

ARTS1900

; YbXYfYX'K cfXg. *blfcXi Wf]cb'hc'; YbXYf'Ghi X]Yg

HYfa 'H\fYY''##'&\$&\$

7ci fgY^{*}Cj Yfj]Yk

GhJZZ 7 cbhJWh 8 YhJ] g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
NcfU [·] G]a]W		Hi YgXUm‱l‱ž K YXbYgXUm&l' cf VmUddc]bha Ybh	A cfj Yb 6fck b	\$&`-' ,) %+' *

Hi hcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
Gcd\]Y`FcV]bgcb	gcd\]Y"fcV]bgcb4 i bgk "YXi "Ui	6mUddc]bha Ybh! Ya U]`hc`a U_Y UffUb[Ya Ybhgžcf WcbhUWiNcfU		

GW.cc`7cbhJWh′±bZcfa Uh]cb

 $\label{eq:GWCCCCZ} $$GW.cc``cZ< i\ a\ Ub]h]Yg`UbX`@Ub[i\ U[\ Yg]$$

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y 7ci fgY

Gi VYWn5fYU. 9W VWdEfgV[We"

 $H\ [g'Wc'\ fgY'WUb'U\ gc'VY'gh'\ X]YX']b'h\ Y'Zc``ck\ [b[\ 'gdYW]U\]gUh]cb.':\ [efack]$

H\]g`Wci fgY`]g`Ub`]bffcXi Wfjcb'hc`h\Y`ghi XmcZk ca Ybž'[YbXYf`UbX`gYI i U`]h]Yg`h\fci [\`h\Y`ZfUa Yk cf_g`cZ k cfX`\]ghcfmUbX`Wfcgg!W`hi fU``UbU`mg]g"`Mci `k]``VY[]b`k]h\`ei Yghjcbg'h\Uh'UfY`Z bXUa YbhU``hc`[YbXYf ghi X]Yg`—'gi W\`Ug`'k\Uh']g`[YbXYf3'`—'UbX`hfUWY`h\Ya `UWfcgg'ha Y`UbX`d`UWY`hc`XYa cbghfUhY`\ck `[YbXYf]bhYfgYWfg`k]h\`fUWYžWUggžgYI i U`]hmUbX`ch\Yf`a Uf_Yfg`]b`X]ZZYfYbh\]ghcf]WU``UbX`W``hi fU``WcbhYI hg''`Mci k]```Wcbg]XYf`\ck `fY`][]cbž'Wc`cb]U`]ga ž'bUhJcbU`]ga `UbX`Wfcgg!W``hi fU``WcbhUWg`\Uj Y`g\UdYX`UbX]bZi YbWYX`[YbXYf`fc`Yg`UbX`fY`UhJcbg''H\Y`Wci fgY`k]```U`gc`]bhfcXi WY`mci `hc`_YmXYVUhYg`UbXXXYj Y`cda Ybhg`]b`[YbXYf`ghi X]Yg`UbX`[YbXYf`\]ghcfmžZfca `h\Y`ei YghJcb'cZh\Y`fc`Y`cZ[YbXYf`]b`YUf`m a cXYfb`k]hW!\i bhgž'h\fci [\`>i X]h\`6i hYf'g`]bZi YbhJU``h\Ycfg]g]b[`cZ[YbXYf`UbX`Ya VcX]a Ybhž'UbX`h\Y Ya Yf[YbW'cZhfUbg`ghi X]Yg`UbX`]bhYfgYWfjcbU``h\Ycfm'

7 ci fgY @YUfb]b['Ci hWca Yg

- %" FYWc[b]gY`\ck`WUggžfUWY`UbX#cf`gYIiU`]hmi]bhYfgYWhg`k]h\`[YbXYf`]b`X]ZZYfYbh\]ghcf]WU``UbX W`hifU``WcbhYIhq"
- &" XYbhZm_YmWcbWdhg`]b`[YbXYf`UbX`ZYa]b]ghighi X]Yg`]b`k f]hhYb`UbX#cf`j YfVU``Zcfa"
- ' " 7 cbgffi WiUf[i a Ybfg:XfUk]b[:cb:ZYa]b]ghUbX:[YbXYf:gWkc:Ufg\]d:]b:k f]ffYb:UbX#cfij YfVU::Zcfa "

HYUWX]b[GhfUhY[]Yg

H\]g`Wci fgY`]g`Ub`]bffcXi Wfjcb'hc`; YbXYf`Ghi X]Yg`]b`k\]W\`mci `k]```HJ_Y`U`\]ghcf]W\``UbX`Wfcgg!W``hi fU`UddfcUW\`hc`\Y`d`mci `i bXYfgHJbX`\ck `[YbXYf`fc`Yg`UbX`XmbUa]\Wg`\Uj Y`Ya Yf[YXž'W\Ub[YX`UbX'j Uf]YX UWfcgg'ha Y`UbX`d`UW\"'=n]g`U`gc`XYg][bYX`hc`XYYdYb`mci f`g_]``g`]b`Wf]hei]b[`gW\c`Uf`m`]hYfUhi fYž Wcbglfi Wf]b[`mci f`ck b`Uf[i a Ybhg`UbX`WcbXi Wf]b[`fYgYUfW\"`@YWi fYg`k]``dfcj]XY`Wfi WJU``VUW_[fci bX]b\Zcfa Uhjcb'UbX`]blfcXi W\`mci 'hc`_YmXYVUhYgž'h\]b_Yfg`UbX`h\Ya Yg`]b`; YbXYf`Ghi X]Yg''K YV`WcbhYbhk]``dfcj]XY`; YbXYf`Ghi X]Yg'']b`U`hYfbUhj] Y`Zcfa Uhgžgi W\`Ug`dcXWJghg`UbX`XcW a YbhUf]Yg'''±b`hi hcf]U`gž'mci `k]``dUfhj\WjdUhY`]b`]b!XYdh\`X]gW gg]cb`cZh\Y`gW\c`Ufg\]d`UbX``YWi fY`a UhYf]U`ž'[i]XYX`Vmhi hcf]U`ei Yghjcbg'''H\Y`hYUW\]b[`ghfUhY[m]g`hc`YbWci fU[Y`mci f`Wf]hj\WJ``Yb[U[Ya Ybhk]h\`Wci fgY`a UhYf]U``UbX`hc\Y`d`XYj Y`cd`mci f`k f]hhYb`UbX'j YfVU``Wca a i b]\WJhjcb`h\fci [\`WUgg`X]gW gg]cbžk f]hhYb`fYgdcbgYgž'UbX]bXYdYbXYbhfYgYUfW\"

5ggYgga Ybh

 $BchY. `U```UggYgga Ybhg`UfY`a Uf_YX`ci hcZ%$$ži g]b[`dUggž`WfYX]hž`X]gh]bWf]cbž`\][`X]gh]bWf]cb`[fUXYfUb[Y"``H\YgY`[fUXYg`UfY`YI d`U]bYX`\YfY.$

\hdg.#ghi XYbh'i bgk "YXi "Ui # fUXY

FYZYfYbWjb[gmghYag.

K Y dfYZYf'h\Y: cchbchY#V]V`]c[fUd\mgmghYa ž'U`gc _bck b Ug C | ZcfX cf 7 \]WU[c"GYY.

\hdg.#gh XYbh'i bgk "YXi "Ui #cchbchY!V]V`]c[fUd\mcf!cl ZcfX!fYZYfYbW]b[!gmghYa

<ck Yj Yfžk Y fYU`]gY a UbmcZmci 'UfY a cfY i gYX hc`]b!hYl higmghYa g"'±b`h\Uh\WJgYžk Y fYWca a YbX mci
i gY 'Ub']b!hYl hifYZYfYbW]b['ghmY h\Uh']bWi XYg'dU[Y bi a VYfgžgi W\ 'Ug' < Ufj UfX cf'5D5.</pre>

\hdg.##qh XYbh'i bgk "YXi "Ui #\Ufj UfX!fYZYfYbW]b[

\hdg.#ghi XYbh'i bgk "YXi "Ui #JdU

DfYgYbhJh]cb.

: cfik f]hYbiUggYgga Ybhgžik YifYWca a YbXi‱idc]bhZcbhži%")!Xci ViYigdUWJb["i5ik UmgighUhYih\Yiei Ygh]cbii dhcdžidfYZYfUVim]biVciX"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
Hi hcf]U``F YgdcbgYg'	(\$1	Bch5dd`]WUV`Y	%ž&
F YgYUfVX 'YggUm'	(\$1	\$*#%%\$\$\$`\$(.\$\$`DA	

UggYgga Ybh']g'(\$i "

: YYXVUW_j]U`]bX]j]Xi U``Wa a Ybhgžfi Vf]WUbX`]b!WUgg`X]gW gg]cbžk]h\]b`U`k YY_cZgi Va]gg]cb`Zcf`h\Y ZfghfYgdcbgY`UbX`k]h\]b`hYb`XUmg`Zcf`gi VgYei YbhcbYg"

.

5XX]h]cbU`XYhU]`g.

EiYgh]cbg. "H\Y"eiYgh]cbg"UfY"dcghYX"cb"AccX`Y"]b"h\Y"kYY_`m'Zc`XYfg"

FYZYfYbW]b[. KYXcfYei]fYfYzYfYbWgZcfth]g'UggYggaYbh!]XYU`mZcchbchYgžVihWb'U`gciigY]b!hYlhždfcj]XYX'mcii]bWiXY'dU[Y'biaVYffYZg"'=Zig]b[ZcchbchYgžmciiXcbchbYYX'hcigidd`mU'V]V`]c[fUd\mUgh\YZcchbchY'WcbhU]bg'h\Y']bZcfaUh]cb"'=Zig]b[`]b!hYlhžd`YUgY'dfcj]XY'U'fYZYfYbW'']gh'

F YgYUfW. Mci 'UfY'bchYl dYWfYX'hc'fYUX'#`]ghYb'VYmcbX'h\Y'gYhWcbhYbhZcf'h\Y'k YY_ž'bcf'UfY'mci Yl dYWfYX'hc'i gY'U``'h\Y'a UhYf]U`'hc'Ubgk Yf'U'ei Ygh]cb'! i gY'h\Y'a UhYf]U`'a cghfY`Yj Ubhhc'h\Uhei Ygh]cb'' Gca Y'ei Ygh]cbgž Zcf'Yl Ua d`Yž'a UmVY'UVci hcbY'gdYW]ZJWhYl hcf'dcXWUghcf'XcW a YbhUfm'

.

Gi Va]gg]cb'bchYg.Gi Va]hmci f'h hcf]U`fYgdcbgY'hc'h\Y'WcffYgdcbX]b['k YY_!'Y"["']Zmci 'Ubgk Yf'U'K YY_ ('ei Ygh]cbžgi Va]hhc'h\Y'K YY_'('Hi fb'\\=\^]b_"

Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh]g'gi Va]hhYX'h\fci [\'Hi fb]hjb'UbX'gh XYbhg'Xc'bchgYY'Hi fb]hjb g]a]'Uf]hmfYdcfhg"

5ggYgga Ybh& FYgYUfW YggUm

GHJfh'XUhY.

@Yb[h. &\$\$\$ k cfXg

8YW]`q.

Ghi XYbhgʻk f]hYʻUbʻYggUmifUddfcl]a UhY`mi&\$\$\$`k cfXgŁ`VUgYX`cb`]bXYdYbXYbhifYgYUfWX"'E i Ygh]cbgʻUfY XfUk bʻZfca 'Wci fgY'h\Ya Yg"

: YYXVUW_j jUjbXjj jXi U`Wca a YbhgʻUbX fi VfjWzk jh\jb hYb XUmgʻcZgi Va jggjcb"

5XX]h]cbU`XYhU]`q.

 $\label{eq:continuous} E \ i \ Ygh]cbg. \ 'H\ Y \ YggUmei \ Ygh]cbg'k \]```fY`UhY 'hc'h\ Y`YWh' fY\#h' hcf]U``hcd] \ Vg'`UbX`k \]```UddYUf']b'h\ Y`k \ YY_m \ Zc`XYfg'cb'A \ ccX`Y''`E \ i \ Ygh]cbg'fY`UhY 'hc'K \ YY_g'&!, \ ''`$

FYgYUfW.: Gh XYbhg UfY:YI dYWhX:hc:XfUk icb:U:a]b]a i a icZ*!, igci fWrg Zcf:U:fYgYUfW.:YggUm!:]"Y" ci fbU:Ufh]WYgžVcc_:W.UdhYfgž:gYWhcbg:cZVcc_g":\DhYfbYhfYgYUfW.:fY"[":K]_]dYX]UL:g\ci :X:VY:_Ydhhc:U a]b]a i a if&\$i ia Ul L":@Y[Ubhc:k]::WcbhU]b:YI hfU:fYUX]b[:Zcf:YUW.:k YY_fb:hcd]Wgc:mci :k]::\Uj Y'd:YbhncZ W fUhYX:a UhYf]U:hc:XfUk :cb"

