

ARTS3450

5Xj UbWX 7\]bYgY 5

GYa YghYf'C bY''##'&\$%

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m				

cf][]bU``cf`VWffjZJYX`WcdmcZU`a YX]WU``WVffjZJWUHY`cf`ch\Yf`Zcfa `cZUddfcdf]UHY`Yj]XYbWV"

5 '7 ci fgY '5 i h\cf]hma UmYl W gY 'U'ghi XYbhiZica 'WUggYg'cf'UWji jhjYg'Zcfi d'hc'cbY 'a cbh\"'< ck Yj Yfžh\Ym a UmUgg][b'UXX]hjcbU``UbX#cf'U`hYfbUhj Y'HUg_g'hc 'Ybgi fY'Wca d`]UbW\"'5 '7 ci fgY '5 i h\cf]hm\%bg]XYf]b['h\Y [fUbh]b['cZUVgYbW' 'a i gh'VY'gUhjgZ]YX'U'ghi XYbhk]``'gh]``'VY'UV'Y'hc'a YYhh\Y'Wci fgY'g'`YUfb]b['ci h\can Yg UbX#cf' c`i a Y'cZ'YUfb]b["'5 'ghi XYbhigYY_]b['Uddfcj U`'hc'VY'UVgYbhiZcf'a cfY'h\Ub'cbY'a cbh\'a i gh'Udd`m]b'k f]hjb['hc'h\Y'8YUb'UbX'dfcj]XY'U``cf][]bU`'cf'WYfh]ZYX'gi ddcfh]b['XcW a YbhUhjcb"

: cfa cfY`]bZcfa Uhjcb`UVci hih\Y`UhhYbXUbWY`dfchcWc`g`]b`h\Y`: UW\`hmcZ5fhg`UbX`GcWJU` GWJYbWYg.`\hhdg.#k_k_k_"Ufhg"i bgk "YXi "Ui #W ffYbHghi XYbhg#UWUXYa]W]bZcfa Uhjcb#dfchcWc`g![i]XY`]bYg#

5 WUXYa]W = b Zcfa Uh]cb

: cf`YggYbh]U``ghi XYbh`]bZcfa Uh]cb`fY`Uh]b['hc.`fYei Yghg`Zcf`Yl hYbg]cb/fYj]Yk 'cZa Uf_g/cWW dUh]cbU``\YU`h\
UbX`gUZYhm'Yl Ua]bUh]cb`dfcWXi fYg/gdYV]U``Wcbg]XYfUh]cb`]b'h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
ghi XYbhYei]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbh]U``UWUXYa]W]bZcfa Uh]cbž
gYY`\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhghi XYbhg#JWUXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

7ci fgY'8YW]`g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y'7ci fgY

GIVYWn5fYU.5Z[WeWEfgV[We

H\]g'Wci fgY'dfcj]XYg'UXj UbWXX'hi]hJcb']b'7\]bYgY'`Ub[i U[Y'fA UbXUf]bE'gh Xmzi g]b['U'hYl hVcc_XYg][bYX gdYW]Z]WU'`mZcf'UXj UbWXX'7\]bYgY''YUfbYfg'Uni b]j Yfg]hJYg'ci hg]XY'cZ7\]bU"±iWca V]bYg'`Ub[i U[Y'gh X]Yg k]h. Ub Yl d`cfUhJcb'cZ7\]bYgY'W 'hi fY''UbX'gcWJYhmzi]bWi X]b['ifUX]hJcbU'W'hi fY'''H\Y'a YX]i a 'cZ]bghfi Whjcb']g'A UbXUf]b"'Mci 'WcbhJbi Y'hc'XYj Y`cd'UbX'Yl hYbX'mci f'i bXYfghUbX]b['UbX'i gY'cZ7\]bYgY 'Ub[i U[YžYgdYW]U'`ma cfY'gdYWJU']nYX'j cWUvi 'UfmUbX'Wca d'Yl 'gYbhYbW'ghfi Whi fYg"'H\Y'Ya d\Ug]g']g'cb h\Y'hfUbgZcfa UhJcb'cZ']b[i]ghJW_bck 'YX[Y']bhc'UWhj Y'g_]`g'h\fci [\'cfU'`UbX'k f]hhYb'UWhj jhJYgžZcW g]b['cb Ui h\YbhJW7\]bYgY 'HYl hg'Vmi hj`]g]b['j Uf]ci g'fYUX]b['gHUhY[]Yg"'H\Y'Wc]W'cZhYl h'UbX'fY'UhYX'a UhYf]U'g UfY'VUgYX'cb'tk c'Wf]hYfJU.'%\H\Ymg\ci 'X'gYfj Y'Ug'U'gUa d'Y'cZVYgh`Ub[i U[Y'Zcf'gh XYbhg'hc'YUfb'UbX]a]hUhYz'UbX'&E'H\Y'WcbhYbhg'cZYUW\hYl hg\Ci 'X'\Uj Y'U'gHfcb['fY'Yj UbW'k]h\ WffYbh7\]bYgY'gcWJYhm UbX'W'hi fU''hYl hg'Wcj Yf'U'k]XY'fUb[Y'cZhcd]Wg']b'7\]bYgY''Ub[i U[YžW'hi fY'UbX'gcWJYhmz'bWifYZ]''mWXcgYb'hYl hg'k]'`gYfj Y'Ug'U'k]bX'h\Y'BcVY`df]nY'k]bbYf'Zcf'`]hYfUhi fYz'A f"'A c'MJb'33"' H\YgY'WJfYZ '`mWXcgYb'hYl hg'k]``gYfj Y'Ug'U'k]bXck 'h\fci [\'k\]W\'mci 'k]``gYY''7\]bU'Uhk cf_z'ZYY']b[']hg'c['UbX'[fYUhYYg'k\]'Yufy'U'gc'Uk UfY'cZ]hg'WU''Yb[Yg'UbX'dfcV'Ya g"

5 hh Y WcbWi g]cb cZh]g Wci fgY h Y gh XYbhk] ` VY UV Y hc

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5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
7 i 'h fU` 7 ca dcbYbh	&\$I	Ei]n'%cb'%+#\$(#&\$% 'fk YY_ +Ł/Ei]n'&'cb'&&#\$)#&\$% fk YY_'%&Ł	(
; fci d'F YgYUfWX'Dfc'YWfg'	(\$1	HfUbg`Uh]cbg`Xi Y`]b`k YY_g ' ž*ž-ž‰'7`Ugg`dfYgYbhUh]cb Xi Y`]b`k YY_g`%\$ž`‰'cf`‰/ UbX`fYgYUfW`dUdYf`Xi Y`cbY k YY_`UZhYf`h\Y`dfYgYbhUh]cb"	%ž&Ž Ž)
HYgh%	&\$I	&\$#\$(#&\$% `\$\$\$`5A	%ž&Ž Ž(
HYgh'&	&\$ı	\$%#\$*#&\$% `\$\$\$`5A	%ž&Ž Ž(

5ggYgga Ybh'8YhJ]`g

5qqYqqa Ybh% 7i `hi fU` 7ca dcbYbh

GhUfh'XUhY.

8 YHJ]`g. `Gh XYbhg`UfY`[]j Yb`hk c`]b!WUgg`ei]nnYg`fl \$`a]bi hYg`YUW\<u>L</u>": YYXVUW<u>'</u>j]U`fi Vf]WUbX`]b!WUgg X]gW gg]cb"

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh&; fci d F YgYUfW Dfc YWg

GhUfh'XUhY.

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$

5ggYgga Ybh' . 'HYgh'%

GhUfh'XUhY.

8 YhJ]`g. '7 ca dfY\Ybg]j Y`hYghcZh\Y`Z|fghZYk 'W\UdhYfg`f%") '\ci fg\!": YYXVUW_'j]U`]bX]j]Xi U``Wca a Ybhgž fi Vf]WžUbX`]b! WUgg`X]gW gg]cb"

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5ggYgga Ybh(. HYgh&

GhUfh'XUhY.

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gh XYbhgʻUfYʻYl dYWYX`hcʻdi hh\Y]fʻbUa YgʻUbXʻgh XYbhibi a VYfgʻcbʻYj YfmdU[YʻcZh\Y]fʻUgg][ba Ybhgʻ

Hi fb]h]b Gi Va]gg]cb

Imagi YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-',) "''' %cf'Ya U] hYa 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u> "Gi ddcfh \ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'" Imai UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y'rc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'YI hYbg]cbž'Vi hmci a i ghfYhJ]b'mci f'h]\W_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgt'rc]bWi XY'Ug'Yj]XYbW' 'rc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WJhjcb"'\Imai 'Ya U]''9I hYfbU''Gi ddcfhmci 'k]'`Ui hca Uh]\WJ'`mfY\W']j Y'U'h]\W_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'`'bYYX'rc'gdY\WZ\WJ'`mUg_'Zcf'cbY" Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU`mž'UggYgga YbhitUg_g'a i ghVY'gi Va]hhYX'Y`YWfcb]WU``mj]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bghUbWfg'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'ghUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYhU]`g"

@UhY 5ggYgga Ybh'DYbU`h]Yg

5b'UggYggYX'HUg_']g'XYYa YX``UhY']Z]h']g'gi Va]hhYX'UZhYf'h\Y'gdYW]Z[YX'h]a Y'UbX'XUhY'Ug'gYhci h']b'h\Y Wci fqY'@YUfb]b['A UbU[Ya YbhGmqhYa 'f@A GŁ"

$$\label{thm:condition} \begin{split} &\text{H}^{\text{U}$$

K cf_'gi Va]hhyx'zci fhyyb'fl%(EXUmg'Uzhyf'h\y'Xi Y'XUhy'k]``VY'a Uf_YX'UbX'zYYXVUW_'dfcj]XYX'Vi hbc'a Uf_ k]``VY'fYWcfXYX"'=Zh\Y'k cf_'k ci `X'\Uj Y'fYWr]j YX'U'dUgg'a Uf_'Vi hzcf'h\Y'`UhybYgg'UbX'h\Y'k cf_']g'U Wca di `gcfm'Wci fgY'Wca dcbYbhz'U'gh XYbh'k]``VY'XYYa YX'hc'\Uj Y'a Yhih\UhfYei]fYa Ybh''H\]g'XcYg'bch Udd`mhc'U'hUg_'h\Uh']g'UggYggYX'Vi hbc'a Uf_']g'Uk UfXYX"

K cf_'gi Va]hh/X'hk YbhnlcbY'f&&XUmg'UZhYf'h\Y'Xi Y'XUhY'k]```bchVY'UWWdhYX'Zcf'a Uf_]b['cf'ZYYXVUW_UbX'k]```fYWY]j Y'bc'a Uf_'cf'[fUXY"'=Zh\Y'UggYgga YbhHUg_']g'U'Wca di `gcfmWca dcbYbhcZh\Y'Wci fgY'U gh XYbhk]```Ui hca Uh]WU``mZU]`'h\Y'Wci fgY"

GdYWJU`7cbg]XYfUhJcb55dd`]WUhJcbg

 $\label{eq:linear_continuity} $$Mc_i = W_0 - W_$

G]W_bYggža]gUXj Ybhi fY'cf'ch\Yf'W]fW a ghUbWrg'VYmcbX'mci f'Wcbhfc``a Um

ł DfYj Ybhimci Zfca Wca d`Yhlb[U.Wci fqY fYei]fYa Ybhž

ł? YYd mci Zfca 'UthYbX]b['Ub 'UggYggUV`Y 'UWhj]tmž

ł Ghod moi gi Va]hh]b['UggYggUV'Y'k cf_ Zcf'U'Wci fgYž

ł `G][b]ZWUbhmUZYWimci f`dYfZcfa UbWi`]b`UggYggUV`Y`k cf_ž`VY`]hU`Zcfa U``YbX!cZgYa YghYf`Yl Ua]bUh]cbž`U`WUgg`hYghž`U``UVcfUhcfmhYghž`U`gYa]bUf`dfYgYbhUh]cb`cf`Ubmch\Yf`Zcfa 'cZUggYgga Ybh'

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UVW]XYbhU`mWcdm]b['Zfca `U`gci f\W' k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

±DUddfcdf]UhY`dUfUd\fUg]b[.'W\Ub[]b['U'ZYk'kcfXg'UbX'd\fUgYg'k\]`Y`acghmfYhJ]b]b['h\Y`cf][]bU`ghfi WhifY`UbX`]bZcfaUh]cb`k]h\ci hUW_bck`YX[YaYbh'H\]g'U`gc'Udd`]Yg`]b'dfYgYbhUh]cbg'k\YfY`gcaYcbYdUfUd\fUgYg`Ubch\Yf'g`]XYUg`cf'kcfXg'k]h\ci hWfYX]h''±iU`gc'Udd`]Yg`hc'd]YW]b['hc[Yh\Yf'ei chYg'UbXdUfUd\fUgYg`]bhc'U'bYk'k\c`Yžk]h\ci hfYZ/fYbW]b['UbX'U'ghi XYbh'g`ck b'UbU`mg]g`hc'Vf]b['h\Y`aUhYf]U`hc[Yh\Yf''

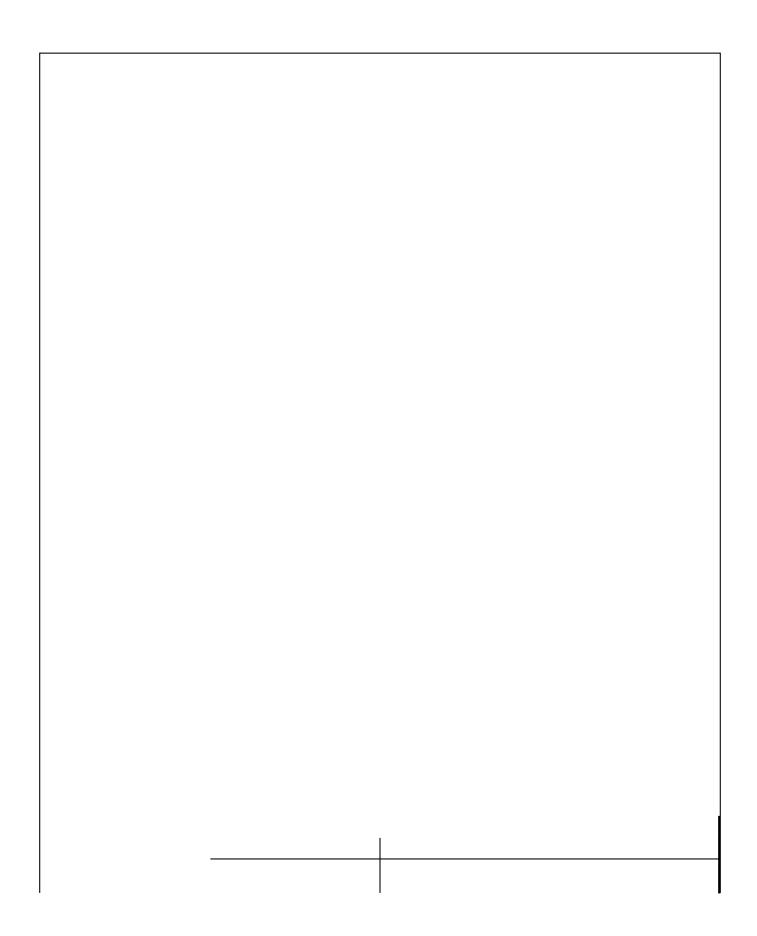
7 c``i g]cb. 'k cf_]b['k]h\ 'ch\ Yfg'Vi h'dUgg]b['c\Z'h\ Y 'k cf_'Ug'U'dYfgcb'g']b\X]j]\Xi U``k cf_"'7 c``i g]cb'U`gc]b\Wi XYg'dfcj]\X]b['mci f'k cf_'hc'Ubch\ Yf'ghi XYbh\ VYZcfY'h\ Y 'Xi Y 'XUhYzcf'Zcf'h\ Y' di fdcgY'cZh\ Ya d`U[]\Uf]g]b['Uh\Ubmh]a Yz'dUm]b['Ubch\ Yf'dYfgcb'hc'dYfZcfa 'Ub'U\WUXYa]\Wh\Ug_z'ghYU']b['cf'U\\ello i]f]b[Ubch\ Yf'dYfgcb'g' U\WUXYa]\Wk cf_'Ub\X'\ello dm]b[']hz'c\ZYf]b['hc'\\ello a d`YhY'Ubch\ Yf'dYfgcb'g' k cf_'cf'gYY_]b[d\una Ybh\Zcf'\\ello a d`Yh\beta d`Y

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[ˈh\Y`″gYWcbXUfm″gci fW/Zfca k \]WX`_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ff`gY`Zld`U[]\Uf]ga "Ł`gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h`\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UqqYqqa Ybhcf\Wci fgY`UhI_BGK_`cf`Ubch\Yf`i b]\Yfq]\m"

7 cffYWhfYZYfYbWb[dfUWhWyg.

• VYHYfa UbU[Ymcifh]a Y



L	

F Ygci fWYg

DfYgWf]VYX F Ygci fWYg