

# **ARTS3368**

5Xj UbWYX'GcWJU`'UbX'Dc`]hJWU`'D\]`cgcd\m

GYa YghYf'C bY''##'&\$%

# 7ci fgY<sup>·</sup>Cj Yfj ]Yk

## GHJZZ:7cbHJWh:8YHJ]`g`

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUh]cb	D\cbY

### 7ci fgY 8YHJ] g

7 fYX]hDc]bhg \* \*

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. BZ['aeabZk H\]g:Wci fgY:WUb:U`gc:VY`ghi X]YX`]b:h\Y:Zc``ck ]b[ 'gdYW]U`]gUh]cb. Ba:{ff[Ue

H\]g`Wci fgY`dfcj ]XYg`Ub`]b!XYdh\ Yb[ U[ Ya Ybhk ]h\ `gY`YWhYX`h\Ya Yg`]b`gc\]U``UbX`dc`]h]\WJ``d\]cgcd\m'`±h k ]``XYj Y`cd`mci f`i bXYfgHJbX]b[ `cZgca Y`cZh\Y`Z bXUa YbhJ``gc\]U``UbX`dc`]h]\WJ``W\U``Yb[ Yg`ci f \\cb\Ya dcfUfmgc\]Yh]Yg`ZJ\WY`ZUbX`\ck `hc`UXXfYgg'h\Ya `d\]`cgcd\]\WJ``m'`: cf`YI Ua d`Yžk Y`k ]``X]g\W gg'h\Y \\Zci bXUh]cbU``fc`Y`cZfY\\cb[ b]h]cb`Zcf`\i a Ub`dYfgcb\ccXzgc\]U`]\mz`Ug`k Y```Ug`]bX]j ]Xi U``UbX`\ck``Y\\f]j Y\]XYbh]\m'h\Y`a YhJd\mg]\\g`cZgc\]U``fYU`]\m'dfcV`Ya g`cZUi h\cf]\mz`dck Yf`UbX`\ZfYYXca `]b`h\Y`a cXYfb`k cf`X/dgn\X\cUbU`mg]g`UbX`h\Y`\cbgh]h h]cb`cZgi VY\\f]j ]\mz`Ua cb[ `ch\Yfg"

#### 5 hith Y WcbWi g]cb cZth ]g Wci fgY th Y ghi XYbhik ]``VY UV Y the

%" 91 d`U]b`Z bXUa YbhU``h\Ya Yg`UbX`WcbWYdhg`]b`gc\WJU``UbX`dc`]h]\WJ\`d\]`cgcd\m' &" 5dd`m\Wf]h]\WJ\`UbX`Wcb\Wdhi U``h\ci [\hhc`\WcbhYa dcfUfmgc\WJU\`UbX`dc`]h]\WJ\`fYU\`]hm' ..."

kl``XYiY`c ayiy c YX

### 5ggYgga Ybh

5Xj ]gY'cb'h\Y'UggYga Ybh'HUg\_g'k ]``'VY'dfcj ]XYX']b'WUgg"

#### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'HJg_	K Y][ \h	8i Y <sup>.</sup> 8UhY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
G\cfhiYggUmi	(\$1	&, #\$' #&\$% `\$(.\$\$`DA	% <u>ž&amp;</u> Ž
@cb[ 'YggUm'	* \$1	' \$#\$) #&\$% `\$(.\$\$`DA	%ž&Ž

#### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% G\cfhYggUm

GHJfh'XUhY. '&' #\$' #&\$% '\$(.\$\$'DA

@Yb[h\. A UI " %) \$\$ k cfXg

8YHJ]`g. `G\cfhYggUmža UI ]a i a '%) \$\$ k cfXg": YYXVUW\_ ]g dfcj ]XYX j ]U ]bX]j ]Xi U``Wta a Ybhg cf`fi Vf]W

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' Hi & fb \end{tabular} $$ h' \end{tabular}$ 

5ggYgga Ybh& @cb[ YggUm

GhUfhiXUhY. '&' #\$) #&\$% '\$(.\$\$ DA

@Yb[ h\. `A UI " '&) \$\$ 'k cfXg"

**8YHJ]`g**. '@cb[ Yf`YggUmž'&) \$\$`k cfXg"`: YYXVUW\_`]g`dfcj ]XYX`j ]U`]bX]j ]Xi U``Wca a Ybhg`cf`fi Vf]W"H\]g`]g`h\Y ZbU``UggYgga YbhZcf`UhYbXYbWY'di fdcgYg"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$ 

#### Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gh XYbhg UfY Yl dYWYX hc di hh\Y]f bUa Yg UbX gh XYbhbi a VYfg cb Yj YfmdU[Y cZh\Y]f Ugg][ ba Ybhg"

#### Hi fb]h]b Gi Va ]gg]cb

Imagi YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b[ 'rc'gi Va ]hmci f'Ugg][ ba Ybht\fci [ \'Hi fb]r]bž'd'YUgY hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-', ) "''' %cf'Ya U] hYa 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u> "Gi ddcfh \ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'" Imai UfY'i bUV'Y 'rc'gi Va ]hmci f'Ugg][ ba YbhXi Y'rc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'YI hYbg]cbž'Vi hmci a i ghfYhJ]b'mci f'h]\W\_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgt'rc ]bWi XY'Ug'Yj ]XYbW' 'rc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WJhjcb"'\Imai 'Ya U]''9I hYfbU''Gi ddcfhmci 'k ]'`Ui hca Uh]\WJ'`mfY\W']j Y'U'h]\W\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]'`'bYYX'rc'gdY\WZ\WJ'`mUg\_'Zcf'cbY" Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk ]hYf"

; YbYfU`mž'UggYgga YbhitUg\_g'a i ghVY'gi Va ]hhYX'Y`YWfcb]WU``mj ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bghUbWfg'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'ghUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYhU]`g"

#### @UhY 5ggYgga Ybh'DYbU`h]Yg

5b'UggYggYX'HUg\_']g'XYYa YX``UhY']Z]h']g'gi Va ]hhYX'UZhYf'h\Y'gdYW]Z[YX'h]a Y'UbX'XUhY'Ug'gYhci h']b'h\Y Wci fqY'@YUfb]b['A UbU[Ya YbhGmqhYa 'f@A GŁ"

 $\label{thm:condition} $$HY^UHY^dYbU^m]g^hY^cgg^cZ)_L^cZ^hY^hcHU^dcgg]VY^aUf_g^Zcf^hYY^HUg_Z^cf^YUW^XUmcf^dUfhhYfYcZhY^kcf_]g^UHY^@UhYbYgg^k]^*]bW^XY^kYY_YbXg^UbX^di_V^]WYc^]XUmg^"HY]g^XcYg^bchUdd^mhc^U'HUg_'hWh]g^UqqYqqYX^Vi_hbc^aUf_]g^Uk_UfXYX"$$$ 

K cf\_'gi Va ]hhYX'Zci fhYYb'fl%( Ł'XUmg'UZhYf'h\Y'Xi Y'XUhY'k ]```VY'a Uf\_YX'UbX'ZYYXVUW\_'dfcj ]XYX'Vi hbc'a Uf\_ k ]```VY'fYWcfXYX"'=Zh\Y'k cf\_'k ci `X'\Uj Y'fYWl]j YX'U'dUgg'a Uf\_'Vi hZcf'h\Y'`UhYbYgg'UbX'h\Y'k cf\_']g'U Wca di `gcfmWci fgY'Wca dcbYbhz'U'gh XYbhk ]```VY'XYYa YX'hc'\Uj Y'a Yhh\UhfYei ]fYa Ybh''H\]g'XcYg'bch Udd`mhc'U'hUq\_'h\Uh]g'UggYggYX'Vi hbc'a Uf\_']g'Uk UfXYX"

K cf\_'gi Va ]hh/X'hk YbhnlcbY'f&&XUmg'UZhYf'h\Y'Xi Y'XUhY'k ]```bchVY'UWWdhYX'Zcf'a Uf\_]b[ 'cf'ZYYXVUW\_UbX'k ]```fYWY]j Y'bc'a Uf\_'cf'[ fUXY"'=Zh\Y'UggYgga YbhHUg\_']g'U'Wca di `gcfmWca dcbYbhcZh\Y'Wci fgY'Ugh XYbhk ]```Ui hca UhJWU``mZU]`'h\Y'Wci fgY"

#### GdYWJU`7cbg]XYfUhJcb55dd`]WUhJcbg

 $\label{eq:linear_continuity} $$Mc_i = W_0 - W_$ 

G]W\_bYggža ]gUXj Ybhi fY'cf'ch\Yf'W]fW a ghUbWrg'VYmcbX'mci f'Wcbhfc``a Um

ł DfYj Ybhimci Zfca Wca d`Yhlb[ U.Wci fqY fYei ]fYa Ybhž

ł? YYd mci Zfca 'UthYbX]b[ 'Ub 'UggYggUV`Y 'UWhj ]tmž

ł Ghod moi gi Va ]hh]b[ 'UggYggUV'Y'k cf\_ Zcf'U'Wci fgYž

ł `G][ b]Z]\WbhmUZZY\Wimci f`dYfZcfa Ub\Wr`]b`UggYggUV`Y`k cf\_ž`VY`]h'U`Zcfa U``YbX!cZgYa YghYf`Yl Ua ]bUh]cbž`U \WUgg`hYghž`U``UVcfUhcfmhYghž`U`gYa ]bUf`dfYgYbhUh]cb`cf`Ubmch\Yf`Zcfa 'cZUggYgga Ybh'

#### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UVW]XYbhU`mWcdm]b[ 'Zfca `U`gci f\W' k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]uf]ga ]bhch\YZc``ck]b[ WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a ]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k ]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci h'WfYX]h'

±DUddfcdf]UhY`dUfUd\fUg]b[.'W\Ub[]b['U'ZYk'kcfXg'UbX'd\fUgYg'k\]`Y`acghmfYhJ]b]b['h\Y`cf][]bU`ghfi WhifY`UbX`]bZcfaUh]cb`k]h\ci hUW\_bck`YX[YaYbh'H\]g'U`gc'Udd`]Yg`]b'dfYgYbhUh]cbg'k\YfY`gcaYcbYdUfUd\fUgYg`Ubch\Yf'g`]XYUg`cf'kcfXg'k]h\ci hWfYX]h''±iU`gc'Udd`]Yg`hc'd]YW]b['hc[Yh\Yf'ei chYg'UbXdUfUd\fUgYg`]bhc'U'bYk'k\c`Yžk]h\ci hfYZ/fYbW]b['UbX'U'ghi XYbh'g`ck b'UbU`mg]g`hc'Vf]b['h\Y`aUhYf]U`hc[Yh\Yf''

7 c``i g]cb. 'k cf\_]b[ 'k ]h\ 'ch\ Yfg'Vi h'dUgg]b[ 'c\Z'h\ Y 'k cf\_'Ug'U'dYfgcb'g']b\X]j ]\Xi U``k cf\_"'7 c``i g]cb'U`gc ]b\Wi XYg'dfcj ]\X]b[ 'mci f'k cf\_'hc'Ubch\ Yf'ghi XYbh\ VYZcfY'h\ Y 'Xi Y 'XUhYzcf'Zcf'h\ Y' di fdcgY'cZh\ Ya d`U[ ]\Uf]g]b[ 'Uh\Ubmh]a Yz'dUm]b[ 'Ubch\ Yf'dYfgcb'hc'dYfZcfa 'Ub'U\WUXYa ]\Wh\Ug\_z'ghYU']b[ 'cf'U\\ello i ]f]b[ Ubch\ Yf'dYfgcb'g' U\WUXYa ]\Wk cf\_'Ub\X'\ello dm]b[ ']hz'c\ZYf]b[ 'hc'\\ello a d`YhY'Ubch\ Yf'dYfgcb'g' k cf\_'cf'gYY\_]b[ d\una Ybh\Zcf'\\ello a d`Yh\beta d`Y

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW\_bck`YX[]b[ˈh\Y`″gYWcbXUfm″gci fW/Zfca k \]WX`\_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ff`gY`Zld`U[]\Uf]ga "Ł`gi Va]\h]b[`mci f`ck b`k cf\_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h`\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UqqYqqa Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]\Yfq]\m"

7 cffYWhfYZYfYbWb[dfUWhWyg.

• VYHYfa UbU[Ymcifh]a Y

	-	
1		

	5ggYgga Ybh	: ]fghiYggUmXi Y': f]XUmžA UfWX'' \$ž'(da 'j ]U'Z]`Y i d`cUX'h\fci [ \'h\Y'fY`Yj Ubh`]b_']b'h\Y'A ccX`Y'dU[ Y Zcf'h\]g'Wti fgY"
6fYU'&'5df]`'!', '5df]`		10
K YY_'*.'- '5df]`'!'%) '5df]`	@YWhifY	FYWc[b]h]cbžai`h]W`hifU`]ga'UbX'Yh\b]Wa]bcf]h]Yg
	Hi hcf]U`	Hi hcf]U`cb'U'WUgg]WhYI hcb'h\YgY'h\Ya Yg"
		F Yei ]fYX fYUX]b[. 'HUmcfž7\UfYgž'H\Y 'dc`]h]Wg cZ fYWc[b]h]cb'ž]b'; i ha Ubž5a mfYX"½"? g f[Ug fgc6 fe_ 7 j S_[[ [ Y fZWBa ff[Ue aXD WUaY` [f[a` žDf]bWhcb   b]j Yfg]hmDfYggž% - ( ždd" &)!+' "
K_YY_"+."%*"5df]`"!"&&	@YWhifY	FYWc[b]h]cb'UbX'dgmWkcUbU`mg]g
5df]`	Hi lcf]U`	Hi hcf]U``cb`U`W\UdhYf`Zfca`U`\i[Y`m`]bhYfYgh]b[`Vcc_cb`cV^YWhfY`Uh]cbU``dgmW\cUbU`mg]g`h\Uhi h]`]nYg`h\Y <y[y`]ub`z][ify`czh\y`aughyf`ubx`h\y`g`ujy"< td=""></y[y`]ub`z][ify`czh\y`aughyf`ubx`h\y`g`ujy"<>
		F Yei ]fYX fYUX]b[ . 6Yb Va ]bž >Ygg]WUž 4a` Ve aX >ahWž DUbh Ycbž %,, ž ddž) %, ( fgYWcbX W UdhYfŁ"
K_YY_', . '&' '5df]`'! '&- 5df]`	@YWnifY	5   Y`` <cbbyh\`ubx`h\y`fyw¢[b]h]cb!dufux][a`]b Wf]h]WU`gcW]U`d\]`cgcd\m"</cbbyh\`ubx`h\y`fyw¢[b]h]cb!dufux][a`]b 
out j	Hi hcf]U`	Hi hcf]U``cb`gYW]cbg`Zca `cbY`cZh\Y`a cgh]bZi Ybh]U` Vcc_g`]b`gcW]U``UbX`dc`]h]WU``d\]`cgcd\mUbX h\Ycfm]b`h\Y`fYWYbhXYWUXYg"`H\Y`Vcc_'i h]`]nYg VUg]W]XYUg`Zca ` <y[y`ž`ug`k y```ug`="">Ygg]WU 6Yb^Ua]bž`Ua cb[`ch\Yfg"</y[y`ž`ug`k>
		F Yei ]fYX fYUX]b[ . '< cbbYh\ž5 I Y`žFZWE fcgYY WXad D WUaY` [f[a` žA +H'DfYggž% - *ždd"' - &!% %ždd" %* \$!% \$"
K YY_'" \$'5df]`!**A Um	@YWi fY	B UbWn: fUgYffg Wf]njei Y cZ < cbbYh\žUbX \Yfck b kk c!X]a Ybg]cbU`UddfcUW\nc gcWjU`^ gnjW"=g < cbbYh\fg fYWc[b]njcb!dUfUX][a 'j i 'bYfUV'Y'hc : fUgYffg Wf]njei Y3
	Hi hcf]U`	Hi hcf]U``cb`h\Y`cdYb]b[ `cZ: fUgYffg`XYVUhY`k]h\ <cbbyh\"< td=""></cbbyh\"<>
		F Yei ]fYX fYUX]b[ .: fUgYfžBUbWm/ '< cbbYh\ž51 Y`ž DW[efd[Tgf[a` adDWJaY` [f[a`13Ba1ff[US/Ž BZ['aeabZ[US^7j UZS`YWfJ Yfgcž'&\$\$' ž-!*-"
K YY_'%\$.'+'A Um!'% A Um	@YWn fY	: fca 'h\Y'fYWt[b]h]cb!dUfUX][a 'hc'U'fY\UV]]hUh]cb cZfgcW]U'tZYYXca 'Ug'h\Y'_YmWtbWYdh']b'Wf]h]WU' gcW]U'd\]cgcd\m''FYWU]a ]b['h\Y'WtbWYdhcZ ZYYXca 'Zfca 'h\Y'`]VYfU'']bX]j ]Xi U`]ghg'UbX'h\Y ? Ubh]Ub'a cfU`]ghg"
	Hi hcf]U`	Hi hcf]U``cb`h\Y`cdYb]b[ `cZ <cbbyh\fg`a a<br="" bi="" u[="">cdi g"</cbbyh\fg`a>
		F Yei ]fYX'fYUX]b[.'8dWWa_

## Fygci fWyg

DfYgWf]VYX F Ygci fWYg

 $\label{eq:hammer_model} $$H\Y'kYY_mfYUX]b[g'k]^*VY'UjU]^UV'Y'cb'AccX'Y''$$ 

FYWca a YbXYX FYgci fWYg

@]ghcZfYWca a YbXYX gYWcbXUfma UhYf]U``k ]```VY`Uj U]`UV`Y`cb`A ccX`Y"

7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y`cda Ybh