

7ci fgY 8YW] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

H\Y`Wci fgY`dfcj]XYg'i ddYf`]bhYfa YX]UhY``Yj Y``hi]hJcb`]b`>UdUbYgY``Ub[i U[Y`ghi Xm'`G]a]`Uf`hc`h\Y`Wci fgY f\$Xj UbW'X`>UdUbYgY`5fz`U`dfYfYei]g]hY`Zcf`h\]g`Wci fgYz`mci `Wcbh]bi Y`hc`XYj Y`cd`mci f`]bhYfW``hi fU` Wca a i b]WUhJcb`g_]``g`]b`Ub`UXi `h`>UdUbYgY`gdYU_]b[`Ybj]fcba Ybhk\]`Y``YUfb]b[`a cfY`UVci h`>UdUb`UbX h\Y`>UdUbYgY`'Mci `U`gc``YUfb`bYk``>UdUbYgY`ghfi Whi fYgž`YI dfYgg]cbgž`UbX`j cWUVi `Ufm''±b`YUW\`hcd]Wz`mci `YUfb`h\Y`W`ffYbhgcW]c!W``hi fU``]ggi Yg`]b`>UdUb"`±b`j cWUVi `Ufm`YUfb]b[`z`mci ``YUfb`&\$\$`_Ub']"`H\YmUfY hUi [\hh\fci [\Zci f`gcW]U`#W`hi fU``hcd]Wg''

7ci fqY @YUfb]b[Ci hWca Yq

HYUWX]b[GlfUhY[]Yg

@YWi fYg`XY`]j Yf`gcWJU``UbX`W`hi fU``]ggi Yg`fY`UhYX`hc'h\Y`hcd]WcZh\Y`k YY_"'BYk `ghfi Wi fYg#[fUa a Ufg`UfY U`gc`YI d`U]bYX"'Nci `dUfh]WJdUhY`]b`dfUWfJWYg`cZbYk `YI dfYgg]cbgž'j cWJVi `Uf]Ygž'UbX`? Ub']`UbX`]bhYfUWfik]h\ZY``ck `ghi XYbhg`Uhh\Y``YWi fYg"'@YWi fYg`UfY`XY`]j YfYX`i g]b[`>UdUbYgY`hc`Ugg]ghmci `]b`XYj Y`cd]b[`h\Y]f `]ghYb]b[`UV]`]hJYg`UbX`VYWca]b[`ZJa]`]Uf`k]h\`bUhjj Y`>UdUbYgY`gdYU_YfgfihU`_"'H\Y`WcbhYbhk\]W\`mci `UfY bchUVY`hc`Wca dfY\YbX`]b`>UdUbYgY`a UmVY`YI d`U]bYX`]b`9b[`]g\"

Hi hcf]U`g`dfcj]XY`mci `k]h`cddcfh b]h]Yg`hc`dfUW]\W`]bhYfUWf]cbžfYUX]b[ž'UbX`k f]h]b[`]b`>UdUbYgY`Zc``ck]b[h\Y`]bZcfa Uh]cb`]bhfcXi \WX`Xi f]b[``Y\Wi fYg`UbX`dfcj]XYX`]b`h\Y`hYI \Wcc_"`\Mci fi`XYj Y`cd`i bXYfghUbX]b[UVci hh\Y`>UdUbYgY`dYcd`Yž\W`h fYž`UbX`gc\W]Yhm'`H\]g`]g`a UXY`h\fci [\`X]g\W gg]cbg`cb`U`hcd]\WcZh\Y \\diff{gY"'9a d\Ug]g`]g`U`gc`d`U\WX`cb`XYj Y`cd]b[`]bhYfU\Wi]cb`g_]``g`]b`Ub`UXi `h>UdUbYgY`gdYU_]b[Ybj]fcba Ybh''5`a YX]i a `cZ`Ub[i U[Y`]b`h hcf]U`g`]g`>UdUbYgY`]b`cfXYf`hc`XYj Y`cd`mci f`cfU``UbX`Ui fU` g_]``g"`\Mci `UfY`ghfcb[`mYb\Mci fU[YX`hc`i gY`>UdUbYgY`Xi f]b[`WUggYg"

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[CihWca Ygʻ5ggYggYX
:]bU`YI Ua]bUh]cb'	(\$1	: cfa U``9I Ua 'DYf]cX	%ž&Ž
GdYU_]b['HYgh'	&\$I	Bch5dd`]WUV`Y	%Z
Dfc YWi	(\$1	Bch5dd`]WUV`Y	%₹

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh%:]bU`Yl Ua]bUh]cb

 $\textbf{GhUfhXUhY}. \ \texttt{Bch5dd`]} \textbf{WUV`Y}$

@Yb[h\.'&\f

8 YW] g.

&!\ci f`Yl Ua `]b`h\Y`Zcfa U``Yl Ua]bUh]cb`dYf]cX"`H\]g`]g`h\Y`Z]bU``UggYgga YbhZcf`UhhYbXUbW'di fdcgYg" Bi a Yf]WU`a Uf_"`'=bX]j]Xi U``ZYYXVUW_k]``VY`[]j Yb`i dcb`fYei Ygh''

5XX]h]cbU`XYHJ]`g.

:]bU``YI Ua]bUhjcb`]g`\Y`X`Xi f]b['Z]bU``YI Ua]bUhjcb`dYf]cX"

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& GdYU_]b[HYgh

GhUfhXUhY. Bch5dd`]WUV`Y

8 YW] g.

9UWX gh XYbhk]``dYf7cfa 'U'fc`Y'd`Umk]h\'Ubch\Yf`gh XYbhž'W'U"')!* 'a]bi hYg'dYf'dU]f''K f]hhYb'2YYXVUW<u>"</u>"

5XX]h]cbU`XYhJ]`g.

 $\label{eq:continuity} $$GdYU_]b[\ 'h'gh]g'' Y'X'Xi \ f]b[\ 'k \ YY'' - 'h' \ hcf]U'''GYY'A \ ccX'Y'Zcf'XYhU]'g'' $$$

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5ggYgga Ybh' . Dfc YWh

GhUfhXUhY. Bch5dd`]WUV`Y

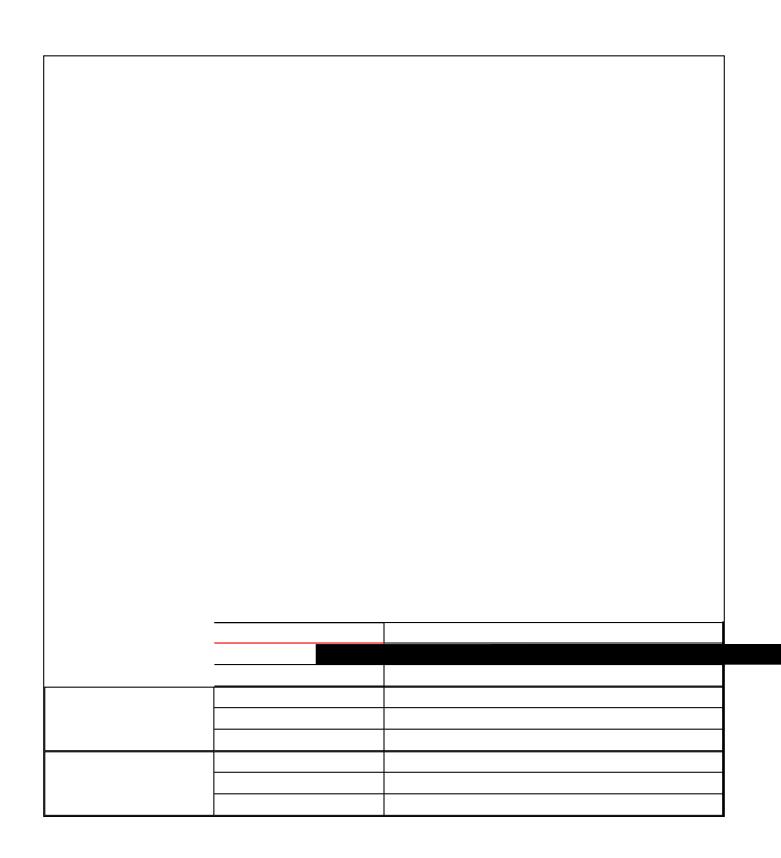
8YHJ]`g.

 $\label{eq:hamiltonian} $$HY''di fdcgY'cZh\]g''UggYgga Ybh']g'hc''YUfb'>UdUbYgY'W'hi fYž'`Ub[i U[Yž'UbX']bhYfW'hi fU''Wcaai b]WUh]cb g_]``g'VmdUfh]W]dUh]b[']b'U'j Uf]YhmcZUWhj]h]Ygz'Wcbg]gh]b['cZW'U''' $`a]bg$ YY_cb'Uj YfU[Y'''H\Y'UWhj]h]Yg]bWi XY'Ub']bX]j]Xi U'# fci d'dfYgYbhUh]cb'f('a]bg$ by XY'Ub']bX']bYfUWh]b['k]h\ >UdUbYgY'dYcd'Y#gdYU_Yfg'Uh'Ub'Yj Ybh\Y'X'Uh'I BGK "'Ghi XYbhg'k]`''h\Yb'fYZYWh'UbX'k f]hY'U'fYdcfh']b >UdUbYgY'f($$!)) $`W\UfUWhYfg#YdcfhL'cb'h\Y]f'YI dYf]YbW'g''K f]hhYb'ZYYXVUW_"$

5XX]h]cbU`XYhU]`g.

GYY'A ccX`Y'Zcf'XYhU]`g"

Hi fb]hb gYhb[. H\]g]g bchU Hi fb]hb Ugg][ba Ybh



F Ygci fWYg

DfYgWf]VYX F Ygci fWYg

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi V'YWfi |XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W±bZcfa Uhjcb