 $\label{eq:continuity} F YZYfYbWJb[...9ggUmg'a i ghVY'Z\ ``mifYZYfYbWX'!'ZcchbchY#V]V`]c[fUd\mCF']b!hYl h#fYZYfYbWY``]ghfk Ufj UfX cf'5D5<math>z$ cbY'cZh\Y'gmghYa g'k \]W\']bWi XYg'dU[Y'bi a VYf'fYZg']b']b!hYl hfYZk \YfY'fY`Yj UbHz''

 $BchY. \ \ \ Z fh Yf Ygg Um [i] XUbW k] ``VY' dfcj] XYX'] b ``YW fYg Z'] b'h hcf] U'g' UbX' cb'A ccX`Y'] b'h Y Ygg Um Zc`XYf gYW f cb''$

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM & gYM & gYY & fb & gYY &$

5ggYgga Ybh' . 7`Ugg hYgh

GhUfh'XUhY.

8 YHJ] g.

 $Gh XYbhg g]hU WUgg hYgh]b hY ZbU h hcf]U hYhYghk] ```]bW XY a i hJd Y W.c]W UbX U g ch YggUmUbX k] ``hYgh_bck YX[Y cZh Y Wci fqY Ug U'k \c`Y"$

Gh XYbhg k]``fYWY]j Y'U'[fUXY'j]U'A ccX`Y'k]h\]b hYb XUmg cZg]hh]b['h\Y'hYgh'

5XX]h]cbU`XYhU]`g.

: i fh\Yf`XYHJ]`g`hc`VY`dfcj]XYX`cb`A ccX`Yž]b`h hcf]U`g`UbX`]b`h\Y`K YY_`%\$``YWh fY''8c`[Yh]b`hci W\`k]h\NcfU`k Y```VYZcfY`h\Y`hYgh]Zmci `\Uj Y`UbmgdYV]U``fYei]fYa Ybhg'': cf`ghi XYbhg`k \c`\UhY`hYghgž`YhNcfUfYUggi fY`mci .`h\]g`hYgh]g`bchXYg][bYX`hc`di b]g\`mci ž'Vi hhc`YbWci fU[Y`mci `hc`fYZYWhcb`k\Uhmci Yb'cmYX``YUfb]b[`UVci hfcfX]Xbfil\z`hc`h\]b_`UVci hh\Y`WcbWrdhg`mci `Zci bX`a cghi gYZ\``UbX`h\Y`k]XYfUdd`]WUV]`]hmcZh\Y`Wci fgY"

Gi Va]gg]cb bchYg. ⇒ WUgg UggYgga Ybh

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

 $H\Y'WcbhYbhZcf'YUWX'k YY_`ffYUX]b[g^{z'}XcW'a YbhUf]Yg`UbX`dcXWUghgŁ'k]```VY``]ghYX`cb`A ccX`Y"`5```fYUX]b[gk]```VY`UjU]`UV`Y`j]U'@Y[Ubhc''cf'hXfci[\``]b_g`cbA ccX`Y"$

FYWca a YbXYX FYgci fW/g

7 ci fgY 9j U`i Uhcb UbX 8Yj Y`cda Ybh

H\]g`]g`U`\][\`dYfZcfa]b[`WcifgY`]b`hYfag`cZgh XYbhZYYXVUW_žVihk Y`aU_Y`gifY`hc`aU_Y`bYk`W\Ub[YgYjYfmmYUf`hc`_YYd`h\Y`WcifgY`ZfYg\`UbX`fY`YjUbhUbX`hc`fYgdcbX`Zfca`hYff]ZJWgi[[Ygh]cbg`Zfca`gh XYbhg" KY`kY`WcaY`]bZcfaU``ZYYXVUW_`UhUbmghU[Y`Xif]b[`h\Y`hYfa`UbX`YbWcifU[Y`Ug`aUbmgh XYbhg`Ugdcg])V`Y`hc`dfcj]XY`ZcfaU``ZYYXVUW_`j]U`Am9IdYf]YbW'"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa] W + b Zcfa Uhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/