

ARTS3270

FYZYW]b['cb'<]ghcf]Yg'UbX'<]ghcf]Ubg.'7 UdghcbY

HYfa 'H\fYY''##'&\$&\$

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
B]Wc`Ug`FUgaiggYb			' %('A cfj Yb 6fck b	

GWXcc`7cbHJWh = bZcfa Uh]cb

GWkcc`cZ<iaUb]h]Yg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

 $C\,dYb]b[~<\,ci~fg.~A~cbXUm!~:~f]XUm\ddot{z}-Ua~!~)~da$

D\cbY.'Ž*%'&'-',)'%*,%

: UI . Ž*%'&'-' ,) ', +\$)

9a U]`. \<u>\U`4 i bgk "YXi "Ui</u>

7ci fgY⁻8YHJ]^{*}g

7 fYX]h'Dc]bhg '* '

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. <]ghcfmi

 $\pm 0.1 \text{ h.}] g. \text{WJdghcbY.} \text{Wci. fgY.} \text{mci. i.k.}] \text{````} \text{`c.}] b. \text{U```h.} \text{Y.} \text{c.h.} \text{Yf.} \text{ghi. XYbhg'a. U'c.f.} \text{b.} [\text{``]b' <]} \text{ghc.fm}] b. \text{U'k. YY_`mgYa.}] \text{bUf.} \text{XYg} [\text{bYX'hc.dYfZYWhiubX'fYZ$

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8i Y [.] 8UhY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
±b'WUgg'Ei]nnYg'	%) ।	Bch5dd`]WUV`Y	% <u>*</u> & <u>*</u> Ž)
AigYia j]g]hUbX'YggUm	&\$ı	%*#%\$#&\$&\$`\$(.\$\$`DA	%ž&Ž Ž)
F YgYUfW. '9ggUm'	*)।	&' <i>#%#</i> &\$&\$`\$(.\$\$`DA	%ž&ž(ž)

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5ggYgga Ybh% 🕏 WUgg E i]nnYg

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

 $7 `Ugg`X]gW `gg]cbg`k]```VY`gHfi WhifYX`Ufci bX`U`gYf]Yg`cZei YgHjcbg`]bHYffc[UHjb[`H\Y`Ugg][bYX`fYUX]b[g"] bYX`fYUX]b[g'] bYX`fYUX]bYX`fYUX bYX`fYUX bYX`fYUX bYX`fYUX bYX`fYUX bYX`fYUX bYX`fYUX$

 $9 \ UW \ gYa \]b \ Uf \ fV \ Zfghzk \]``VY \ Zc ``ck \ YX \ VmU \ g\ cfh \ Ubgk \ Yf \ ei \]n \ hYgh]b \ [\ 'mci \ f'i \ bXYfghUbX]b \ [\ 'cZh\ Y \ Zh \ VmU \ gYm \ Mbghy \ dhg \ X]gW \ ggYX \]b \ WUgg"$

 $; fUXYg"k]```VY'dfcj]XYX'j]U"A ccX`Y'UbX'kYfi`X]W'gg'h\Y'WcffYWhUbgkYfg'Uh'h\Y'VY[]bb]b[~cZh\Y'Zc``ck]b[$

 $\label{thm:cb-dynamics} $$YI \setminus [V] \cdot fV \cdot h \cdot cb^* fV \cdot h \cdot cb^* fV \cdot h \cdot gY^* \cdot \cdot h \cdot gY^$

 $\begin{tabular}{ll} Gi Va]gg]cb bchYg. H\g`Ugg][ba Ybh']g`i gi U``m'Wca d`YhYX`VmdU]fg`cZghi XYbhgžk \c`a i ghYUWk`gi Va]h H<9`G5A 9`YggUm \\ \end{tabular}$

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM &$

5ggYgga Ybh'' . 'F YgYUfVX '9ggUm

GHJfhXUhY. Bch5dd`]WUV`Y

5 https://bxubwww.Fyei]fya Ybhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj]Yk '`YWhi fY`fYWcfX]b[g"

