# **ARTS3991**

5 fhg 'UbX'GcW]U`GW]YbWYg'7 UdghcbY

Gi a a Yf "# 8\$&%

# 7ci fgY<sup>-</sup>Cj Yfj ]Yk

## GHJZ7cbHJWh8YHJ]`g

## 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
>Ua Yg?YUŊb[	^*_YUŊb[4 i bgk "YXi "Ui	-	F cca <sup>*</sup> &, \$ž A cj Yfb 6fck b	

## GWXcc``7cbHJWhi±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b[ '<ci fg. 'A cbXUm'! : f]XUmž - Ua '! ') da

D\cbY.'Ž\*%'&'-',)'%\*,%

: UI . 'Ž \* %'&'-' , ) ', +\$)

9a U]`. '<u>\U`4 i bgk "YXi "Ui</u>

# 5ggYgga Ybh

 $Gi \ [\ [\ YghYX`fYgYUfW.YggUmhcd]Wg`k\ ]```VY`dfcj\ ]XYX`cb`A\ ccX`YžVi\ hid`YUgY`ZYY``ZfYY`hc`[\ Yh]b`hci \ WX`k\ ]h\`a\ Y\ g\ci``X`mci \ k \ Ubhhc`hk\ YU_'h\Ya \ `cf`XYj\Y`cd`mci \ f`ck\ b"$ 

5 bm/Wcbj Ybh]cbU``UWUXYa ]WfYZYfYbW]b[ 'gmghYa ']g'UWWYdhUV`Yž'gc ``cb[ 'Ug'mci 'Udd`m']hWcbg]ghYbhmUbX WcffYWhm'

## 5ggYgga Ybh'HUg\_g

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
GYa]bUffYZYWjcb	&\$ı	&\$#\$%#&\$&%`%%) - `DA	' ž'(
F YgYUfWX 'YggUm'	) \$1	&-#\$%#&\$&%`%%) - `DA	%ž`&ž`'
HU_Y!\ca Y`Ugg][ ba Ybh	' \$1	%#\$%#&\$&%`%%) - `DA	%ž <sup>:</sup> &ž <sup>:</sup> (

## 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% GYa ]bUf fYZYWh]cb

 $\textbf{GhUfhXUhY}. \ \texttt{Bch5dd']WUV'Y}$ 

@Yb[h\.

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. '&ž) \$\$ k cfXg

8YHJ]`g.

5 Wf]h]WU`z`UbU`mf]WU`z`UbX`g\W.c`Uf`mU\WUXYa ]WYggUmcZ&z) \$\$`k cfXg": YYXVU\\\\_i ]U`]bX]j ]Xi U``\\cappa a Yb\rg"

#### 5XX]h]cbU`XYhU]`g.

Gi [[YghYX`YggUmhcd]Wg`UbX`U`XYhU]`YX`fi Vf]Wk]```VY`a UXY`Uj U]`UV`Y`cb`A ccX`Y"`D`YUgY`ZYY``ZfYY`hc XYj Y`cd`mci f`ck b`]b`Wcbgi `hUh]cb'k]h\`a Y"`5bmWcbj Ybh]cbU``UWUXYa ]WfYZYfYbW]b[`gmghYa `]g`UWWdhUV`Y gc``cb[`Ug`mci `Udd`m]h'Wcbg]ghYbhm"

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b"gYh]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"gh XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \end{tabular}$ 

5ggYgga Ybh' . HU\_Y!\ca Y Ugg][ ba Ybh

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`q.

5 Wf]r]WU``UbU`mg]g`cZUgg][bYX`Ufr]WYg`fl%2&)\$ k cfXgl:":YYXVUW\_j ]U`]bX]j ]Xi U``Wca a Ybhg"

#### 5XX]h]cbU`XYhU]`q.

5ft]WYg'Zcf'Wf]t]WU`'UbU`mg]g'UbX'th\Y'YggUmei Ygt]cb'UbX'fi Vf]Wk]`''VY'a UXY'Uj U]`UV`Y'cb'A ccX`Y'Ug'gccbUg'th\Y'Wci fgY'Wca a YbWYg"

 $\label{thm:limit} \begin{array}{ll} \mbox{Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ ba Ybh]g"gi Va ]hhYX"h\fci [\ \ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a ]"Uf]hmfYdcfhg" \\ \end{array}$ 

