

IEST6001

 $F \ Yghcf Uhj \ Y \ '9 \ Vc`c[\] Yg. \ 'DY fa \ UW' \ 'hi \ fY \ 'Df] b W d \ 'Yg' \\ Df UW J W$

HYfa 'Hk c''##'&\$&%

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUhjcb	D\cbY
>UWei] 6Yg[fcjY		6mUddc]bha Ybh Xif]b[Wcbgi Huh]cb \cifgfK YXbYgXUmg (.'\$da -).'\$da L" +bdYfgcbcfcb]bY"		В5

GWXcc`7cbHJWhi±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b['<ci fg. 'A cbXUm!': f]XUmž'-Ua '!') da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . Ž * % & - ' ,) ', +\$)

9a U]`. \<u>\U`4 i bgk "YXi "Ui</u>

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

H\]g'Wti fgY'k]```]brfcXi WY Zci bXUh]cbU``WtbWYdhg'cZh\Y'Yh\]Wg'UbX'df]bW]d'Yg'cZdYfa UW 'hi fYž'U fY[YbYfUh]j Y'gmghYa g'UddfcUW\Zci bXYX']b'5i ghfU']U'UbX'[`cVU``]b']a dUW'' \(\frac{1}{2} \) th\]g'Wti fgYž'mci 'k]``[U]b WtbZ]XYbW'']b'dYfa UW 'hi fY'fYghcfUh]j Y'ZfUa Yk cf_g'UbX'a Yh\cXc`c[]Yg']b'h\Y'WtbHYl hg'cZYbj]fcba YbhU`a UbU[Ya Ybhz'YUXYfg\]dž'UbX'gi ghU]bUV]`]hm''DYfa UW 'hi fY'XYg][b'gYY_g'hc'fYWtbbYWidYcd'Y'k]h\ YWt'c[]WU''dfcWyggYgz']bhYf!fY'Uh]cbg\]dgz'UbX']bhYfXYdYbXYbW''cZ']j]]b['h\]b[g'UbX'hc']bhY[fUhY'h\YgY_bck'YX[Yg'hc'hYbX'hc'YwtgmghYa g']b'k Umg'h\UhWUfY'Zcf'h\Y'9Uff\Z'WufY'Zcf'dYcd'Yz'UbX'i gY'U'ZU]f'g\UfY cZfYgci fWyg''DYfa UW 'hi fY'\Ug'Ub']a dcfhUbhfc'Y'hc'd'Um]b'h\Y'fUd]X'hfUbg]h]cb'hc'nYfc WUfVcb'cf'bYh bY[Uh]j Y'WUfVcb'h\fci [\'fYghcfUh]j Y'UbX'fY[YbYfUh]j Y'Ywt'c[]WU'zgcW]U'z'UbX'W 'hi fU''dfUW]Wyg'"H\Y Wti fgY'k]'`\Uj'X'U'ZcW'g'cb']a d'Ya Ybh]b['dYfa UW 'hi fY'gc'i h]cbgzXYa cbghfUh]b['\ck 'h\Ycfm'YUfbYX Xi f]b['gYa]bUfg'Udd']Yg'hc'fYU''k cf'X'gmghYa g''' \(\frac{1}{2} \) ww'yggZ ``m'Wta d'Yh]b['h\]g'Wti fgYz'mci 'k]``XYj Y'cd Udd']YX'g_]'`g'hc'XYg][b'UbX']a d'Ya Ybhi fVUb#gi Vi fVUb'dYfa UW''hi fY'gc'i h]cbg'UbX'mci 'k]``YUfb'U dYfa UW''hi fY'XYg][b'WYfh]ZWUhY''

7ci fgY @YUfb]b[Ci hWca Yg

- %" 91 d`U]b`YWc`c[]\WJ`'UbX`gc\WU`'Wca dcbYbhg'cZdYfa UW 'hi fY
- &" ⇒XYbhjZm\ck 'dYfa UW 'hi fY'Yh\]Wg'UbX'df]bW]d`Yg`]bZcfa 'XYg][b'Zcf'fYWcbbYWf]b['\i a Ubg'k]h\#Ug "bUh fY"'k \]'Y'dfcj]X]b['UVi bXUbWY
- "8 Ya cbqffUlY h\Y Z bXUa YbHJ`g cZYWcgmghYa glfi Wi fY UbX Z bWjcb
- (" +bHY[fUHY'h\Y'_Ymdf]bWdYg'cZkUHYfž'gc]'ž'W]aUHYž'HYYgž'd'Ubhg'UbX'gcWJU''dYfaUW'h fY
-) " 5dd mdYfa UW h fy XYg][b a Yh\cXc c[mhc Ybi]fcba YbhUbX gcWYhmq]h Uhlcbg UbX gYhlb[g

HYUWX]b['GhfUhY[]Yg'