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'ha YhUV'Y

H]a YtUV`Y

8 UHY	Hmd Y	7 cbhYbh		
K YY_'%'%('GYdhYa VYf'! % 'GYdhYa VYf	GYa]bUf	±btfcXiWt]cb."K\Uh]g`\]gtcfmž'UbX'k\Uh]g`\]gtcf]c[fUd\m3		
		H\Y`Z]fgha YYh]b[`k]``]bj c`j Y`\ci gY_YYd]b[`a UhhYfgž]bWi X]b[`Ub`YI d`UbUh]cb`cZh\Y`a U`cf`UggYgga Ybhg UbX`h\Y`XUhYg`Xi Y"`A cfY`]a dcfhUbhmžk Y`k]``Wca Y hc`VY`U`dfUWh]WU``UbX`h\YcfYh]WU``i bXYfghUbX]b[`cZ k\Uh`\]ghcf]Ubg`Xc`UbX`k\Uhih\Y`X]ZYfYbhk Umg`h\Ym Xc`]h!!`Wc``YWh]j Y`mžfi]ghcf]c[fUd\mfi!!`UfY"		
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K YY_'&.'&%'GYdhYa VYf'!	<ca cf_<="" td="" yk=""><td>7 ci fgY fYUX]b[g]b @Y[Ubhc</td></ca>	7 ci fgY fYUX]b[g]b @Y[Ubhc		
&) GYdhYa VYf	5ggYgga Ybh	±b!WUgg ei]n cb Ugg][bYX fYUX]b[
	GYa]bUf	DfcZYgg]cbU``\]ghcfmUbX`di V`]Wa Ya cfm		
		DfcZYgg]cbU``\]ghcf]Ubg'UfY'cZhYb'hfYUhYX'Vmh\Y di V`]WUg']j cfmhck Yf']bhY``YWi U'g'k \c'`UVci f'Uk Um cb'hf]j]U`hcd]Wg']b'h\Y'XYgYfj YX'cVgWi f]hmcZXi ghm UfW\]j Yg"'MYhh\Y'di V`]W!! Yj Yb'h\YgY'gUa Y'Wf]h]Wg !!'\UfVci f'\]ghcf]WU``VY`]YZg'h\Uh'UfY']a dcfh'Ubh'nc h\Ya 'UbX'k\]W\'h\YmYI dYWh'c'VY'gi ddcfhYX'Vm \]ghcf]Ubg'UbX'\]ghcf]WU``Yj]XYbWY'': fYei Ybhmh\YgY W\Yf]g\YX'\]ghcf]WU``bUffUh]j Yg'WcbZ]Whk]h\'k\Uh dfcZYgg]cbU``\]ghcf]Ubg'HU_Y'fgca Yha Yg i bWcbhfcj Yfg]U``mz'Ug'YghU\`]g\YX'ZJWh'		
		H\YgY`WcbZ]Wjb[`i bXYfgHUbX]b[g`cZh\Y`dUghcWWg]cbU``mWcaY`]bhc`h\Y`cdYbžYgdYWjU``mk\Ybghcf]Yg`Zci bXUhjcbU``hc`W``hi fU``cf`bUhjcbU``]XYbhjhmUfY`i dXUhYX`UWWcfX]b[`hc`dfcZygg]cbU``gHUbXUfXg`]bdi V`]WZcfUžgi W.`Ug`gWkcc``W`ff]W``U`cf`a i gYi aYI\]V]hg"`H\YmfU]gY`ei Yghjcbg`UVci hh\Y`\]ghcf]Ub'ggcWjU``fc`Y`UbX`Xi hjYg"`K Y`k]```X]gW`gg`h\Y`fc`Yg\]ghcf]Ub'g gcWjU``fc`Y`UbX`Xi hjYg"`K Y`k]```X]gW`gg`h\Y`fc`Yg\]ghcf]Ubg'd`Um]b`di V`]W]b`h\]g`gYa]bUfž`ZcW`g]b[`cbh\Y`Wcbhfcj Yfgmcj Yf`h\Y`9bc`U`; UmYI\]V]hjcb`Uhh\Y`Ga]h\gcb]Ub`A i gYi a `]b`%-)"`		
K YY_" ."&, "GYdhYa VYf"!	GYa]bUf	H\Y`]b!WUgg`ei]nnYg`k]```U`gc`VY[]b`h\]g`kYY_" <]ghcf]WU``gW\c`Ufg\]d`UbX`UXjcWU\m		
&°C WcVYf		13.2.1.2 3.4.2 5.3 12 557 679 5 757		

K YY_'(.') 'C WcVYf'!'- C WcVYf	5fY`\jgrcf]ubg`gi VYWirc Yh\jWuqcbcZthY]f`k cf_]b k\Uh]g`fYei]fYX Zcf`hY'di V'jWuqcbcZthY]f`k cf_]b dYYfIfYj]Yk YX Zcf`hY'di V'jWuqcbcZthY]f`k cf_]b dYYfIfYj]Yk YX Zcf`hYg`fY"["'ff ih\J" 'UbX'Wca d'YhY fYdcfibli cZdfja UfmYj]XYbWi*gcf fWgz*c[]WU Uf[i a YbtulicbzWjfuljcbcZ]bZi YbtjU`cf`chXfk]gY]a dcftUbhgYWcbXUfmgci fWgz3: cf`YI Ua d'Yz g\ci Xih\mdi V'jWmX]gW gg]ggi Yg`cb`mk]h\]b`h\Yj Xca U]b`cZgWc`UfmYl dYffigY3 A i ghih\YmWcbg]XYf h\Y`gcWjU`]a dUWicZh\Y]f`gWc`Ufmk cf_]b di V']g\]b[`cb`WffU]b`ei Ygf]cbg`UbXicf`i g]b[`WffU]b gci fWg3 C bWi*jh]g`di V']g\YxZXc`h\YmYUf`Ubm ch\Yf`fYgdcbg]y]jh\g`jb`XYgWf]V]b[`]h]b`di V']W j Ybi Yg`gi W\"Ug`h\Y Wc`i fffcca 3 H\Y`dc]bmYbX`cZgi W\"ei Ygf]cbg`\Ug`VYb Yl dYf]YbWYX`Vma YX]WU^\\]jghcf]Ubg`k cf_]b[`cb hc\UWWc**-b`h\]g`WUgg k Y k\]``Wcbg]XYf\]grcf]Ubgfi fc`Yg`]b`h\Y`nc\UwWc``jhj`Ujcb WugYg`cZh\Y`% - \$g UbX &\$\$\$g`UbX XXYVUh\`[yhr]fJbcfYh\]W' fYgdcbg]y]jh\Yg`nc`h\Y]f`gi VYWgz`Ui X]YbWigz`UbX dfcZYgg]cbU`Wca a i b]jh\Yg` 7ci fgY`fYUX]b[`g`]b`@Y[Ubhc -b!WUgg`ei]n cb`Ugg][bYX`Wc`i fgY`fYUX]b[Bc`WUgg`gYgg]cb'!`@UVci f`8 Um

