; fUa a Uf`f%)ı Ł"7cbhYbhWcbj YfYXk YY_g`)!%\$
		5 WW gg A ccX Y Zcf Z fth Yf]bcZfa Utjcb UbX tcdfYdUfY Zcf th Y thygh
		5 ZhYf'h\Y'hYgh'g\cfh'VfU_Y'Zc``ck YX'Vm'CfU` Yl Ua "
K YY_'‱'&) Bcj Ya VYf'! &- Bcj Ya VYf	Hi hcf]U`	H\]g`]g`U`a U_Y`i d`h hcf]U``Zcf`k YY_`(`Xi Y`hc`U`di V`]W \c`]XUmcb`A cbXUm+`C WhcVYf

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

 $\label{eq:cbiby} $$Cb^*]bY^7cUW_b[..difW_Ug]b[..difW$

%"; c'hc'\hdg.#k k k "`]b[i Ua YYh]b["Wca #Y[]ghfc"d\d

- &" GY`YWhici f`i b]j Yfg]hmžgY`YWhimci f`]bghfi Whcf#WUgg`gYWhjcbžYbhYf`mci f`dYfgcbU``]bZcfa Uhjcb`UbX gY`YWhimci f`dUma Ybhicdhjcb.
- ' " 7\ccgY`mci f'gYgg]cb'Zfca 'h\Y`Uj U]`UV`Y'gYgg]cb'h]a Yg"'Mci f'gYgg]cb'k]``'VY'cb'h\Y'gUa Y'XUm'UbX

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5WUXYa]W±bZcfaUh]cb

 $: \ cf `YggYbh]U``gh XYbh]bZcfa Uh]cb `fY`Uh]b[\ 'hc.$

• fYei Yghg Zcf Yl hYbg]cb/

cfixill hm bX -aYbbilliab#dfahbby hai7cfiYlbYbodiab/ hlcb\$