

# **MODL5103**

A i `ha YX]U HfUbg`Uhcb

HYfa 'Hk c''#'&\$&%

# 7ci fgY<sup>-</sup>Cj Yfj ]Yk

### GHJZ7cbHJWh8YHJ]`g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
GYUb <sup>.</sup> 7\Yb[	gYUb"W 4 i bgk "YXi "Ui	Hi YgXUm	A cfj Yb 6fck	b Ž*%'-',)
		%\$Ua!%&Ua	&+,	&' %(

#### Hi hcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
FYVYWU <sup>.</sup> DUfYXYg	f"dUfYXYgb]Yhc4 i bgk "YXi "Ui	H65	B <i>#</i> 5	B#5
9`YbU'A ]_\U]`]_	Y"a ]_\U]`]_4 i bgk "YXi "Ui	H65	B#5	B#5
FcgU">cc"Mb"MUb[	°cc"mUb[4 i bgk "YXi "Ui	H65	B <i>#</i> 5	B#5
Gi nUbʻD]dYf	g"d]dYf4 i bgk "YXi "Ui	H65	B <i>#</i> 5	B#5
< ]XYHJ_U <sup>-</sup> 5VY	\"UVY4 i bgk "YXi "Ui	H65	B <i>#</i> 5	B#5
5Xf]Ub <sup>.</sup> 6i nc	U"Vi nc4 i bgk "YXi "Ui	H65	B <i>#</i> 5	B#5

### GWXcc`7cbHJWh =bZcfa Uh]cb

GWkcc``cZ<iaUb]h]Yg'UbX'@Ub[iU[Yg

CdYb]b[ '< ci fg. 'A cbXUm! ': f]XUmž'- Ua '!') da

D\cbY.'Ž\*%'&'-',)'%\*,%

: UI . 'Ž \* %'&'-' , ) ', +\$)

9a U]`. \<u>U`4 i bgk "YXi "Ui</u>

# 7ci fgY 8YHJ] g

#### 7 fYX]hDc]bhg \*

### Gi a a UfmcZh\Y 7ci fgY

 $H\[g'WcifgY'dfcj]XYg'mci'k]h\ Vch\ h\ YcfYh]WU`\_bck\ YX[Y'UbX'dfUWf]WU``YI\ dYf]YbW']b'a\ i\ ha YX]U fUbg`Uh]cb'Uh'U'dfcZYgg]cbU```Yj\'''H\]g']bWi\ XYg.$ 

%" cb`]bY`HYI hg`fVi g]bYggždc`]hJVgžW`hi fYžUbX`gcVJU``UZZU]fgŁ/
&" Ui X]c!j ]gi U``HfUbg`UhJcb`fWUdhJcb]b[ 'UbX`gi VhJh]b[ Ł/
' " a i `Ha YX]U`V\cbHYbhfl ]XYc`[ Ua Y``c\W]gUhJcb`UbX`gcZhk UfY``c\W]gUhJcbŁ"

H\Y`Wci fgY`ZfUa Yg`a i `h]a YX]U`hfUbg`Uh]cb`k]h\]b`Ub`]bhYfbUh]cbU``dYfgdYWf]j Y`k]h\ Ya d\Ug]g`cb`h\Y W h]b[!YX[Y`hYW\bc`c[]Yg`Ya VYXXYX`]b`h\Y``Ub[i U[Y`gYf]]WYg`]bXi ghfmcZhcXUm"

#### 7 ci fgY @YUfb]b[ Ci hWca Yg

%" UbU`mgY``]b[i]gh]WZYUhifYg`h\Uhi]a dcgY`lfUbg`Uh]cb`X]Z]W``h]Yg`]b`ai`h]aYX]U`lfUbg`Uh]cb &" Zcfai`UhY`lfUbg`Uh]cb`ghfUhY[]Yg`hc`WcbZ]XYbh`miffUbg`UhY`ai`h]aYX]U`WcbhYbh`]b`Vch\```Ub[iU[Y X]fYWh]cbq

'" 1 ahlZmifUbg`Uhlcb`VX.c]WYg`VUgYX`cb`h\YcfmUbX`fYgYUfVX`

("fY`UHY`WcbHYI Hi U``\_bck `YX[Y`cZh\Y``Ub[i U[Y`qYfj]WYq`]bXi qHfmhc`HfUbq`UHcb`qHfUHY[]Yq

) " WUdhch Ub X gi Vhh Y a i ha YX]U Wch Y bh

### HYUWX]b[ 'GhfUhY[]Yq'

@YWi fYg`UbX``Ub[i U[Y!gdYW]Z]Whi hcf]U`g`UfY`HJi [\hiVmUWJXYa]WgHJZUbX`dfcZYgg]cbU``hfUbg`Uhcfg`]b`h\YfY`Yj UbhZ]Y`Xg"`H\]g`Wca V]bUh]cb`Ybgi fYg`h\YcfYh]WU``\_bck `YX[Y`]g`dU]fYX`k]h\`dfcZYgg]cbU``dfUW]WY"

# 5ggYgga Ybh

 $HYUggYggaYbhHUg\_gZcWgcbhhfYYUgdYWgcZhfU]b]b[...$ 

- Cb[c]b['kYY\_mfubg`uhcb'!'dfuWnW'ubX'XYjY`cd'h\Y'WcfY'ffubg`uhcb'g\_]`g'ubX'hYW\b]eiYg
- 5i X]c!j ]gi U`HfUbg`UhJcb`dfc^YWh! dfUWhJWY`UbX`XYj Y`cd`Zci bXUhJcbU`a YX]U`gi VhJh`]b[ `g\_]``g`UbX hYVXb]ei Yg`]bWi X]b[ `Udd`]WUhJcb`cZU'gi VhJh`]b[ `hcc``UbX`Wca dfY\Ybg]j Y`i bXYfgHUbX]b[ `cZ gi VhJh`]b[ `k cf\_Zck g"`
- Ei]n'!'Wcbgc`]XUhjb['h\Y'ibXYfghUbX]b['cZZcibXUhjcbU`\_bck`YX[Y'UbX'h\YcfYhjWU``WcbWYdhg'''

### 5ggYgga Ybh HUg\_g

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[ CihWca Ygʻ5ggYggYX
K YY_`mlfUbg`Uh]cb Ugg][ ba Ybhg`	) \$1	k YY_`%\$	%ž'&ž' ž'(
5i X]c!j ]gi U``lfUbg`UlfJcb dfc^YWi	(\$1	k YY	%ž &ž ( ž )
Ei]n	%\$ı	k YY_ <sup>:</sup> %\$	%ž'' ž'(

#### 5ggYgga Ybh8YhJ]`g

 $5ggYgga Ybh% K YY_mhfUbg`Uh]cb Ugg][ba Ybhg$ 

GhUfh'XUhY. k YY\_'%

8YW]`q.

("HJg\_g"dYf"X]fYWf]cbž"Ufci bX"(\$\$!)\$\$"k cfXg"dYf"HJg\_"

: YYXVUW\_dfci ]XYX^]b!WUggžcfij ]U^Hi fb]hjb"

#### 5XX]h]cbU`XYhJ]`g.

• C b`mit\Y``UghitfUbg`Uhjcb'tUg\_']b'YUV\X'X]fYWtjcb'k ]```VY'a Uf\_YX"

#### 8 YW] g.

&ij ]XYcgidYfiX]fYWfjcbž(ij ]XYcgi]bihchUi/UfcibXi) \$\$ik cfXgicZiYb[h\idYfij]XYc"

: YYXVUW\_'dfcj ]XYX'j ]U'Hi fb]h]bž']b! WUgg'cfU``ZYYXVUW\_"

#### 5XX]h]cbU`XYhU]`q.

- Hkcij]XYcgiVhjh]b['HJg\_q'dYf'`Ub[iU[Y'X]fYWhjcbž('HJg\_q']b'hchJ''
- C b`mih\Y`gYWcbX'gi Vhjh`jb[ 'HUg\_']b'YUW\XX]fYWhjcb'k]``'VY'a Uf\_YX"'
- Bchy. `X]ZZYfYbh`Ub[i U[Y!gdYW]Z]WWUggYg`k]``\UjY`X]ZZYfYbhkYY\_`mgi Va]gg]cb`XYUX`]bYg"'H\Y i b]j YfgU``gi Va]gg]cb``]b\_g`]b`AccX`Y`Xc`bchgdYW]ZmXYUX`]bYg`Zcf`X]ZZYfYbh`Ub[i U[Y`ghfYUagž`gcd`YUgY`XciV`Y!W\YW\_h\Y`XYUX`]bYg`k]h\`mcif`hhcfg`UbX`Zc``ck'h\Y]f`UffUb[YaYbhg"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$ 

5ggYgga Ybh' . Ei]n

8YHJ]`q.

&\$ "ei Ygh]cbg"cb`]bY "ei ]n"

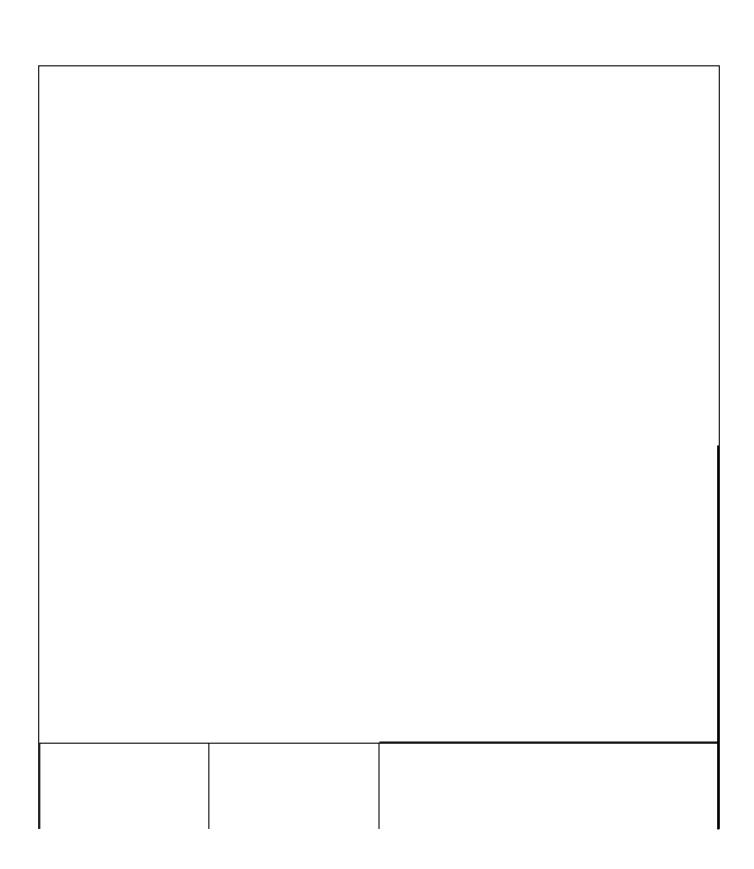
A ccX`Y`Ui hca Uh]Wa Uf\_]b[`UbX`ZYYXVUW\_

#### 5XX]h]cbU`XYhJ]`g.

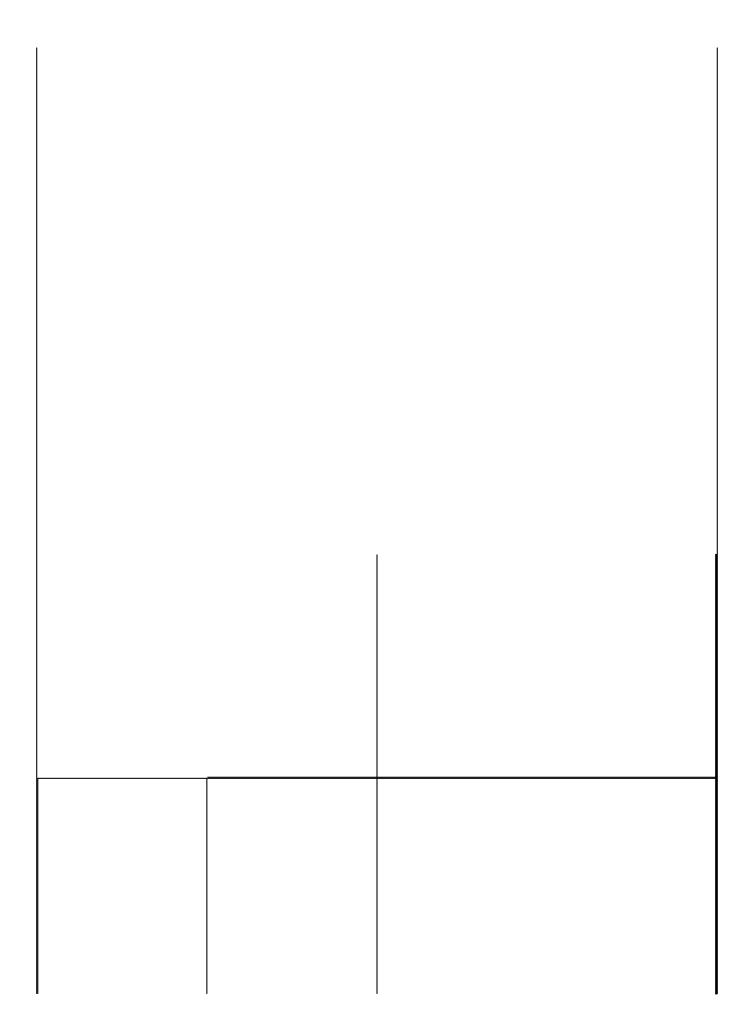
H\Y'ei ]n'k ]``VY'a UXY'Uj U]`UV`Y'cb'A ccX`Y'Zcf'gh XYbhg'hc'Wca d`YhY']b'k YY\_'%\$"

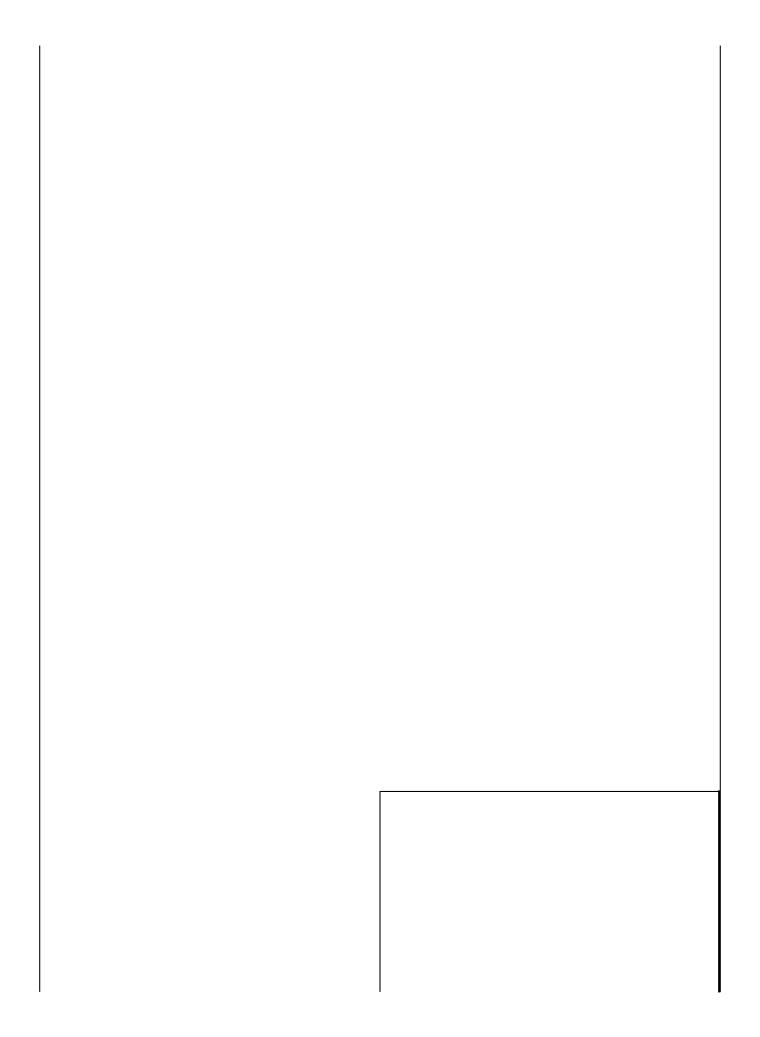
D'YUqY VXYW\_A ccXY UbX Ubbci Wa YbhZcf XYHJ] "]bglfi Wicb"

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh



	UggYgga Ybh
	%"`6Ug]WhfUbg`Uh]cb`k cf_Zck g
	&"'HfUbg`Uhjcb'ei U`jhmUggYgga YbhfHE 5Ł
	' "`±bXi glfmiglUbXUfXg
1	<b> </b>





g`][\hmX]ZZYfYbhUffUb[Ya Ybhg"`D`YUgY
Xci V`Y!W\YW_`k]h\`h\Y`hi hcfg`VYZcfY`k YY_`&"

# Fygci fWyg

# DfYgWf]VYX F Ygci fWYg

91 dYWYX FYUX]b[g.

- H\Y'Fci hYX[Y'<UbXVcc\_cZ5i X]cj ]gi U`'HfUbg`Uh]cb'Vm@ ]g'DÅfYn!; cbn½Yn
- H\Y'F ci hYX[Y'< UbXVcc\_'cZHfUbg`Uhjcb'Gh X]Yg'f&\$% E'YX]hYX'Vm7 Ufa Yb'A ]``Ub'UbX': fUbWYgWU 6Ufhf]bU

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

5WUXYa]W<cbYghmUbXD`U[]Uf]ga

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY