

# **ARTS2383**

9h\]Wg. H\YcfmUbX DfUW]WY

# 7ci fgY'Cj Yfj ]Yk

### GHJZ7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUh]cb	D\cbY
>YfYa mA cgg	å cgg4 i bgk "YXi "Ui		A 6' &&	- ' , ) &' ) +

#### @YWhi fYfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D∖cbY
8Ub]Y``6i f_Yth	XUb]Y`"Vif_Yhn4 ibgk"YXi"Ui	6mUddc]bha Ybh	A 6' %	-',)&\$'(

#### GWXcc``7cbHJWh'±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@cWUhjcb. GVXcc``CZJVYžAcfjYb.6fckb.6i]X]b[ž@YjY`.&ž&),

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D\cbY. 'Ž\*%'&'-',)'%\*,%

: UI . 'Ž \* %'&'-', ) ', +\$)

9a U]`. \<u>\U`4 i bgk "YXi "Ui</u>

# 7ci fgY 8YHJ] g

#### 7 fYX]hDc]bhg\*\*

#### Gi a a UfmcZh\Y 7ci fgY

#Dirk ]g'Wci fgY'mci 'k ]```Wcbg]XYf'gca Y'cZh\Y'a cghdfYgg]b['dfUWf]WD``Yh\]WD``]ggi Yg'ZJWWX'VmgcW]Yhm'UbX Vm]bX]j ]Xi U``dYcd`Y']b'h\Y]f'Yj YfmXUm`]j Yg"'K Y'k ]```UbU`mgY'h\Y'Yh\]WD``Uf[i a Ybhg'VY\]bX']ggi Yg'gi W\`Ug fVi hbch`]a ]hYX'hck'W]a UhY'W\Ub[Yž]bhYfbUh]cbU``U]Xžh\Y'a Uf\_YhzbYk 'hYW\bc`c[]Yg'cf'X]gUV]`]hm''H\Y Wci fgY'k ]```]bhfcXi WV'mci 'hc'\_YmWcbWdhg']b'Yh\]Wg'gi W\`Ug'fYgdcbg]V]`]hmž'\ gh]WV'UbX'k Y``!VY]b["'Mci 'k]`` VY'YbWci fU[YX'hc'YI Ua]bY'mci f'ck b'Yh\]WD``VY`]YZg']b'h\Y'`][\hcZh\YgY'WcbWdhg'UbX'Wcbg]XYf'h\Y'fc'Yh\UhYh\]Wg'WUb'd'Um]b'Yj YfmXUm`]Zy'"

#### 7 ci fgY @YUfb]b[ 'Ci hWca Yg

- % = XYbhjZmUbXYId`U]b`hXYaU]bYhX]WU``]ggi Yg`hXUhUf]gY`kXYb`Wcbg]XYf]b[`hcd]Wg`]b`Udd`]YX`YhX]WgUbX`ci f`]bX]j ]Xi U``Wc``YWf]j Y`cV`][Uhjcbg`hc`dfcXi WY`YZZYWf]j Y`WXUb[Y"
- &" 5dd`md\]`cgcd\]WU``a Yh\cXg`hc`Yl Ua ]bY`UbX`Yj U`i UhY`j Uf]ci g`X]a Ybg]cbg`cZUb`]ggi Y`dYffU]b]b[hc`hcd]Wg`]b`Udd`]YX`Yh\]Wg"
- ' " 8 ]gh]b[ i ]g\`UbX`Yj U`i UhY`ZfUa Yk cf\_g`Zcf`dYfgcbU``UbX`Wc``YWh]j Y`XYW]g]cb`a U\_]b[ i]b`fY`Uh]cb`hc hcd]Wg`]b`Udd`]YX`Yh\]Wg"
- ("FYUX'UbX'Wca dfY\YbX'a U'cf'd\]cgcd\]WJ``HYI hgž\]ghcf]WJ``UbX'WcbHYa dcfUfmžYI Ua ]b]b[dYfgcbU``UbX'Wc``YWfj Y'a cfU``cV`][Uh]cbg"
- ) " K f]hY 'WYUf'UbX 'Wc[YbhUbU'mgYg'cZj Uf]ci g']ggi Yg']b'a cfU'd\]'cgcd\m'

#### HYUWX]b[ 'GhfUhY[]Yg'

## 5ggYgga Ybh

#### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'tUg_	K Y][ \h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
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±b!WUgg"hYgh"	%\$ı	&-#\$-#&\$&\$`\$\$\$`5A	%ž&ž)
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#### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh%: ]bU`YggUm

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

Ghi XYbhgʻk f]hYʻUʻ&ž) \$\$`k cfX`YggUmʻGhi XYbhgʻk ]``fYWY]j Yʻ]bX]j ]Xi U``Wca a Ybhg''`H\]gʻ]gʻh\YʻZ]bU`UggYgga YbhZcf`UhhYbXUbWYdi fdcgYg''

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYH & fb & gYYH & fb &$ 

5ggYgga Ybh& ±b!WUgg hYgh

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g.

