

# **LING5029**

H\Y'Dgm\\\c`c[mcZ@Ub[iU[Y

HYfa 'Hk c''##'&\$&%

## 7ci fgY'Cj Yfj ]Yk

### GHJZ7cbHJWh8YHJ]`g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
G]ħ\Ya V]b_cg]'8i VY	9	6mUddc]bha Ybhj ]U Y!a U]`	,	6m Uddc]bha Ybh i ]U'Y!a U]`

#### @YWhi fYfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D∖cbY
G]h\Ya V]b_cg] <sup>.</sup> 8i VY	g"Xi VY4 i bgk "YXi "Ui	6miUddc]bha Ybhij ]U	A cfj Yb 6fck b	6m
		Y!a U]`		Uddc]bha Ybh
				j ]UʻY!a U]`

### GWXcc`7cbHJWh′±bZcfa Uh]cb

GWkcc``cZ<iaUb]h]Yg'UbX'@Ub[iU[Yg

@c\\U|rgcb. G\\\cc``C\Z]\\\\ž'A\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y`'&\z'&),

CdYb]b[ '< ci fg. 'A cbXUm! ': f]XUmž'- Ua '!') da

D\cbY.'Ž\*%'&'-',)'%\*,%

: UI . Ž\*%'&'-',)', +\$)

9a U]`. \<u>U`4 i bgk "YXi "Ui</u>

## 7ci fgY'8YW]`g

### 7 fYX]hDc]bhg\*\*

### Gi a a UfmcZh\Y 7ci fgY

### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" XYg\f]\Y`UbX`YI d`U]b`h\Y`a cgh\\fi \\]U``]ggi Yg`]bj c`j YX`]b`h\Y`]bh\fU\\f\gh\Y`\Yb`h\Y`\VfU]b`UbX``Ub[i U[Y`]b`fY`\Uh]cb`hc`\ck ``Ub[i U[Yg`UfY`U\\equiv i]fYX\text{z}'dfc\\f\ggYX`UbX`ghcfYX\"`
- &" Udd`m[YbYfU``df]bW]d`Yg`cZ`]b[i]gh]Wg`hc``Ub[iU[Y`UWei]g]h]cbž`Ub[iU[Y`WcadfY\Ybg]cb`UbX dfcXiWf]cbž'UbX`hc`Uddcg]hY`bYifc`c[]WU`dfcWfggYg`]bjc`j]b[``Ub[iU[Y"
- ' "YI d`]\W\\Y'\V\Ug]\WfYgYUf\\\'cb'\VfU]b`XUa U[Y'\UbX``\Ub[i U[Y']a dU]fa Ybhz`\VfYU\_Xck b`\UbX``cggz\\\UbX``\Ub[i U[Y'\UbX`h\ci [\h''
- ("Wf]h]WU``m`YI Ua ]bY`UbX`Yj U`i UhY`a cXYfb`UddfcUVXYg`]b`dgmVXc`]b[i ]gh]Wgž]b`hYfa g`cZh\Y]f`dck Yf hc`UVWci bh'Zcf`h\Y`dgmVXc`c[]\W``dfcWYggYg`i bXYf`m]b['h\Y'dfcXi Wf]cbž'dfcWYgg]b['UbX fYdfYgYbhUh]cb`cZ\i a Ub``Ub[i U[Y"`

### HYUW ]b[ 'GhfUhY[ ]Yg'

H\]g`Wci fgY`i gYg`Ub`]bhYfUWnjj Y`gYa ]bUf`Zcfa Uhhc`]bhfcXi WY`h\Y`VUg]WWcbWYdhi U``X]ghjbWnjcbg`UbX dfYgYbhh\Y`Z bXUa YbhU``fYgYUfW\``k \YfY`h\Y``YWni fYf`k ]```YbXYUj ci f`hc`a U\_Y`cf][ ]bU``hYW\b]WU``fYgYUfW\ a cfY`a UbU[ YUV`Yž`" \

## 5ggYgga Ybh

## 5ggYgga Ybh HUg\_g

5ggYgga Ybh'HJg_	K Y][ \h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
DfcV`Ya 'gYh'	&) ı	% #\$*#&\$&%`\$(.\$\$`DA	%ž <sup>*</sup> (
=b!WUgg'hYgh	&) ı	Bch5dd`]WUV`Y	%ž'&ž''
DfYgYblUljcb	%\$I	5g <sup>-</sup> fcgh/fYX	%ž''
9ggUmi	(\$1	\$-#\$,#&\$&%`\$(.\$\$`DA	%ž.' ž.(

## 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh'% 'DfcV'Ya 'gYh

GHUffmXUh%%\*#\$\*#&\$&%\*\$(.\$\$\*DA

**% % %** 

‰

 $@Yb[h\.\ \&) \ g\cfh\Ubgk\ Yf\ ei\ Ygh]cbg$ 

8YHJ]`g.

 $\label{eq:continuous} \textbf{Ghi XYbhg'k ]``VY'fYei ]fYX'hc'UbU'mgY'gca Y'WUgY'ghi X]Yg'fl! \textbf{\%} dfcV'Ya gŁ']b'fY`Uh]cb'hc'dUfh]W'`Uf Yg] \\ dgm\Xc`]b[i]gh]W]ggi Yg'''H\Ymk]```UXXfYgg'h\YgY']ggi Yg'Vm'Ubgk Yf]b['j YfmgdYW]Z]Wei Ygh]cbg'XYg][bYX'hc''' hand a start of the start of t$ 

#### 8YHJ]`g.

%\$!%) 'a ]bi hYg"'Ghi XYbhg'k ]``'VY'Ugg][ bYX'U'dfYgYbhUh]cb'cb'U'gdYV]Z]WWci fgY'hcd]W'

**Gi Va ]gg]cb`bchYg.** `DfYgYbhUh]cb`Ug`fcghYfYX`UbX`ghi XYbhg`a i ghigi Va ]hih\Y]f`Dck YfDc]bhig`]XYg`VYZcfYh\Y]f`gW\YXi `YX`dfYgYbhUh]cb

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ $H \ fb \end{tabular} $$ fo \end{tabular} $$ Hi & fb \end{tabular} $$ fo \end{tabular} $$ fo \end{tabular} $$ fo \end{tabular} $$ Hi & fb \end{tabular} $$ fo \en$ 

5ggYgga Ybh(. 9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h. 5ddfcl]a UhY m&ž) \$\$ k cfXg

8 YHJ]`g.

 $\label{eq:localized-loca$ 

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYH & fb & gYY &$ 

## FYgci fW/g

### DfYgWf]VYX F Ygci fWYg

 $D Y U g Y f Y Z Y f h C h Y F Y U X ] b [ G Y e i Y b W Y g ``] b _ i c b `A c c X `Y `Z c f `X Y h U] `g ` c Z Y U W `k Y Y _ f g ` f Y e i ] f Y X `f Y U X ] b [ " '5 `` f Y U X ] b [ g `U f Y `U j U] `U V `Y `j ] U `A c c X `Y `U b X `h Y `I B G K `@] V f U f m'$ 

- < cZž9" f&\$%( Ł" @Ub[i U[Y XYj Y`cda Ybhf) h\ YX"Ł" 6Y`a cbhž75. K UXgk cfh\ 7 Yb[U[Y @YUfb]b[" "
- < UfYmžH"f&\$%( Ł"H\Y'dgm\%c`c[mcZ`Ub[i U[Y.: fca 'XUhU'hc'h\Ycfmf( h\'YX"Ł"< cj YžI ?. Dgm\%c`c[mDfYgg"</li>

#### FYWca a YbXYX FYgci fW/g

- ;  $Ug_Y^*\check{z}A''$ ; "'f&\$\$+Ł"'H\Y'C|ZcfX'\UbXVcc\_'cZdgm\%c`]b[i]gh]\\g''C|ZcfXž|?.'C|ZcfX'| b]j Yfg]\text{lm} DfYgg"
- : i fth\Yf'fYWca a YbXYX'fYgci fWYg'UfY'Uj U]`UV`Y'cb'A ccX`Y"`

### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Ghi XYbhZYXVUW\_`]g'[Uh\YfYX'i g]b['I BGK fig'a m9I dYf]YbWY'gmghYa `]b'UXX]h]cb'hc'cb![c]b['ZYXVUW\_hfci [\ci hh\Y'Wci fgY'UbX'j]U']b!WUgg'UWhj jh]Yg'UbX'UggYgga Ybhg"'5```Zcfa g'cZghi XYbhZYXVUW\_k j``'VYUWhX'i dcb'hc']a dfcj Y'h\Y'ghi XYbh`YUfb]b['YI dYf]YbWY]b'h\Y'Wci fgY'UbX']hg'Z hi fY']hYfUh]cbg"'

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g Hi fb]h]b Gi Va ]gg]cb

## 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±i\WbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiW\YUh]b[ihciU\W]XYbhU`m\wcdm]b[iZfca iUigci f\Wik ]h\ci hiU\w\_bck iYX[Ya Ybh'

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

± Uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]'Y 'a cghmfYhU]b]b['h\Y'cf][]bU' ghfi WhifY'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa UhJcb'k]h\ci hUW\_bck 'YX[Ya Ybh''H\]g'U'gc Udd']Yg']b'dfYgYbhUhJcbg'k\YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk'k\c'Yž'k]h\ci h'Uddfcdf]UhY'fYZYfYbW]b['"

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth\Yfg'Vi hdUgg]b[ 'cZZth\Y'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZth\Ya d`U[ ]Uf]g]b[ 'UhUbmh]a YždUm]b[ 'Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa ]WhUg\_žghYU`]b[ 'cf'UWei ]f]b[ Ubch\Yf'dYfgcb'g'UWUXYa ]Wk cf\_'UbX'Wcdm]b[ ']hz'cZYf]b[ 'hc'Wca d`YfY'Ubch\Yf'dYfgcb'g'k cf\_'cf'gYY\_]b[

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wcaaib]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

## 5 WUXYa] W + D Zcfa Uh]cb

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb[ 'hc.

- fYei Yghg Zcf Yl hYbg]cb/`UhY gi Va ]gg]cbg [ i ]XY`]bYg/
- fYj ]Yk cZa Uf\_g/