## 5 https://bxubwww.Fyei ]fya Ybhg

D`YUgY`bchY`h\Uh`YWh fY`fYWcfX]b[g`UfY`bch'Uj U]`UV`Y`Zcf`h\]g`Wci fgY"`Ghi XYbhg`UfY`ghfcb[`mYbWci fU[YX`hc UhhYbX`U```WUggYg`UbX`WcbhUWh\Y`7 ci fgY`5i h\cf]hmhc`a U\_Y`U`hYfbUh]j Y`UffUb[Ya Ybhg`Zcf`WUggYg a ]ggYX"

# 7ci fgY'GWXYXi `Y

#### J]Yk 'WUgg'ha YHUV'Y

## H]a YHJV`Y

8 UhY	HmdY	7 cbhYbh
( '>Ubi Ufm!', '>Ubi Ufm	GYa ]bUf	GYa ]bUf %f( '>Ubi UfmŁ '7 ci fgY ]blfcXi Wh]cb / h\Y'cf][ ]bg'cZK YghYfb'gW]YbWY'UbX'd\]`cgcd\m  FYUX]b[? Uf 'DcddYfz'f&UW_'hc'h\Y'DfY!GcWfUh]Wgfi f%-) - Ł"  \[ \daggerup UXX]h]cb'hc'ZJa ]`]Uf]g]b[ 'ci fgY'] Yg'k ]h\'h\Y gYa ]bUf'Zcfa Uh'UbX'Wci fgY'UggYgga Ybhgz'k Yfi VY[ ]b'h\Y'Wci fgY'Vmh\]b_]b[ 'h\fci [\'h\Y'cf][ ]bg'cZ k YghYfb'd\]`cgcd\m'UbX'gW]YbW'''H\]g'a ca Ybhz k \Yb'a mh\c`c[ ]Yg'k YfY'X]gWUfXYX']b'ZUj ci f'cZ fYUgcbz'a ][ \h\Y'h\ci [\hcZUg'h\Y'VY[ ]bb]b[ 'cZh\Y W]h]WU''ffUX]h]cb'UbXz'Ug'gi W\Z'Ub']XYU'cZYXi WUh]cb h\Uh'\Ug'dYfg]ghYX'h\fci [\'hc'h\]g'XUm'

GYa ]bUf	GYa ]bUf" 'ff' '>Ubi UfmŁ 'H\Y'di fdcgYg'cZ YXi WUh]cb
	FYUX]b[7":D":Gbck ž:ffH\Y:Hkc:7i `hi fYgfz:fl%)-Ł/K ":9":A UggYmz:f7":D":Gbck:UbX:h\Y:Hkc:7i `hi fYgz G]I hm:MYUfg:@UhYffif&\$% Ł"
	#b 'h\]g 'gYa ]bUf 'k Y 'k ]``X]gW gg 'h\Y '6f]hjg\ 'gWjYbhjgh UbX 'bcj Y`]gh7 "D" Gbck fig Wcbhfcj Yfg]U``UbX ]bZi YbhjU`'%-) - ``YWi fYžfill\Y'Hk c '7 i 'hi fYgfiz]b k\]W. '\Y'fYZYWhYX'cb'h\Y' Z bXUa YbhU``X]j ]g]cb']bh\Y'k YghYfb']bhY``YWi U`'HfUX]h]cb'VYlk YYb'h\Y gWjYbWfg'UbX'\i a Ub]hjYg"'H\Y'`YWi fY'k Ug']bhYbXYX Vch\'Ug'U'Wf]hjei Y'cZh\Y'6f]hjg\'a ]X!Wfbh fm YXi WUhjcb'gmghYa 'Vi h'U'gc'k\UhGbck'gUk'Ug'h\Y 'cgg'cZU'Wca a cb']bhY``YWi U`'W'hi fY'UbX']hg'YZYWfg cb'YZcfhg'hc'gc'j Y'f[i'cVU'fidfcV'Ya g"'Gbck fig Wf]hjei Y UbX'fYgdcbgYg'hc']h\Uj Y'fYa U]bYX'Ub'i bXYfWi ffYbh ]b'XYYUhYg'UVci h'h\Y'i b]j Yfg]hmz'Ug'k Y'g\U``gYY k\Yb'k Y'fYUX'A UggYmfig g\cfhfYZYWfjcb'cb'Gbck']bh\Y'&%ghWfbh fm'
GYa ]bUf	GYa]bUf'('f+'>Ubi UfmŁ'BUh]cbU`]ga ž dfcdU[UbXUž']bXcWf]bUh]cb  FYUX]b[.'; Ycf[Y'CfkY``ž'fBchYg'cb'BUh]cbU`]ga fi f%-() ŁZ' '5fi bX\Uh]'Fcmž'fH\Y'5`[YVfU'cZ+bZ]b]hY
GYa ]bUf	>i ghj\\\fif\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
	F YUX]b[.:K "'?":7`]ZZcfXžfH\Y'9h\]Wg'cZ6Y`]YZfi f% ++tž'K]``]Ua '>Ua YgžfH\Y'K]``hc'6Y`]Yj Yfif% -*t"
	K \miXc'k Y'VY`]Yj Y'k \Uhk Y'VY`]Yj Y'UbX'\ck 'Xc'k Y_bck 'k \Yb'k Y'g\ci `X'a cX]Zmcf'[ ]j Y'U'VY`]YZi d3 HcXUmk Yfi'X]gW gg'h\Y'a Uh\Ya UhJWJUb'K ]``]Ua 7`]ZZcfXfg`ZJa ci g'YggUmž'k \]W\'gHJfhYX'U d\]`cgcd\]WJ`'HUX]hJcb'!'h\Y'Yh\]Wg'cZVY`]YZ' K \YfYUg'7`]ZZcfX'Uf[ i YX'h\Uhk Y'UfY'U'k Umg'cV`][ YXhc'\Uj Y'gi ZZW]YbhYj ]XYbWY'Zcf'Yj YfmcbY'cZci f VY`]YZgž'h\Y'd\]`cgcd\Yf'K ]``]Ua '>Ua Yg'gYhci hU

	dfU[aUh]Wdcg]h]cb`]b`fYgdcbgY"`< Y`Uf[iYX`h\Uhh\YfY`UfY`gcaY`WcbhYIhg`]b`k\]W\`]h`]g`Z]bY`hc`Zcfa`U VY`]YZYjYb`h\ci[\`kY`Xcbfhi\UjY`giZZJW]Ybh Yj]XYbWY`Zcf`]hžUbX`YjYb`h\ci[\`kY`_bck`h\Uh'kY

	FYUX]b[7`]bhcb: YfbUbXYgžfH\Y'k]X'a Ub']b'h\Y k]b[g.BcUa '7\cag_mff&\$\$)Ł/BcUa 7\cag_m'GcaY'H\ci[\hg'cb'\bhY`YWi U'g'UbX'h\Y GW\cc`gfif% **Ł/BcUa '7\cag_mžfH\Y FYgdcbg]V]]micZ\bhY``YWi U'gfž% *+fh\YgY'UfY'U``g\cfhd]YW'g'Ł"  HcXUmk Y'Wcbg]XYf'h\Y'bchJcb'h\Uhi b]j Yfg]hJYg g\ci `X'UWiUg'VUghJcbg'cZWJhJWU`h\]b_]b[']b'k YghYfb gcWJYhJYg"'K\Uhi]g'WfJhJWU`h\]b_]b['UbX'g\ci `X'h\]g VY'Uhih\Y'WcfY'cZh\Y'i b]j Yfg]mfg'a ]gg]cb3'Hc Ubgk Yf'h\YgY'ei YghJcbg'k Y'h fb'hc'h\Y'k cf_'cZh\Y'5a Yf]WUb'`]b[i]ghž'd\]`cgcd\Yfž'UbX'UWfj]ghBcUa 7\cag_mž'Wcbg]XYf]b['\]g'k f]hJb['cb'h\Y'cV'][Uh]cb cZh\cgY'Uhi b]j Yfg]hJYg'hc'Yb[U[Y'k]h\'W ffYbh

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g Hi fb]h]b Gi Va ]gg]cb

-Zmci 'YbWci bhYf'U'dfcV`Ya 'k \Yb'UhhYa dh]b[ 'hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [ \'Hi fb]h]bž'd`YUgY hY`Yd\cbY'9I hYfbU``Gi ddcfhcb'-',)'''' %cf'Ya U]`'h\Ya 'cb'YI hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

# 5 WUXYa ]W + DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb[ 'hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va ]gg]cbg'[i ]XY`]bYg/
- fYj ]Yk cZa Uf\_g/

•