Ghi XYbhgʻWca dʻYhYʻUʻhYghik ]h\ʻg\cfhiUbgk Yfʻei Ygh]cbgžYei ]j UʻYbhihcʻWUʻʻʻ) \$\$`k cfXgʻ]bʻhchU`''Ghi XYbhgʻk ]``fYWY]j YʻUʻa Uf\_ʻUbXʻZYYXVUW\_ij ]Uʻ]bʻWUggʻX]gW gg]cb''```

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

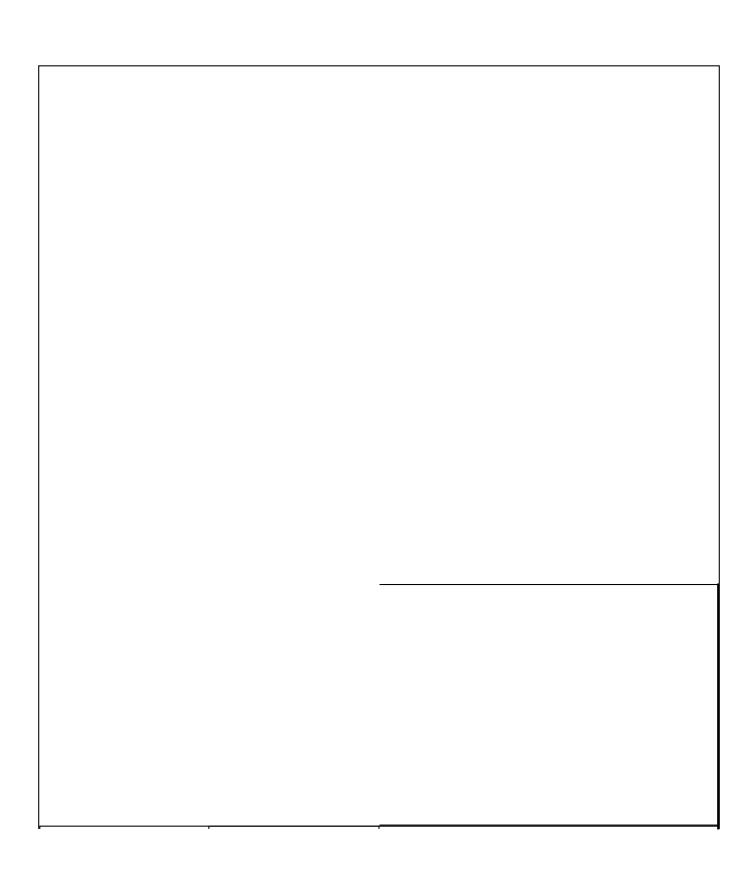
5ggYgga Ybh' . 9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

8YW]`q.

Gh XYbhg k f]hY U %) \$\$ k cfX YggUm Gh XYbhg k ]``fYW]j Y ]bX]j ]Xi U Wca a Ybhg

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb \end{tabular} $$ h' \end{tabular} $$ fb \end{tabular} $$ h' \end{tabular} $$ f' \end{tabular} $$ h' \end{tabular} $$ f' \end{tabular}$ 



K YY_', .`&Bcj Ya VYf'!'* Bcj Ya VYf	K YY_`, `!`; `cVU``Dcj Yfhm '5'X]gW gg]cb'cZh\Y Yh\]WU``]ggi Yg'gi ffci bX]b[ '[`cVU``dcj Yfhmž ]bWi X]b[ 'h\Y'cV`][ Uh]cbg'h\Uhk Y'YUW\'\Uj Y'hc fYgdcbX'hc'gi W\'Wf]gYg'Vm[]j ]b[ 'U]X"
K YY_''- 'Bcj Ya VYf'! % 'Bcj Ya VYf	K YY_'- '! 'K Uf 'UbX 'Hcfhi fY. '5 'Vf]YZX]gW gg]cb 'cZ h\Y 'Yh\]Wg 'cZk Ufž 'Zc``ck YX 'VmUb ']b! XYdh\ Wcbg]XYfUh]cb 'cZk \Yb '! ']ZYj Yf '! 'hcfhi fY 'WUb 'VY a cfU``m'^ gh]Z]YX"
K YY_`%\$.`%*`Bcj Ya VYf'! &\$`Bcj Ya VYf	K YY_'%\$'!'H\Y'9h\]\W\`'HfYUha YbhcZ'5b]a U`g.'5 \Wbg]XYfUh]cb'cZk\Uh!']ZUbm!']bhYfYghg'bcb!\i a Ub Ub]a U`g'a][\h\Uj Yž'UbX'k\Uhfc`Y'h\YgY']bhYfYghg

# FYgci fWyg

DfYgWf]VYX F Ygci fWyg

Bch'Uj U]`UV`Y

FYWca a YbXYX FYgci fWyg

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

: YYXVUW\_.

Ghi XYbhg`k ]```VY`dfcj ]XYX`k ]h\`U`a Uf\_`Zcf`h\Y`]b!WUgg`hYgh'`E i Ygh]cbg`k ]```VY`g\cfh'UbX`]b`WUgg ZYYXVUW\_k ]```VY`[]j Yb"`Ghi XYbhg`k ]```fYWY]j Y`]bX]j ]Xi U``Wca a Ybhg`UbX`U`[ fUXY`Zcf`h\Y`YggUmWca dcbYbhg cZh\Y`Wci fgY"`H\Y`[ fUXY`k ]```VY`Wcbg]ghYbhk ]h\`h\Y`YggUmfi Vf]Wdfcj ]XYX`]b`h\]g`XcW`a Ybh'

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY