

ARTS3330

@Ub[i U[Yg.7 UdghcbY. HfUbg`Uh]cb UbX +bhYfdfYh]b[

GYa YghYf'C bY''##'&\$%

7ci fgY⁻Cj Yfj]Yk

'GhJZZ'7cbhJWh'8YhJ]`g'

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
@ Xa]`U'GhYfb	`"gHYfb4 i bgk "YXi "Ui	A cbXUm'&!('da	A 6 '&) +	-',)`&',&

GWXcc`7cbHJWh′±bZcfa Uh]cb

GWkcc``cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

CdYb]b[< ci fg. A cbXUm!: f]XUmž-Ua !!(.() da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . Ž*%'&'-',)', +\$)

9a U]`. <u>\U`4 i bgk "YXi "Ui</u>

5 http://bxubww.F.Yei]fYa Ybhg

UbX`gUZ/my'Yl Ua]bUh]cb`dfcWXi fYg/gdYW]U``Wtbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/ghi XYbhYei]lmiUbX`X]gUV]`]lmy'UbX`ch\Yf`YggYbh]U``UWJXYa]W]bZcfa Uh]cbž gYY`\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhghi XYbhg#JWJXYa]W]bZcfa Uh]cb#dfchcWt`g![i]XY`]bYg#

7ci fgY⁻8YHJ]`g

7fYX]hDc]bhg**

Gi a a UfmcZ'h\Y'7ci fgY

 $\label{eq:control_gradient} \text{Gi VYWh} \text{5 fY U. `5 Z [` WeWE fg V [Well 8 dW UZ E fg V [Well 9 Wd_S` E fg V [Well < SbS` WeWE fg V [Well = a dWS` E fg V [Well = b]] } \\$

5ggYgga Ybh' : cfhb][\hmUgg][ba Ybhg

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g. :: cffb][\hmUgg][ba Ybhg`f&) \$`hc`) \$\$`k cfXgŁ']bj c`j]b['h\Y`UbU`mg]g`UbX`Wca dUf]gcb`cZ a cbc`]b[i U``UbX`V]`]b[i U``a i `h]a YX]U`k]h\ fYZYWf]j Y`Wca a Ybhg"9UW\`Ugg][ba Ybh]g`k cfh\ '%\$i "`A Uf_]b[k]h\ 'U`fi Vf]Wk]h\ ']bX]j]Xi U``k f]hHYb`ZYYXVUW_'UbX`[fci d!VUgYX`cfU``ZYYXVUW_']b`h hcf]U`g"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gh XYbhg UfY Yl dYWYX hc di hh\Y]f bUa Yg UbX gh XYbhbi a VYfg cb Yj YfmdU[Y cZh\Y]f Ugg][ba Ybhg"

Hi fb]h]b Gi Va]gg]cb

Imagi YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-',) "''' %cf'Ya U] hYa 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u> "Gi ddcfh \ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'" Imai UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y'rc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'YI hYbg]cbž'Vi hmci a i ghfYhJ]b'mci f'h]\W_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'YYj UbhXcW a Ybhgt'rc]bWi XY'Ug'Yj]XYbW' 'rc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WJhjcb"'Imai 'Ya U]''9I hYfbU''Gi ddcfhmci 'k]'`Ui hca Uh]\WJ'`mfY\W']j Y'U'h]\W_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'``bYYX'rc'gdY\WZ\WJ'`mUg_'Zcf'cbY'' Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf''

; YbYfU`mž'UggYgga YbhitUg_g'a i ghVY'gi Va]hhYX'Y`YWfcb]WU``mj]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bghUbWfg'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'ghUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYhU]`g"

@UhY 5ggYgga Ybh'DYbU`h]Yg

5b'UggYggYX'HUg_']g'XYYa YX``UhY']Z]h']g'gi Va]hhYX'UZhYf'h\Y'gdYW]Z[YX'h]a Y'UbX'XUhY'Ug'gYhci h']b'h\Y Wci fqY'@YUfb]b['A UbU[Ya YbhGmqhYa 'f@A GŁ"

 $\label{thm:condition} $$HY^UHY^dYbU^m]g^hY^cgg^cZ)_L^cZ^hY^hcHU^dcgg]VY^aUf_g^Zcf^hYY^HUg_Z^cf^YUW^XUmcf^dUfhhYfYcZhY^kcf_]g^UHY^@UhYbYgg^k]^*]bW^XY^kYY_YbXg^UbX^di_V^]WYc^]XUmg^"HY]g^XcYg^bchUdd^mhc^U'HUg_'hWh]g^UqqYqqYX^Vi_hbc^aUf_]g^Uk_UfXYX"$$$

K cf_'gi Va]hhyx'zci fhyyb'fl%(EXUmg'Uzhyf'h\y'Xi Y'XUhy'k]``VY'a Uf_YX'UbX'zYYXVUW_'dfcj]XYX'Vi hbc'a Uf_ k]``VY'fYWcfXYX"'=Zh\Y'k cf_'k ci `X'\Uj Y'fYWr]j YX'U'dUgg'a Uf_'Vi hzcf'h\Y'`UhybYgg'UbX'h\Y'k cf_']g'U Wca di `gcfm'Wci fgY'Wca dcbYbhz'U'gh XYbh'k]``VY'XYYa YX'hc'\Uj Y'a Yhih\UhfYei]fYa Ybh''H\]g'XcYg'bch Udd`mhc'U'hUg_'h\Uh']g'UggYggYX'Vi hbc'a Uf_']g'Uk UfXYX"

K cf_'gi Va]hh/X'hk YbhnlcbY'f&&XUmg'UZhYf'h\Y'Xi Y'XUhY'k]```bchVY'UWWdhYX'Zcf'a Uf_]b['cf'ZYYXVUW_UbX'k]```fYWY]j Y'bc'a Uf_'cf'[fUXY"'=Zh\Y'UggYgga YbhHUg_']g'U'Wca di `gcfmWca dcbYbhcZh\Y'Wci fgY'U gh XYbhk]```Ui hca Uh]WU``mZU]`'h\Y'Wci fgY"

GdYWJU`7cbg]XYfUhJcb55dd`]WUhJcbg

 $\label{eq:linear_continuity} $$Mc_i = W_0 - W_$

G]W_bYggža]gUXj Ybhi fY'cf'ch\Yf'W]fW a ghUbWrg'VYmcbX'mci f'Wcbhfc``a Um

ł DfYj Ybhimci Zfca Wca d`Yhlb[U.Wci fqY fYei]fYa Ybhž

ł? YYd mci Zfca 'UthYbX]b['Ub 'UggYggUV'Y 'UWnji]tmž

ł Ghod moi gi Va]hh]b['UggYggUV'Y k cf_ Zcf'U Wci fgYž

ł `G][b]Z]\WbhmUZZY\Wimci f`dYfZcfa Ub\Wr`]b`UggYggUV`Y`k cf_ž`VY`]h'U`Zcfa U``YbX!cZgYa YghYf`Yl Ua]bUh]cbž`U \WUgg`hYghž`U``UVcfUhcfmhYghž`U`gYa]bUf`dfYgYbhUh]cb`cf`Ubmch\Yf`Zcfa 'cZUggYgga Ybh'

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UVW]XYbhU`mWcdm]b['Zfca `U`gci f\W' k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

±DUddfcdf]UhY`dUfUd\fUg]b[.'W\Ub[]b['U'ZYk'kcfXg'UbX'd\fUgYg'k\]`Y`acghmfYhJ]b]b['h\Y`cf][]bU`ghfi WhifY`UbX`]bZcfaUh]cb`k]h\ci hUW_bck`YX[YaYbh'H\]g'U`gc'Udd`]Yg`]b'dfYgYbhUh]cbg'k\YfY`gcaYcbYdUfUd\fUgYg`Ubch\Yf'g`]XYUg`cf'kcfXg'k]h\ci hWfYX]h''±iU`gc'Udd`]Yg`hc'd]YW]b['hc[Yh\Yf'ei chYg'UbXdUfUd\fUgYg`]bhc'U'bYk'k\c`Yžk]h\ci hfYZ/fYbW]b['UbX'U'ghi XYbh'g`ck b'UbU`mg]g`hc'Vf]b['h\Y`aUhYf]U`hc[Yh\Yf''

7 c``i g]cb. 'k cf_]b['k]h\ 'ch\ Yfg'Vi h'dUgg]b['c\Z'h\ Y 'k cf_'Ug'U'dYfgcb'g']b\X]j]\Xi U``k cf_"'7 c``i g]cb'U`gc]b\Wi XYg'dfcj]\X]b['mci f'k cf_'hc'Ubch\ Yf'ghi XYbh\ VYZcfY'h\ Y 'Xi Y 'XUhYzcf'Zcf'h\ Y' di fdcgY'cZh\ Ya d`U[]\Uf]g]b['Uh\Ubmh]a Yz'dUm]b['Ubch\ Yf'dYfgcb'hc'dYfZcfa 'Ub'U\WUXYa]\Wh\Ug_z'ghYU']b['cf'U\\ello i]f]b[Ubch\ Yf'dYfgcb'g' U\WUXYa]\Wk cf_'Ub\X'\\ello dm]b[']hz'c\ZYf]b['hc'\\ello a d`YhY'Ubch\ Yf'dYfgcb'g' k cf_'cf'gYY_]b[d\una Ybh\Zcf'\\ello a d`Yh\beta d`

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[ˈh\Y`″gYWcbXUfm″gci fW/Zfca k \]WX_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ff`gY`Zld`U[]\Uf]ga "Ł`gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h`\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UqqYqqa Ybhcf\Wci fgY`UhI_BGK_`cf`Ubch\Yf`i b]\Yfq]\m"

7 cffYWhfYZYfYbWb[dfUWhWyg.

