

7ci fgY 8YW] g

7 fYX]hDc]bhg**

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H\Y`Wci fgY`dfcj]XYg'i ddYf`]bhYfa YX]UhY``Yj Y``hi]hJcb`]b`>UdUbYgY``Ub[i U[Y`ghi Xm'`G]a]`Uf`hc`h\Y`Wci fgY f\$Xj UbW'X`>UdUbYgY`5fz`U`dfYfYei]g]hY`Zcf`h\]g`Wci fgYz`mci `Wcbh]bi Y`hc`XYj Y`cd`mci f`]bhYfW``hi fU` Wca a i b]WUhJcb`g_]``g`]b`Ub`UXi `h`>UdUbYgY`gdYU_]b[`Ybj]fcba Ybhk\]`Y``YUfb]b[`a cfY`UVci h`>UdUb`UbX h\Y`>UdUbYgY`'Mci `U`gc``YUfb`bYk``>UdUbYgY`ghfi Whi fYgž`YI dfYgg]cbgž`UbX`j cWUVi `Ufm''±b`YUW\`hcd]Wz`mci `YUfb`h\Y`W`ffYbhgcW]c!W``hi fU``]ggi Yg`]b`>UdUb"`±b`j cWUVi `Ufm`YUfb]b[`z`mci ``YUfb`&\$\$`_Ub']"`H\YmUfY hUi [\hh\fci [\Zci f`gcW]U`#W`hi fU``hcd]Wg''

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@YWi fYg`XY`]j Yf`gcWJU``UbX`W`hi fU``]ggi Yg`fY`UhYX`hc'h\Y`hcd]WcZh\Y`k YY_"'BYk `ghfi Wi fYg#[fUa a Ufg`UfY U`gc`YI d`U]bYX"'Nci `dUff]WJdUhY`]b`dfUWfJWYg`cZbYk `YI dfYgg]cbgž'j cWJVi `Uf]Ygž'UbX`? Ub']`UbX`]bhYfUWfik]h\ZY``ck `ghi XYbhg`Uhh\Y``YWi fYg"'@YWi fYg`UfY`XY`]j YfYX`i g]b[`>UdUbYgY`hc`Ugg]ghmci `]b`XYj Y`cd]b['h\Y]f `]ghYb]b[`UV]`]f]Yg`UbX`VYWca]b[`ZJa]`]Uf`k]h\`bUhjj Y`>UdUbYgY`gdYU_YfgfihU`_"'H\Y`WcbhYbhk\]W\`mci `UfY bchUVY`hc`Wca dfY\YbX`]b`>UdUbYgY`a UmVY`YI d`U]bYX`]b`9b[`]g\"

Hi hcf]U`g`dfcj]XY`mci `k]h\`cddcfhi b]h]Yg`hc`dfUW]\W`]bhYfUW]cbžfYUX]b[ž'UbX`k f]h]b[`]b`>UdUbYgY`Zc``ck]b[h\Y`]bZcfa Uh]cb`]bhfcXi \WX`Xi f]b[``Y\Wi fYg`UbX`dfcj]XYX`]b`h\Y`hYI h\cc_"`\Mci fi`XYj Y`cd`i bXYfghUbX]b[U\ci hh\Y`>UdUbYgY`dYcd`Yž\W`hi fYž`UbX`gc\W]Yhm'`H\]g`]g`a UXY`h\fci [\`X]g\W gg]cbg`cb`U`hcd]\WcZh\Y \\ci fgY"`9a d\Ug]g`]g`U`gc`d`U\WX`cb`XYj Y`cd]b[`]bhYfU\Wi]cb`g_]``g`]b`Ub`UXi `h>UdUbYgY`gdYU_]b[\\yb j]fcba Ybh''5`a YX]i a `cZ`Ub[i U[Y`]b`hi hcf]U`g`]g`>UdUbYgY`]b`cfXYf`hc`XYj Y`cd`mci f`cfU``UbX`Ui fU` g_]``g'`\hci `UfY`ghfcb[`mYb\ki fU[YX`hc`i gY`>UdUbYgY`Xi f]b[`WUggYg"

8YHJ]`g.

 $\label{eq:hamiltonian} $$HY''di fdcgY'cZh\]g''UggYgga Ybh']g'hc''YUfb'>UdUbYgY'W'hi fYž'`Ub[i U[Yž'UbX']bhYfW'hi fU''Wcaai b]WUh]cb g_]``g'VmdUfh]W]dUh]b[']b'U'j Uf]YhmcZUWhj]h]Ygz'Wcbg]gh]b['cZW'U''' $`a]bg$ YY_cb'Uj YfU[Y'''H\Y'UWhj]h]Yg]bWi XY'Ub']bX]j]Xi U'# fci d'dfYgYbhUh]cb'f('a]bg$ by XY'Ub']bX']bYfUWh]b['k]h\ >UdUbYgY'dYcd'Y#gdYU_Yfg'Uh'Ub'Yj Ybh\Y'X'Uh'I BGK "'Ghi XYbhg'k]`''h\Yb'fYZYWh'UbX'k f]hY'U'fYdcfh']b >UdUbYgY'f($$!))$ WXUfUWhYfg#YdcfhL'cb'h\Y]f'YI dYf]YbWYg''K f]hhYb'ZYYXVUW_"$

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; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g''

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- fYj]Yk cZa Uf_g/