		H\]g`k YY_`k Y`Wcbg]XYf`h\Y`k Umg`]b`k \]W\`\]ghcf]Ubg\Uj Y`i gYX`[YbXYf`UbX`gYI`Ug`U`a YUbg`cZ YI Ua]b]b['h\Y`Uggi a dh]cbg`h\Uha Uf[]bU`]gYž gi VcfX]bUhYžcf`fYbXYf`]bj]g]V`Y`h\Y`YI dYf]YbWYg`cZ]bX]j]Xi U`g`cf`[fci dg`]b`\]ghcf]c[fUd\m'`6Y[]bb]b[k]h\`h\Y`5a Yf]WUb`\]ghcf]Ub >cUb`K "'GWchfigi gi [[Ygh]cb`h\Uhi\]ghcf]Ubg`a i ghbcha YfY`mUXX k ca Yb`hc`\]ghcfmUbX`figh]ffik Y`k]``Wcbg]XYf`h\Y Yj c`i h]cb`cZ[YbXYf`\]ghcfm]b`h\Y`&\$h\`WYbh`fmUbX]hg`]a d`]WUh]cbg`Zcf`i g`Ug`\]ghcf]Ubg`]b`h\Y`dfYgYbh' 7 ci fgY`fYUX]b[g`]b`@Y[Ubhc
	5ggYgga Ybh	#o!WUggrei]nrcbrugg][bYXrWci fgYrfYUX]b[
K YY_', .'&'Bcj Ya VYf'!'* Bcj Ya VYf		#: Wogg Cr Jireb oggj[brx verigr rroxjb[

&\$`Bcj Ya VYf		<pre><]ghcfmži]b`Wcaacbik]h\acgh\iaub]h]Yg X]gW]d`]bYgži]g`h\Y`gi VYWhcZcb[c]b[idi V`]WUbX UWUXYa]WXYVUhY`UVci hh\Y`bYYX`hc`XYWc`cb]gY W]ff]W``U`UbX`fYgYUfW\dfUW]WYg" \dot\Y`Z]bU``gYa]bUf kY`k]``YI Ua]bY`h\Y`df]bW]d`Yg`cZXYWc`cb]nUh]cbž YI d`cfY`k\Uhh\Y`acj YaYbh\cdYg`hc`UW\]Yj Yž'UbX k]h\n\Y`\Y`d`cZfYWfbhgW\c`Ufg\]džX]gW gg`h\Y dcgg]V]`]h]Yg`UbX`Zi hifY`cZXYWc`cb]gYX`\]ghcfm'</pre>
	< ca Yk cf_	7 ci fgY fYUX]b[g]b @Y[Ubhc
	5ggYgga Ybh	±b!WUgg ei]n cb Ugg][bYX Wci fgY fYUX]b[

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FYWca a YbXYX FYgci fW/g

8 UHUVUgY#g`flYgd"`Zcf`h\Y`fYgYUfVX`YggUmEk]```VY`]blfcXi WYX`UbX`X]gW ggYX`]b`ci f`Z]fghgYa]bUfž`Ug`k]```h\Y A i gYi a `5gg][ba Yblf': i fl\Yf`]bglfi Wf]cbg`UbX`h\Y`gYhhcd]WZcf`h\Y`F YgYUfVX`9ggUmk]```VY`dcghYX`Vm K YY_'' "``=a UmdcghfY`Yj Ubh[i]XYgžg\cfhd]YWYg`cf``]b_g`]b`A ccX`Y"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

c``ck]b['ZcfaU``UbX']bZcfaU``ghiXYbhZYXVUW_``UghmYUfž`=fiY`hU_Yb`ghYdg`hc`YbgifY`h\Y`fYUX]b[``cUX`]g a cfY`a UbU[YUV`Yž`UbX`W\Ub[YX`h\Y`Z]bU``gYa]bUf`giVYWhhc`cbY`h\UhfYZYWhg`a cfY`WYUf`mcb`h\YX]gW]d`]bYfig`Z hifY"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hzil BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUv`Y`I gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY