

ARTS1690

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7 ci fgY Cj Yfj]Yk

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'7cbj Ybcfg'

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5 hmy b X Ub Wy F Yei] f Ya Yb hg

5 qh XYbh]q Yl dYWYX hc UhYbX U``WUqq WcbhUWi\ci fq Zcf U ZUW!hc! ZUW! fl & L'cf V YbXYX Wci fqY UbX WeadYhY'U``'UWAj]hYq'Zef'U'VYbXYX'cf'Z ``micb`]bY'WeifqY"

5 gh XYbhk\c'Uff]j Ygʻa cfY h\Ub′%) a]bi hYgʻ`UhYʻa UmVY dYbU`]gYX Zcf'bcb!UthYbXUbVW" Zgi VX U'dYbU`hm]g^]a dcgYXz*h\Y`ghi XYbha i ghVY`]bZcfa YX`j YfVU``mUhh\Y`YbX`cZWUgg^UbX`UXj]gYX`]b`k f]h|b['k]h\]b`&(\ci fg"

ZUʻgh XYbhYl dYf]YbW'gʻ]`bYggža]gUXj Ybh fYʻcfʻch\YfʻcVW ffYbW h\Uha U_YgʻUVgYbW Zfca ʻU WUgg#JWNj jmi buj cjXUV`Yžcf`YI dYWmg`hc`VY`UVgYbhZfca`U`Zcfh\Wcajb[`WUgg#JWNj jmz`h\Ymg\ci`X`gYY_ dYfa]qq]cb Zfca 'h\Y'7 ci fqY'5i h\cf]|mz'UbX'k\YfY'Udd`]WUV`Yz'h\Y]f`fYei Yqhq\ci `X'VY'UW\can dUb]YX'VmUb cf][]bU`cf`WfflZYX`WcdmcZU'a YX]WU`WfflZWJHY`cf`ch\Yf`Zcfa 'cZUddfcdf]UHY`Yj]XYbWY"

5 7 ci fqY 5 i h\cf|lma UmYl W qY U qhi XYbh2fca WUqqYq cf UWlij |h|Yq Zcf i d hc cbY a cbh\" < ck Y j Yfz h\Ym a UmUgg][b'UXX]hcbU`UbX#cf'U`hYfbUh]j Y'hUg_g'hc'Ybgi fY'Wca d`]UbWY"'5'7 ci fgY'5i h\cf]hmWcbg]XYf]b['h\Y [fubhb[cZuvqYbWraighvYrguhqZ]YXrurgh XYbhk]```qh]```VYruv`Yrbc'a YYhrh\YrWcifqY'q``Yufb]b[cihWcaYq UbX#cfjciaYcZYUfb]b["5 gh XYbhgYY_]b['Uddfcj U`hc VY UVgYbhZcfa cfY h\Ub cbY a cbh\ a i ghUdd`m]b'k f]hb['hc'h\Y'8YUb'UbX'dfcj]XY'U```cf][]bU``cf'WYfhZYX'gi ddcfhb['XcW a YbHJhcb"

cfacfy]bzcfaUt]cbUVcihthYUthYbXUbWYdfchcVc`g]bthYY:UW`tmcZ5fhgUbX'GcWJU` GWYbWrg. \htdg.#k k k "Uffig"i bak "YXi "Ui #W ffYbhah XYbha#UWJXYa \\mathbb{W}\delta Zcfa Uhcb#dfchcWc\a![i \]XY\\delta Ya#

5 WUXYa | W = b Zcfa Uh|cb

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7 fYX]hDc]bhg**

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5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

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IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

±DUddfcdf]UhY`dUfUd\fUg]b[.'W\Ub[]b['U'ZYk'kcfXg'UbX'd\fUgYg'k\]`Y`acghmfYhJ]b]b['h\Y`cf][]bU`ghfi WhifY`UbX`]bZcfaUh]cb`k]h\ci hUW_bck`YX[YaYbh'H\]g'U`gc'Udd`]Yg`]b'dfYgYbhJh]cbg'k\YfY`gcaYcbYdUfUd\fUgYg`Ubch\Yf'g`]XYUg`cf'kcfXg'k]h\ci hWfYX]h''±iU`gc'Udd`]Yg`hc'd]YW]b['hc[Yh\Yf'ei chYg'UbXdUfUd\fUgYg`]bhc'U'bYk'k\c`Yžk]h\ci hfYZ/fYbW]b['UbX'U'ghi XYbhg'ck b'UbU`mg]g`hc'Vf]b['h\Y`aUhYf]U`hc[Yh\Yf''

7 c``i g]cb. 'k cf_]b['k]h\ 'ch\ Yfg'Vi h'dUgg]b['c\Z'h\ Y 'k cf_'Ug'U'dYfgcb'g']b\X]j]\Xi U``k cf_"'7 c``i g]cb'U`gc]b\Wi XYg'dfcj]\X]b['mci f'k cf_'hc'Ubch\ Yf'ghi XYbh\ VYZcfY'h\ Y 'Xi Y 'XUhYzcf'Zcf'h\ Y' di fdcgY'cZh\ Ya d`U[]\Uf]g]b['Uh\Ubmh]a Yz'dUm]b['Ubch\ Yf'dYfgcb'hc'dYfZcfa 'Ub'U\WUXYa]\Wh\Ug_z'ghYU']b['cf'U\\ello i]f]b[Ubch\ Yf'dYfgcb'g' U\WUXYa]\Wk cf_'Ub\X'\ello dm]b[']hz'c\ZYf]b['hc'\\ello a d`YhY'Ubch\ Yf'dYfgcb'g' k cf_'cf'gYY_]b[d\una Ybh\Zcf'\\ello a d`Yh\beta d`Y

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[ˈh\Y`″gYWcbXUfm″gci fW/Zfca k \]WX`_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ff`gY`Zld`U[]\Uf]ga "Ł`gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h`\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UqqYqqa Ybhcf\Wci fgY`UhI_BGK_`cf`Ubch\Yf`i b]\Yfq]\m"

7 cffYWhfYZYfYbWb[dfUWhWyg.

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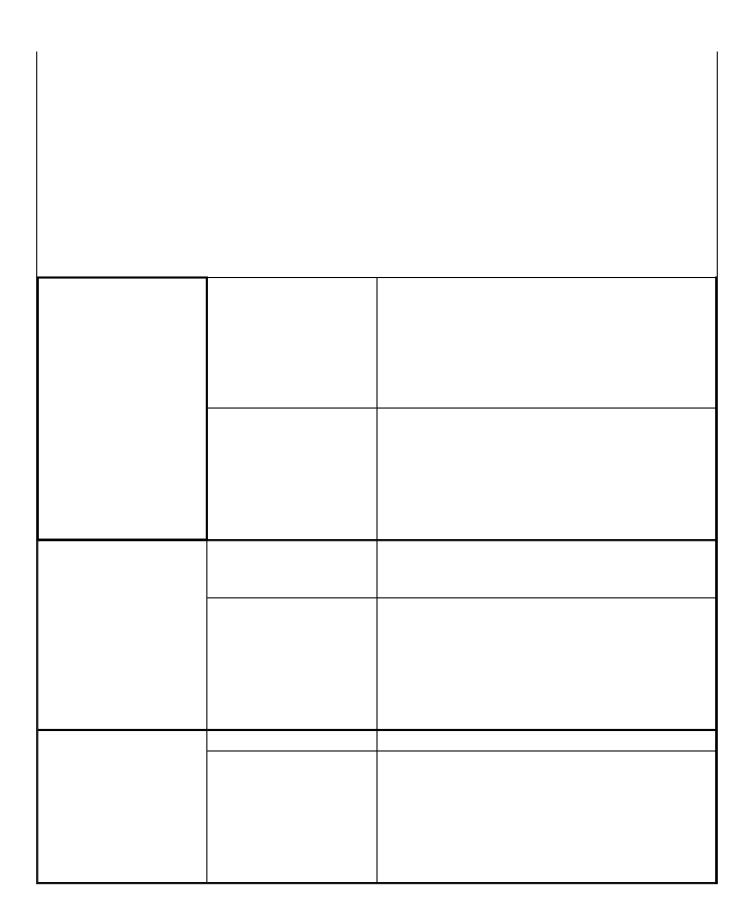
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