

# **ARTS3454**

 $7 \ |bYgY^9b[^]g = bhYfdfYh]b[$ 

HYfa 'H\fYY''##'&\$%-

# 7ci fgY<sup>-</sup>Cj Yfj ]Yk

## 'GhJZZ'7cbhJWh'8YhJ]`g'

## 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
8f"'M]b[`]'Gi b	1 1 1 3		A cfj Yb <sup>·</sup> 6fck b &) )	-',)&*(\$

### @YWhifYfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUHcb	D\cbY
8f""M]b[`]"Gi b	m]b[`]"gi b4 i bgk "YXi "Ui		A cfj Yb 6fck b	-',)&*(\$
			&))	

## GWXcc`7cbHJWhi±bZcfa Uhjcb

GWkcc``cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\Ulrighthgaper: G\\\cc``C\Z]\\Y\Z\A\cfj\Yb`6fck\b'6i\]\X]b[\z'@\j\Y``&z\&),

CdYb]b[ '< ci fg. 'A cbXUm!': f]XUm $\check{z}$ - Ua '!') da

D\cbY.'Ž\*%'&'-',)'%\*,%

: UI . Ž\*%'&'-',) ', +\$)

9a U]`. <u>\U`4 i bgk "YXi "Ui</u>

## 7ci fgY 8YW] g

7 fYX]hDc]bhg \* \*

#### Gi a a UfmcZh\Y 7ci fgY

GI VYWN5fYU. 5Z[ WeWEfqV[We

 $H\g``Wci fgY']blfcXi \W'g`mci 'hc 'fY\W'bh']bhYfdfYh]b['h\Ycf]Yg``UbX`a Yh\cXc`c[]Yg'' Mci 'k ]````YUfb`\ck 'hc ]bhYfdfYhYhk YYb'9b[`]g\`UbX`7\]bYgYži g]b['X]ZZYfYbh']bhYfdfYh]b['hY\Wb]ei Yg'' Mci 'k ]```U\\&i ]fY'`]b[i ]gh]W UbX`W``hi fU``UV]`]h]Yg`fYei ]fYX`Zcf`\\cbgY\W hj Y`]bhYfdfYh]b['UbX`YZZY\\] Y`\wcgg!`Ub[i U[Y\\W\`hi fY \\cdot a i b]\\Uh]cb']b'U'fUb[Y`cZX\mbUa]\\\cby\mci k]``XYj Y`cd'UbX`\cbY`\mci f`g_]``g']b'\\U\\\\\\\\cdot [\\]b\YfdfYh]b['YlYf\\]gYg''$ 

#### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" ⇒XYbhjZmh\YcfYhjWJ`]ggi Ygʻ]bWi X]b[ʻ]bhYfdfYhjb[ʻUddfcUW\YgʻUbXʻghm\YgžYh\]WgʻcZdfcZYgg]cbU` |bhYfdfYhYfgʻUbXʻbcfa q'UbX`a cXY`qʻcZ]bhYfdfYhjb[
- &" 5ggYgg'bYk 'a Yh\cXc`c[]Yg'UbX'hYW\b]ei Yg'Uj U]`UV`Y'Zcf'9b[`]g\'UbX'7\]bYgY']bhYfdfYh]b[
- ' "8Ya cbghfUhY`Wca dYhYbWY`]b`UbX`VUg]Wg\_]``g`UbX`hYWXb]ei Yg`cZWcbgYW'hjj Y`]bhYfdfYh]b[`UbX`di V`]WqdYU\_]b[
- (" ±bHYfdfYh]b'U'fUb[Y'cZdUfUdfcZ/gg]cbU``g]hi Uh]cbg. 'gcW]U``X]U`c[i Ygž'Wca a i b]hmigYfj ]WY X]U`c[i Ygž'HUXY'X]U`c[i Yg`UbX'WcbZ/fYbWY'gdYYWXYg
- ) " +bH/fdfYhigY`YWH/X`fYdfYgYbHJhjj Y`hYl hg`Zfca '9b[`]g\`]bhc'7\]bYgY`Jbhc'9b[`]g\

### HYUW ]b[ GhfUhY[]Yg

 $\label{thm:linear:lin$ 

## 5ggYgga Ybh

5 hh/bXUbW'cZ@YWh fYg#Hi hcf]U`g`]g`a UbXUhcfm]b`h\]g`Wci fgY"I bYI W gYX`UVgYbW'Zfca `a cfY`h\Ub`&\$I cZ@YWh fYg#Hi hcf]U`g`k]``fYgi `h]b`h\Y`Uk UfX`U`ZU]``[fUXY"`+b`@YWh fY#Hi hcf]U`g`mci `k]```UWh]j Y`mYb[U[Y k]h\WcfY`Wci fgY`WcbhYbhzYbUV`]b[`mci `hc`UhfU]b`7@C`%&&(z)"

#### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
±bhYfdfYh]b[ 'ei ]nnYg'UbX Ugg][ ba Ybhg'	')।		%Ž Ž( Ž)
HYghg <sup>*</sup>	(\$1	A ]X! hYfa 'HYghXi Y ']b 'k YY_ '* h hcf]U``f&' #%\$#&\$% \L': ]bU` HYghXi Y ']b 'k YY_ '%\$ 'h hcf]U` f&\$#%#&\$% \L''	%₹ ž( ž)
F c `Y `D `Umi	&) ı	FcYd`UmXiY`]bkYY_; `YWnfYf\$(#%#&\$% ŁzkYY `YWnfYf%#%#&\$% ŁcfkYY_ %\$``YWnfYf%#%#&\$% Ł'	%ž&ž ž(

## 5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% +bhYfdfYh]b[ ei ]nnYg UbX Ugg][ ba Ybhg

#### Ghufh Xuhy.

8YHJ]`g. `Ghi XYbhg`Wca d`YhY`&`]bhYfdfYh]b[ `Ugg][ ba Ybhg`f&\$i \ EcZUddfcl ""') \$\$!+\$\$`k cfXg#M\UfUWYfg YUW\`UbX`%`]bhYfdfYh]b[ `ei ]n`f%) i \ Ek\]W\`Wcbg]ghg`cZ&\$`a i `h]d`Y`W\c]W\`ei Ygh]cbg`UbX`) \$`j cWUVi `Ufm ei Ygh]cbg": YYXVUW\_j ]U`]bX]j ]Xi U``Wca a Ybhg`UbX`]b!WUgg`X]gW gg]cb"

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh'& 'HYghg

#### GhUfh XUhY.

8 YHJ]`g. `Ghi XYbhg`Wca d`YhY`U`a ]X!hYfa `hYghf&\$ı EcZUddfcl "`) \$\$!\*\$\$'k cfXg#W.UfUWhYfg`UbX`U`Z|bU``hYghf&\$ı EcZUddfcl "`) \$\$!\*\$\$'k cfXg#W.UfUWhYfg`UbX`U`Z|bU``hYgh]g`h.Y`Z|bU``UggYgga YbhZcf`UhHYbXUbWYdi fdcgYg": YYXVUW\_`Zcf`a ]X!hYfa `hYghdfcj ]XYX`j ]U`]bX]j ]Xi U``Wca a Ybhg`UbX`]b!WUgg`X]gWi gg]cb"`Ghi XYbhfYWY]j Y`U`a Uf\_`Zcf`Z]bU``hYghUbX`WUb`Wcbgi `hh\Y``YWhi fYf`Zcf`Z fh\Yf`ZYYXVUW\_"

Hi fb]h]b gYh]b[. H\]g ]g bchU Hi fb]h]b Ugg][ ba Ybh

5ggYgga Ybh' . 'F c`Y'D`Um

GhUfh'XUhY.

8YHJ]`g. `Ghi XYbhg`Wca d`YhY`U`[fci d`fc`Y`d`UmfWJ"`, `a ]bi hYg`dYf`dYfgcbŁ`cb`]bhYfdfYh]b[ `g]hi Uh]cbg []j Yb": YYXVUW\_j jU`]bX]j ]Xi U``Wca a Ybhg`UbX`]b!WUgg`X]gW gg]cb"

Hi fb]h]b gYh]b[. H\]g ]g bchU Hi fb]h]b Ugg][ ba Ybh

# 5 hm/b XUb VW F Yei ]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$ 

# 7ci fgY'GWXYXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YtUV`Y

8UhY	HmdY	7 cbhYbh
K YY_'% '%* 'GYdhYa VYf'!	Hcd]W	±bXiWnjcb UbX ±blfcXiWnjcb
&\$`GYdhYa VYf		

# Fygci fWyg

## DfYgWf]VYX F Ygci fWYg

 $5\, ``fYUX]b[\ 'a\ UhYf]U`g`UfY`Uj\ U]`UV`Y`cb`A\ ccX`Y''$ 

## FYWca a YbXYX FYgci fWYg

- \hd.#kkk"YfiX]h"cf[#Yb#YjiY#aYhU#
- \hd.#kkk"bUUh]"Wca"Ui#

## 7 ci fgY '9j U'i Uh]cb 'UbX '8Yj Y`cda Ybh

H\]g`WcifgY`k]``VY`ZcfaU``mYjU`iUhYX`h **‰** 

‰Ac j af]Ubh "

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

\_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb[ 'hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZUi 'hk ]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW\_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k ]``

#### 5 WUXYa | W < cbYghmUbX D`U[ | Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU\_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUiqci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth Yfg'Vi hidUgg]b[ 'cZZth Y 'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

# 5 WUXYa ]W±bZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb'fY`UhJb['hc.