 $HYUWX]b[GfUhY[]Yg^*Zcf^* \rightarrow GH^* $$\%Yb[U[Y]hk c^*Zcfa Uhg.$

%" ±0! WUgg gYa]bUfg]b dYfa UW `h fY XYg][b `\Y`d 'mci `XYj Y`cd 'Ub i bXYfghUbX]b[`cZdYfa UW `h fY Yh\]Wg UbX df]bW]d Yg" ? Ymh\YcfYh]WU` WcbWdhg fY[UfX]b[`k UhYfžgc]`žW]a UhYžhfYYgžd`UbhgžgYYXgžUbX gcW]Yhm U]a YX UhXYj Y`cd]b[`_bck `YX[Y`cZbUh fU``dUhYfbg 'k]``]bZcfa `XYg][b" ? Ymh\YcfYh]WU` XYg][b 'WcbWYdhg UfY Wci d`YX`k]h\ YI Ua d`Yg`cZdYfa UW `h fY`]b UWf|cb"

&"'F YgYUfW.!gYfj]WY!`YUfb]b['Z]Y`X`ghi XmgYa]bUfg`]blfcXi WY`mci 'hc'j Uf]ci g`dYfa UW`hi fY`g]hYg`]bWi X]b[
ga U```i fVUb`dfcdYfh]Ygžgi Vi fVUb`g]hYgž'UbX`Wca a i b]mmUbX`i fVUb`ZJfa 'gWU`Y`g]hYg"'Mci 'k]```VY`YI dYWYX
hc`dUfh]W]dUhY`]b`dfUWf]WU``YI dYf]YbW'`Ug`dUfhcZUb`YbYf[mYI W.Ub[Y`hc`Wca dYbgUhY``UbX`W ghcX]Ubg`Zcf
h\Y]f`h]a Y"'H\Y`Z]Y`X`Wca dcbYbh]g`XYg][bYX`hc`Udd`mUbX`Vi]`X`i dcb'h\Y'h\YcfmUbX`WcbWidhg`Wcj YfYX
Xi f]b['h\Y']b!WUgg`Wca dcbYbhcZh\Y`Wci fgY"

5ggYgga Ybh

5ggYgga Ybh HUg_g

5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
G]hY`UbU`mg]g`UbX`W&bWfdh XYg][b`	' \$I	\$*#\$+#&\$&%`%%\$\$`DA	%ž'&ž''
8 Yg][bʻUbXʻ]a dʻYa YbHJhjcb fYdcfhi) \$1	\$'#\$,#&\$&%'%%\$\$'DA	%ž'&ž' ž'(
8Yg][b'dfYgYbHJhcb'	&\$I	Bch5dd`]WUV`Y	%ž &ž ' ž (ž)

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% G]hY UbU`mg]g UbX WcbWdhXYg][b

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. %\$\$\$ k cfXg fbch]bWi X]b['UHUW\a YbhgŁd`i g WcbW/dhXYg][b XfUk]b[

8YHJ]`g.

Ghi XYbhgʻk]``WkccgY`Ubʻi fVUbžgi Vi fVUbʻcfʻfi fU``g]hYʻh\YmUfYʻdUfh]W``Uf`m`]bhYfYghYX#VcbbYWYXʻhc`UbX k]```Wca d`YhY`U`g]hY`UbU`mg]g`UbX`WcbWYdhXYg][bʻfi dʻhc`%\$\$\$\$`k cfXg`Ž`XYg][b½"`H\]g`g]hY`k]```VY`h\Y gi VYVNcZh\Y`a Uʻcf`XYg][b`UggYgga Ybh'

ai .

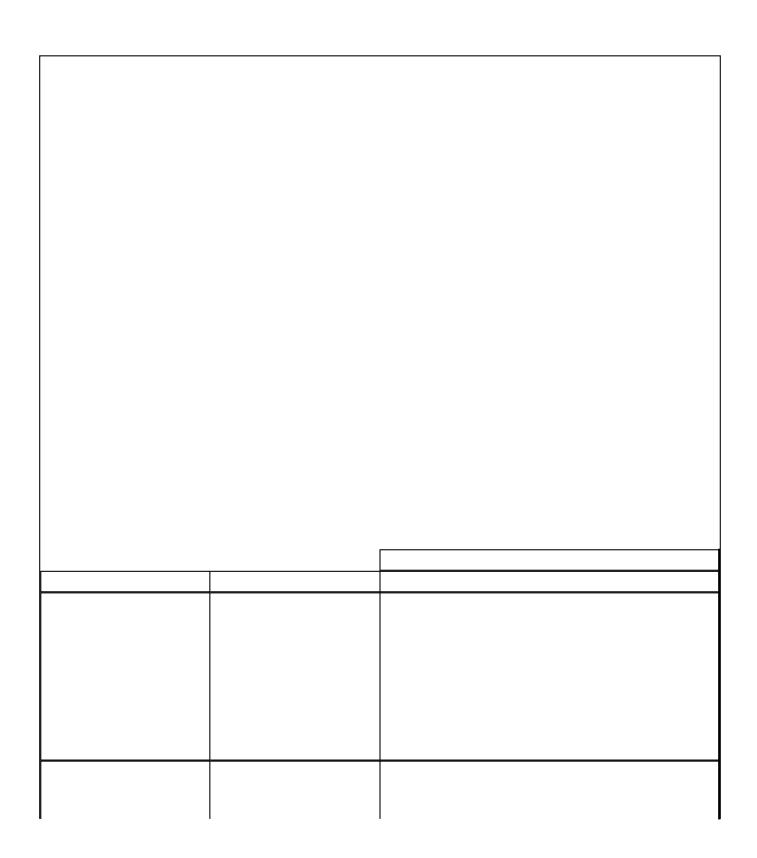
5ggYgga Ybh' . 8Yg][b'dfYgYbhUh]cb

@Yb[h\. B5

8YHJ]`g.

 $H\Y'Z_bU''k YY_cZhY'Wci fgY'k]```U``ck 'ghi XYbhg'hc'dfYgYbh'h\Y]f'XYg][b'hc'h\Y'WUgg'''H\Y'dfYgYbh'Jh]cb g\ci `X'hU_Y'Ufci bX'' $`a]bi hYg'''$

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh



١						

Fygci fWyg

DfYgWf]VYX`F Ygci fWYg`

 $\label{eq:linear_property} $$Mci\ f'_YmfYUX]b[\ g'UbX'chXYf'a\ UhYf]U'g'k\]``'VY'Uj\ U]`UV'Y'j\]U'mci\ f'A\ ccX'Y'g]]hY'''$

FYWca a YbXYX FYgci fWYg

 $\label{thm:condition} \mbox{Mci "UfY"YI dYWMYX" hc "a U_Y" [ccX" i gY" cZh\Y" fYUX]b[g" dfcj] XYX" Zcf" h\Y" Wci fgYz" UbX" hc "fYUX" VY mcbX" h\cgY = 0.$

< YdVi fb 'Gdf]b[gž'J]W". 'A Y``]cXcfU'Di V`]g\]b[ž'&\$%

.

H\Y'ffUbg]hcb: hc'U'gi ghU]bUV'Y'UbX'1 ghk cfX

: "'9"'HfU]bYf

7 UbhYfVi fmžB"G"K ". 9bj]fcVcc_ž'W&\$%\$

7 ci fgY '9j U'i Uh]cb 'UbX '8Yj Y`cda Ybh

H\]g`]g`U`bYk`Wci fgYz`WfYUHYX`Ug`dUfficZh\Y``Uf[Yf`fYj]g]cb`cZh\Y`A UgHYf`cZ9bj]fcba YbHJ``A UbU[Ya Ybh dfc[fUa ``Ui bW\YX`]b`&\$&%"DUghYl dYf]YbWY`UbX`ghi XYbhZYYXVUW_`cb`h\Y`l BGK `cb`]bY`gi fj Ymhcc` a m9l dYf]YbWY`]bX]WUHYg`h\Y`]a dcffUbWY`cZghi XYbhg`Xc]b[`WcgY`UbX`WUfYZ\``fYUX]b[`cZh\Y`k YY_`m Ugg][bYX`fYUX]b[g`VYZcfY`UHYbX]b[`gYa]bUfgžYb[U[YX`ghi XYbhidUff]W]dUh]cb`]b`WUgg`X]gW gg]cbg UbX`ZY`X`hf]dgž'UbX`cZghi XYbhi]bXYdYbXYbhifYgYUfW\"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dhJb['tc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9| hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'#c'i'#c'i'
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'm\Zcf'Ub'Yl hYbg]cb\ZVi hmci
a i ghfYhU]b'mci f'hJ\W_Yhbi a VYf'\Zfca '9| hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj]XYbW''hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WUhjcb'''#mci 'Ya U]''9| hYfbU''Gi ddcfhmci 'k]''

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=iWUbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'i

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = b Zcfa Uh cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg 'Zcf'Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYi]Yk 'cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]\\\Yg/
- YI Ua]bUh]cb dfcWYXi fYg/
- qdYWJU``Wcbq]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]miUbX'X]qUV]`]m/
- UbX'ch\Yf'YggYbhJU`UWUXYa]W]bZcfa UhJcbžgYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

.

⇒a U[Y'7fYX]h

6m>UWei]6Yg[fcjY"Gibf]gY]bKc~cb[cb["

7F = 7CG

7F = 7CG Dfc|]XYf 7cXY. \$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci bhfm

 $K_{Y'UW\underline{b}ck} YX[Y'hY'6YXY[U''dYcd'Y'k \c'UfY'hY'ffUX]h]cbU''W ghcX]Ubg'cZhY''UbXg'cb'k \]WX'I_BGK_{Ybg]b[hcb'WJa_di_q']q''cWJhYX'''$