



HUMS1005

DYfgcbU`]gYX'9b[`]g\'@Ub[i U[Y'9b\UbWa Ybh

HYfa "H\fYY" ## "&\$&\$

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`k]``\Y`d`mci 'hc`Yb\UbW'mci f'9b[`]g\``Ub[i U[Y`g_]``g`]Zmci 'gdYU_'9b[`]g\`Ug U'gYWcbX``Ub[i U[Y"'=b`UXX]h]cbž'h\]g`Wci fgY`dfcj]XYg`mci 'k]h\`h\Y`g_]``g`hc`VY`Ub`Ui hcbca ci g`UbX]bXYdYbXYbh`YUfbYf`gc'h\Uhmci 'WUb`Wcbh]bi Y`hc`Yb\UbW'mci f'9b[`]g\``Ub[i U[Y`g_]``g`]bhc'h\Y`Z hi fY`hc UW\]Yj Y`mci f`UWUXYa]WUbX`dfcZYgg]cbU``[cU`g"'Mci 'k]``]XYbh]Zmk\]W\`UgdYWffgŁ'cZmci f'9b[`]g\`mci k ci `X``]_Y`hc']a dfcj Y`UbX`h\Yb`XYg][b'UbX']a d`Ya Ybhmci f'ck b'dYfgcbU``dfcYWhc'UVWca d`]g\`mci f [cU`g"'

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 5bU`mgY`UbX`]XYbhZmdYfgcbU```Ub[i U[Y`bYYXg
- &" 5dd`mhcc`g`UbX`fYgci fWyg`Zcf``Ub[i U[Y`Yb\UbWa Ybh
- ' "8Yg][b'UbX'WUffmcih'U'dYfgcbU`dfcYWhVUgYX'cb'h\Y'UbU`mg]g'cZdYfgcbU```Ub[iU[Y'bYYXg
- ("FYZYWicb"UbX"UggYgg"h\Y]f"ckb"dfc[fYgg"hc"gigHJ]b"dYfgcbU"`UbX"dfcZYgg]cbU```Ub[iU[YYb\UbWaYbh
-) " 7 ca a i b]\WUhY`\Wf]h]\WU``dYfgcbU```YUfb]b['ci h\care Yg`k]h\ 'dYYfg

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<! A G%\$\$) '\Y`dg`ghi XYbhg`hc`hU_Y`fYgdcbg]V]`]mmZcfYb\UbV\]b['h\Y]f`9b[`]g\``Ub[i U[Y`g_]``g`Vm\Y`d]b[
h\Ya ']XYbh]ZmUbX`UXXfYgg`h\Y]f`ck b`]bX]j]Xi U``bYYXg"'H\]g`]g`VYWUi gY`ghi XYbhg`Uhih\]g`UXj UbV\X`ghU[Y`cZ
9b[`]g\``Ub[i U[Y`UWei]g]h]cb`\Uj Y`X]ZZYfYbhbYYXgž`ghfYb[h\g`UbX`k YU_bYggYg"':]fa `mVUgYX`cb`gcW]c!
W`hi fU```YUfb]b[`h\Ycfmih\UhUf[i Yg`h\Uh`YUfbYfg'Webghfi Whh\Y]f`ck b``YUfb]b[`h\fci [\`a YUb]b[Z\`]
]bhYfUWl]cbg`fWZ''J m[chg_m%, *Lž'h\]g`gY`Z\X]fYWhYX`Ui hcbca ci g`Wei fgY`\Y`dg`ghi XYbhg`hc`Yb\UbW\'h\Y]f
9b[`]g\`g_]``g`VmZ]fghmUbU`mg]b[`h\Y]f`ck b`bYYXg/gYWebX`mXYj Y`cd]b[`dYfgcbU``dfc`YWhg/h\]fX`m
]a d`Ya Ybh]b[`h\Ya /Zci fh\`mYj U`i Uh]b[`h\Y]f`ck b`dfc[fYgg/UbX`Z]bU``mfYZYW]b[`cb`h\Y]f`ck b``YUfb]b[''
H\fci [\ci hh\Y'Wei fgYžghi XYbhg`k]```VY`[i]XYX`k]h\`fY`Yj Ubhh\Ycfmž'dfUW]WU``gi [[Ygh]cbg`UbX`i gYZ\`fYqci fW\q`]b`Vch\``YWfi fY`UbX`h hcf]U``qYqq]cbq"</pre>

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@Yb[h\.'%) \$\$"k cfXg

8YHJ]`g.

Ghi XYbhg`XYg][b'h\Y]f`ckb'dYfgcbU``dfcYWhc'UXXfYgg'h\Y]f`ckb'9b[`]g\``Ub[i U[Y'bYYXg"'H\YmUfYfYei]fYX'hc'gdYW]Zmh\Y]f`dYfgcbU``dfcYWh[cU'žVUW_[fcibXžaYh\cXžfYgcifWygždfc[fYgg'aYUgifYaYbhžh]aY`]bYžgW\YXi `Y'UbX'Ubh]W]dUhYX'cihWcaYg"'H\YmbYYX'hc'XYjY`cd'h\Y]f`DD8'VUgYX'cb'fY`YjUbh`]hYfUh'fY'cZh\Y'Z]Y`X'WcbWfbYX"'H\Y'kcfX``]a]h]g'%)\$\$"

Ghi XYbhg fYVW]j Y k f]hhyb ZYYXVUW_VUgYX cb fi Vf]Wg"....

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b"gYh]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"gh XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \end{tabular}$

5ggYgga Ybh& Dfc YWh`c[Vcc_UbX 'ci fbU`

GhUfh'XUhY.

8YHJ]`g.

 $Ghi XYbhg`fYWcfX`hXY]f``YUfb]b[``UWh]j]h]Yg`UbX`_YYd`fYZYWh]j Y``cifbU``cb`U`fY[i``Uf`VUg]g`Ug`d`UbbYX`]b`hXY]fDD8"`H\YmUfY`fYei]fYX`hc`gdYbX'` $`\cifg`cf`acfY`Zcf`hXY]f`dYfgcbU``dfc`YWh'$

Gh XYbhg fYWr]j Y k f]hhYb ZYYXVUW_VUgYX cb fi Vf]Wg"

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5ggYgga Ybh' . CfU dfYgYbhJh]cb

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@Yb[h\. '%s'a]bi hYg

8 YW] g.

Ghi XYbhgʻdfYgYbhhkY]f``YUfb]b[`\][\`ngʻZ̄ca ˈhkY]fʻdYfgcbU``dfcYWhZcfʻUVci h%\$`a]bi hYgʻhcʻ]bgd]fY dYYfgʻhcʻWcbh]bi YʻhcʻYb\UbWYʻhkY]f``Ub[i U[Yʻg_]``gʻYj YbʻUZhYfʻWca d`Yh]b[ˈhkYˈWci fgY"ˈH\YmUfYʻfYei]fYXʻhc gi Va]hUbʻUVghfUWhfUVci hi%) \$`k cfXgŁʿUVci hik\Uhih\YmUfYʻ[c]b[ˈhcˈdfYgYbhUh]cbʻXUmʻʻGhi XYbhgʻfYWY]j Yʻk f]hhYbʻZYYXVUW_Z̄ca ˈhi hcfʻcbʻUVghfUWhgʻfl%\$i ŁʻUbXʻZ̄ca ˈdYYfgʻcb dfYgYbhUh]cbʻfl%\$i ŁʻVUgYXʻcbʻfi Vf]Wgʻ"

5XX]h]cbU`XYhU]`g.

5 VghfUWgʻUfYʻXi YʻcbʻHi fb]h]bʻVmʻ‰da žʻA cbXUm-ʻBcj Ya VYfʻfK YY_'-Ł"ʻDfYgYbhUh]cbgʻUfYʻXi Yʻ]bʻh\Y WUggʻcZh\Y``Ughik YY_"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYH & fb & gYY & fb &$

5ggYgga Ybh (. FYZYW)j Y YggUm

GHJfhXUhY.

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8YW]`q.

 $Ghi \ XYbhg`UbU`mhJWU``miUbX`Wf]hJWU``mfYZYWhicb'h\Y]f``YUfb]b[\ YI \ dYf]YbW'`cZ]a \ d`Ya \ YbhJb[\ 'h\Y]f`ck \ b'dYfgcbU` dfc YWhUbX'di \ hUWX]Yj \ UV`Y`UWhJcbg`]b'U'd`Ub'Zcf'h\Y'bYI h`YUfb]b[\ 'ghU[Y"'H\Y'k \ cfX``]a]h]g`&$$$"$

 $\label{eq:charge} Gh XYbhg fYWM j Y k f]hhYb ZYYXVUW_VUgYX cb fi Vf]Wg" H jg jg h Y Z]bU UggYgga YbhZcf UhhYbXUbW di fdcgYg"$

5 https://bxubwww.Fyei]fYaYbhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`m'YbWc'i fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk'``YWhfY'fY'WcfX]b[g''' } \\$