• VYHYfa UbU[Ymcifh]a Y

7ci fgY GWXYXi `Y

J]Yk 'WUgg'ha YHUV'Y

H]a YtUV`Y

8 UhY	HmdY	7 cbhYbh	
K YY_'%'&*': YVfi Ufm!'(A UfVX	@YWi fY	HfUbg`Uhjcb`UbX`]bhYfdfYhjb[`]b`UWUXYa]WUbX dfcZYgg]cbU``gYhjb[g"	
		HU_Y`\ca Y`hYb`gYbhYbVWg	
K YY_'&.') 'A UfW\'!'\% A UfW\	Hi hcf]U` GYUfWX]b['Zcf'Yei]j U`YbWY'UWfcgg'W' `hi fYg"		
		6]!`]b[iU``dfYdUfUh]cb`UbX`fYgYUfWX"`5WWfUWmiUbX Yei]jU`YbWY.`]blfU`]b[iU``lfUbg`Uh]cb`fbUaYgž]X]cagŁ"`GYUfWX]b[`Zcf`Yei]jU`YbWY`UWfcgg`W``hifYg"	
K YY_'' .'%&'A UfWX'!'% A UfWX	@YVVni fY	5i X]c!j]gi U```YWi fY. HfUbg`Uh]cb`fGhYd\Yb 8c\Yfmz"	
	5ggYgga Ybh	: cffb][\hm5gg][ba Ybh%hc 'VY'gi Va]hhYX'Vm('da cb'%* 'A UfW. 'HfUbg`UhY'U'gY`YWhYX'dUggU[Y'UbX dfcj]XY'U'Wca a YbhUfmcb'W.U``Yb[Yg'UbX hYW.b]ei Yg'i gYX"	
K YY_'(.'%-'A UfVX'!'&)	Hi hcf]U`	HfUbgWf]dh]cb`UbX`gi Vh]h]b[
A UfWX	@YWhi fY	5i X]c!j]gi U```YWhi fY. `\DhYfdfYh]b['fGUbXfU' <u`ył< td=""></u`ył<>	
		H\]g``YWhifY`]g`gW\YXi``YX`Zcf`KYY_`(ž`]bghYUX`cZ KYY_`)žhc`Ujc]X`U`WUg\`k]h\`h\Y`diV`]W\c`]XUmcb :f]XUm'\$`AUfW\flccX`:f]XUmL"	
K YY_^).^&*^A UfWX!^% 5df]`	5ggYgga Ybh	: cffb][\hmUgg][ba Ybh&'hc'VY'gi Va]h\X'Vm('da cb'&- 'A UfVX.'HfUbgWf]VY'U'@C H9'j]XYc'hc'dfcj]XY]bffU']b[i U`'UbX']bhYf]b[i U`'gi Vh]hYg"	
6fYU'&'5df]`'!', '5df]`			
K YY_'*.'-'5df]`'!'%)'5df]`	Hi hcf]U`	⇒hYfdfYh]b[ˈ]bˈX]ZZYfYbha cXYg	
K_YY_`+.`%*`5df]``!`&& 5df]`	@YWhifY	J]XYc ``YWi fY. '7 fcgg!W' `hi fU``]bhYfUWi]cb fBYVfU 5 UfcbgŁ	
	5ggYgga Ybh	: cffb][\hm5gg][ba Ybh' 'fc VY'gi Va]ffYX'Vm('da &\$'5df]`.'Gi Va]hmci f'ffUbgWf]dhcZh\Y'gci fW' UbX mci f']bfYfdfYfUf]cb']b'H@	
K_YY_', .'&' '5df]`'!'&- 5df]`	Hi lcf]U`	7 ca a i b]\WJrjb['W' 'ri fU``m!Vci bX'rlYfa g'UbX Wcb\Wdrg'fl\UgYX'cb'Zcfrb][\nmUgg][ba Ybrg'UbX fYZYWfj Y'Ya YbriZfca 'rh\Y'gi Vrjh'jb['Ugg][ba Ybrk	
K YY_''' \$'5df]`'!'*'A Um	@YWhifY	J]XYc^YWn fY.^DfcZYgg]cbU^9h\]Wg^]b^HfUbg`Uh]cb// =bhYfdfYh]b[^f@ Xa]`U^GhYfbŁ	
	5ggYgga Ybh	: cftb][\hm5gg][baYbh('hc'VY'giVa]hhYX'Vm('dacb'('AUm'Nci'Wb'WkcgY'mcif'Ugg][baYbhVUgYXcb'UbmcbY'cZh\Y'dfYj]cig'acXYg.'hfUbg`Uh]cbžgiVh]h]b['cf']bhYfdfYh]b['cZUb'UiX]c'cf'U'j]XYc"	

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

• A U`a _^A fž?"'/ `K]bX`Yž?"'&\$%%"`H\Y`C | ZcfX`< UbXVcc_`cZHfUbg`Uh]cb`Gh`X]Yg"`C | ZcfX`| b]j Yfg]hm DfYgg"`3hS['\$T'W\$f? S[`>[Tc\$ck /&#*Z"\$!\$') fi[`fZW: [YZ GeWUa^\WUf[a`/\$`V Xaad >[Tc\$ck 'Si` WhWfž 7 \WUfca`[U Uabk S'ea ShS['\$T W\$f fZWG@E | >[Tc\$ckž

FYWca a YbXYX FYgci fWYg

K YY_'% 'Gi gUb '6UggbYhž'7\UdhYf'+"'HfUbg`Uhcf'Ug'Wfcgg!Wi`hi fU`a YX]Uhcfž'dd"'- (-\%, ž']b'H\Y'C | ZcfX < UbXVcc_'cZHfUbg`Uhjcb'Ghi X]Yg"

5 Wcbj YfgUhjcb VYhk YYb GUbXfU