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'h]a YhUV'Y

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8 UhY	HmdY	7 cbhYbh		
K YY_'%'%('GYdhYa VYf'! %'GYdhYa VYf	@YWi fY	#blfcXi Wljcb hc D5 A cXY ž C dYb@YUfb]b['d`Uhrcfa UbX WcbhYI hUbU mg]g Ug k Y `` Ug X]U[bcgh]W UggYgga Ybhhcc g		
	Hi hcf]U`	7 cbHYl h'UbU`mg]g:Zcf'h\Y'dYfgcbU`'dfcYWh		
K YY_'&.'&%'GYdhYa VYf'! &)'GYdhYa VYf	@YWi fY	→YbhJmb['U'a]gg]b['d]YWY]b Wca a i b]WUhJcb/ YI d`]Wh_bck 'YX[Y / ']a d`]Wh_bck 'YX[Y /]bHcXi WJ dYfgcbU`'dfc YWhXYg][b'fDD8Ł		
	Hi hcf]U`	GYMjb['id'U'[cU``Zcf'h\Y'dYfgcbU``dfcYWh		
	C b`]bY'5Wfjj]lm	8 Yj Y`cd]b['Ub'Y!dcfhZc`]c		
K YY_" .'&, 'GYdhYa VYf'! &'C WcVYf	@YWhi fY	8 Yj Y`cd]b['U'dYfgcbU`'dfc'YWY`Ub[i U[Y'`YUfb]b[fYgci fWg		
	Hi hcf]U`	8 fuZr]b['U'dYfgcbU''dfcYWiXYg][b'fDD8Ł		
K YY_'(.') 'C WhcVYf'!'- C WhcVYf	@YWi fY	@Ub[i U[Y``YUfb]b['a Yh\cXgžfYgci fWfg'UbX dfc[fYgg'a YUgi fYa Ybh		
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K_YY_^).^%&^CWhcVYf^!^%*	@YWhifY	FYZYW¶jY'6ifbU`]b[/Yach]cbg'UbX``YUfb]b[
C WcVYf	Hi hcf]U`	8]gWgg]cb'cb'h\Y'Zcif'hYbXYbV]Yg		
K YY_'*.'%-'CWcVYf'!'&' CWcVYf	C b`]bY'5W]j]hm	: YUhi fYgʻcZUWUXYa]WX]gWti fgY/dfcYWhUbXʻh]a Y a UbU[Ya Ybh		
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K YY_"+."&*"C WhcVYf"!" \$	@YWn fY	5 VglfUWiUbX fYZYWlj Y YggUmk f]ljb[
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K_YY_"%\$."%* "Bcj Ya VYf"!	DfYgYbHJhjcb	@YUfb]b[::Ygh]j U`!_YmbchY`gdYYWXYg		
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Fygci fWyg

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FYWca a YbXYX FYgci fWYg

 $\label{eq:cc_problem} 6\texttt{cc}_\texttt{'-'9ggYbh]U\texttt{'g'cZ9ggUmK}} f]\texttt{h}\texttt{b}\texttt{[.'K \ UhA \ Uf_Yfg'@cc_': cf'f&\$\%+$L}}$

6cc_'-'@YUfbYf'9b[`]g\.'5'HYUW\Yff@'; i]XY'hc'+bHYfZYfYbWY'UbX'Ch\Yf'DfcV'Yag'f&\$\$\lambda

Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX'dfYgYbh]b['h\Ya 'Ug`mci f`ck b"'=h\Wb'hU_Y`a Ubm Zcfa qžZfca 'XY`]VYfUhY'W\YUh|b['hc`UWW]XYbhU`mWcdm]b['Zfca 'U'gci fW'k]h\ci h'UW_bck 'YX[Ya Ybh'

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'] Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`h\I hcf']XYU'k]h\ci hUW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfc'YWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_jb['k]h\ 'ch\ Yfg'Vi h'dUgg]b['cZZh\ Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\ Yf'ghi XYbhVYZcfY'h\ Y 'Xi Y 'XUhYzcf'Zcf'h\ Y' di fdcgY'cZh\ Ya d`U[]Uf]g]b['Uh'Ubmh]a Yz'dUm]b['Ubch\ Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_z'ghYU']b['cf'UWei]f]b[Ubch\ Yf'dYfgcb'g' UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d`YhY'Ubch\ Yf'dYfgcb'g' k cf_'cf'gYY_]b[dUma YbhZcf'Wca d`Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[ˈh\Y`″gYWcbXUfm″gci fW/Zfca ik \]WX`_bck`YX[Y`cZh\Ya`\UgʻVYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł`gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWhfYZYfYbWb[dfUWhWyg.

- DUfUd\fUq]b[žgi a a Uf]q]b[žYggUmk f]h]b['UbX'h]a Y'a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhff]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[`7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Ghi XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``h]a Y`a UbU[Ya Ybh']g`Ub`]a dcfhUbhdUfhcZghi XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`h]a Y`a UbU[Ya Ybh'`Ghi XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'h]a Y`Zcf fYgYUfW\z`XfUZh]b[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fWg`]b`dfYdUf]b[`U```UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@&9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK"`9@&9`]g XYg][bYX`hc`]bHfcXiWY`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcif

- i bXYfghUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'ghi XYbh'Uhil BGK
 VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hžI BGK 'Ghi XYbh'7cXY'cZ7cbXi Wh'UbX'5WW'dhUV`Y'I gY'cZ I BGK '=7 H'F Ygci fWfg'Dc`]Wh