

HUMS1005

DYfgcbU`]gYX'9b[`]g\'@Ub[i U[Y'9b\UbWa Ybh

GYa YghYf'CbY''##'&\$%

7ci fgY Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUhjcb	D∖cbY
@cb[`cb["`]4 i bgk "YXi "Ui	H\i fgXUm'&!' 'da	A cfj Yb 6fck b	- ' ,) '&(%+
		UbX VmUddc]bha YbI	n! '&, \$	

@YWhifYfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJhjcb	D\cbY
@cb['@]	`cb["`]4 i bgk "YXi "Ui			

"Hi hcfg"

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJŊcb	D\cbY
@cp[`cb["`]4 i bgk "YXi "Ui			
>Ugcb < YZZYfbUb	¹Ugcb"\YZYfbUb4 i bgk "YXi "Ui			

GWXcc~7cbHJWh~abZcfa Uhlcb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@cWUhjcb. 'GW\cc`'C ZZWYž'A cfj Yb'6fck b'6i]`X]b[ž'@Yj Y``&ž'&),

CdYb]b[< ci fg. A cbXUm! : f]XUmž - Ua !!(.() da

D\cbY. Ž*%'&'-',) '%*, %

: UI . Ž * % & - ' ,) ', +\$)

9a U]`. \<u>\U`4 i bgk "YXi "Ui</u>

5 h Y b X U b W F Y e i] f Y a Y b h g

5 'ghi XYbh']g'Yl dYWYX'hc'UhhYbX'U```WUgg'WcbhUWi\ci fg'Zcf'U'ZJWY!hc!ZJWY'fl &: Ł'cf'V`YbXYX'Wci fgY'UbX Wca d'YhY'U```UWYj]h'Yg'Zcf'U'V'YbXYX'cf'Z ``mcb`]bY'Wci fgY"

5 'ghi XYbhik \c'Uff]j Ygʻa cfYʻh\Ubʻ%) 'a]bi hYgʻ`UhYʻa UmVYʻdYbU`]gYXʻZcfʻbcb!UhhYbXUbWY" -Zgi W\`UʻdYbU`hm]gʻ]a dcgYXž'h\Yʻghi XYbha i ghVYʻ]bZcfa YXʻj YfVU``mUhih\Y`YbX`cZWUggʻUbX`UXj]gYXʻ]bʻk f]h]b['k]h\]bʻ&(\ci fg"

ZUʻghi XYbhYl dYf]YbWrgʻ]`bYggža]gUXj Ybhi fYʻcfʻch\YfʻcWW ffYbWrʻh\Uha U_YgʻUVgYbWr'z̄ca ʻU WUgg#UWjj]hmi bUj c]XUV`YžʻcfʻYl dYWgʻhcʻVY`UVgYbhZ̄ca ʻUʻZcfh\Wca]b[ˈWUgg#UWjj]hmž'h\Ymg\ci `XˈgYY_ dYfa]gg]cbʻZ̄ca ʻh\Yʻ7 ci fgYʻ5i h\cf]hmžʻUbX`k\YfY`Udd`]WUV`Yžʻh\Y]fʻfYei Yghg\ci `X'VY`UVWca dUb]YX`VmUb cf][]bU``cf`WrfjZJYX`WcdmcZU`a YX]WU``WrfjZJWUhY`cf`ch\Yf`Zcfa `cZUddfcdf]UhY`Yj]XYbWY"

5.7 ci fgY 5 i h\cf]hma UmYl W gY U ghi XYbhTca 'WUggYg cf'UWjj]hJYg Zcf'i d hc cbY a cbh\"' < ck Yj YfZth\Yma UmUgg][b 'UXX]hJcbU`'UbX#cf'U`hYfbUhJj Y 'HUg_g'hc Ybgi fY Wca d`]UbW''' 5.7 ci fgY 5 i h\cf]hm\Wcbg]XYf]b['h\Y

[fUbh]b['cZUVgYbW' a i gh'VY 'gUh]gZ]YX 'U 'gh XYbhk]```gh]```VY 'UV'Y 'hc 'a YYhh\Y 'Wci fgY'g ``YUfb]b['ci hWca Yg UbX#cf' j c`i a Y 'cZ`YUfb]b["'5 'gh XYbhgYY_]b['Uddfcj U``hc 'VY 'UVgYbh'Zcf' a cfY 'h\Ub 'cbY 'a cbh\ 'a i gh'Udd`m]b 'k f]h]b['hc 'h\Y'8 YUb 'UbX'dfcj]XY 'U```cf][]bU``cf'WYfh]ZYX 'gi ddcfh]b['XcW a Ybh'Un]cb"

 $: cf'a\ cfY']bZcfa\ UhJcb'UVci\ hih\ Y'UhHYbXUbWY'dfchcWc`g']b'h\ Y':\ UW\ `hmcZ5fhg'UbX'GcWJU`\\ GWJYbWYg.\ `\underline{hhdg.} \#k\ k\ k''Ufhg''i\ bgk''YXi''Ui\ \#W\ ffYbHghi\ XYbhg\#UWUXYa\]W]bZcfa\ UhJcb\#dfchcWc`g!\ [i\]XY`]bYg\#UWUXYa\]W]bZcfa\ UhJcb#dfchcWc`g!\ [i\]XY`]bYg#UWUXYa\ [i\]XY`]bYyYYA \ [i\]XY`]BYYYA \ [i\]XY`]bYyYYA \ [i\]XY`]bYYYA \ [i\]XY`]bYYYA \ [i\]XY`]bYYYA \ [i\]XY`]bYYYA \ [$

5 WUXYa]W = b Zcfa Uh cb.

: cf`YggYbh]U``ghi XYbh`]bZcfa Uh]cb`fY`Uh]b[`hc.`fYei Yghg`Zcf`YI hYbg]cb/fYj]Yk `cZa Uf_g/cWW dUh]cbU``\YU`h\UbX`gUZ/hm/YI Ua]bUh]cb`dfcWXi fYg/gdYW]U``Wcbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/ghi XYbhYei]hmUbX`X]gUV]`]hm/UbX`ch\Yf`YggYbh]U``UWUXYa]W]bZcfa Uh]cbž gYY`\hdg.#k k k "Ufhq"i bqk "YXi "Ui #W ffYbhqhi XYbhq#UWUXYa]W]bZcfa Uh]cb#dfchcWc`q![i]XY`]bYq#

7ci fgY 8YW] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y'7ci fgY

H\]g`Wci fgY`k]``\Y`d`mci 'hc`Yb\UbW'mci f'9b[`]g\``Ub[i U[Y`g_]``g`]Zmci 'gdYU_'9b[`]g\`Ug U'gYWcbX``Ub[i U[Y"'=b`UXX]h]cbžh\]g`Wci fgY`dfcj]XYg`mci 'k]h\`h\Y`g_]``g`hc`VY`Ub`Ui hcbca ci g`UbX]bXYdYbXYbh`YUfbYf`gc`h\Uhmci 'WUb`Wcbh]bi Y`hc`Yb\UbW'mci f'9b[`]g\``Ub[i U[Y`g_]``g`]bhc`h\Y`Z hi fY`hc UW\]Yj Y`mci f`UWUXYa]WUbX`dfcZYgg]cbU``[cU`g"'Mci 'k]``]XYbh]Zmk \]W\`UgdYWffgŁ`cZmci f`9b[`]g\`mci k ci `X``]_Y`hc`]a dfcj Y`UbX`h\Yb`XYg][b`UbX`]a d`Ya Ybhmci f`ck b`dYfgcbU``dfcYWhc`UVWca d`]g\`mci f [cU`g"'

5 hith Y WcbWi g]cb cZth]g Wci fgY th Y ghi XYbhik]``VY UV Y thc

- %" 5bU`mgY`UbX`]XYbhJZmdYfgcbU```Ub[i U[Y`bYYXg
- &" 5dd`mhcc`g`UbX`fYgci fWyg`Zcf``Ub[i U[Y`Yb\UbWa Ybh
- ' "8Yg][b'UbX'WUffmcih'U'dYfgcbU`dfcYWhVUgYX'cb'h\Y'UbU`mg]g'cZdYfgcbU```Ub[iU[Y'bYYXg
- ("FYZYWicb'UbX'UggYgg'h\Y]f'ck b'dfc[fYgg'hc'gi ghJ]b'dYfgcbU`'UbX'dfcZYgg]cbU```Ub[i U[YYb\UbWa Ybh
-) " 7 ca a i b]\WUhY`\Wf]h]\WU``dYfgcbU```YUfb]b['ci h\care Yg`k]h\ 'dYYfg

HYUWX]b[GlfUhY[]Yg

<! A G%\$\$) `\Y`dg`ghi XYbhg`hc`hU_Y`fYgdcbg]V]`]mmZcfYb\UbW]b[ih\Y]f`9b[`]g\``Ub[i U[Y`g_]``g`Vm\Y`d]b[
h\Ya ']XYbh]ZmUbX`UXXfYgg`h\Y]f`ck b`]bX]j]Xi U``bYYXg"'H\]g`]g`VYWUi gY`ghi XYbhg`Uhih\]g`UXj UbWiX`ghU[Y`cZ
9b[`]g\``Ub[i U[Y`UWei]g]h]cb`\Uj Y`X]ZZYfYbhbYYXgž`ghfYb[h\g`UbX`k YU_bYggYg"':]fa `mVUgYX`cb`gcW]c!
W``hi fU```YUfb]b[i ih\Ycfmih\UhUf[i Yg`ih\Uh`YUfbYfg'Wcbghfi Whih\Y]f`ck b``YUfb]b[i ih\fci [\`a YUb]b[Z]
]bhYfUWh]cbg`fWZ''J m[chg_m%, *Lžih\]g`gY`Z\X]fYWhYX`Ui hcbca ci g`Wci fgY`\Y`dg`ghi XYbhg`hc`Yb\UbWi'h\Y]f
9b[`]g\`g_]``g`VmZ]fghmUbU`mg]b[ih\Y]f`ck b`bYYXg/gYWcbX`mXYj Y`cd]b[idYfgcbU``dfcYWhg/h\]fX`m
]a d`Ya Ybh]b[ih\Ya /Zci fh\`mYj U`i Uh]b[ih\Y]f`ck b`dfc[fYgg/UbX`Z]bU``mfYZYWh]b[icb'h\Y]f`ck b``YUfb]b["
H\fci [\ci hih\Y'Wci fgYžghi XYbhg`k]```VY`[i]XYX`k]h\`fY`Yj Ubhih\Ycfmž'dfUWh]WU``gi [[Ygh]cbg'UbX'i gYZ]`
fYgci fWyg`]b`Vch\``YWhi fY`UbX'hi hcf]U``gYgg]cbg"</pre>

5ggYgga Ybh

5ggYgga Ybh'HUg_g

5ggYgga Ybh'hJg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
DYfgcbU`'dfcYWiXYg][bfDD8Ł'	' \$1	&'#\$(#&\$%_'%%\$\$`DA	% <u>ž</u> &ž

GHJfh'XUhY.

@Yb[h\. '%s'a]bi hYg

8 YHU]`g. `Gh` XYbhg`dfYgYbhh\Y]f``YUfb]b[`\][\hg`Zfca `h\Y]f`dYfgcbU``dfc YWhZcf`UVci h%\$`a]bi hYg`hc
]bgd]fY`dYYfg`hc`Wcbh]bi Y`hc`Yb\UbWY`h\Y]f``Ub[i U[Y`g_]``g`Yj Yb`UZhYf`Wca d`Yh]b[`h\Y`Wci fgY"`H\YmUfY
fYei]fYX`hc`gi Va]hUb`UVghfUWhfUVci h%) \$`k cfXgŁ`UVci hk \Uhh\YmUfY`[c]b[`hc`dfYgYbhU`Wci d`Y`cZk YY_g
VYZcfY`h\Y`dfYgYbhUh]cb`XUm'`Gh` XYbhg`fYWY]j Y`k f]hhYb`ZYYXVUW_`Zfca `h`hcf`cb`UVghfUWhg`f%\$i Ł`UbX`Zfca
dYYfg`cb`dfYgYbhUh]cb`ff%\$i Ł`VUgYX`cbYfX\$f]Wg"

5 XX]h]cbU``XYhU

5XX]h]cbU`XYhU]`g.

GYY'AccX`Y'Zcf'XYHJ]`"

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tabular} $$$ I' \end{tabular} $$$ I' \end{tabular} $$ I' \end{tabular} $$$ I' \end{tabular} $$$ I' \end{tabular} $$$ I' \end{tabular} $$$ I' \e$

5ggYgga Ybh'(. FYZYWhj Y YggUm

GHJfhXUhY.

@Yb[h. &ž\$\$\$ k cfXg

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gh XYbhg UfY Yl dYWYX hc di hh\Y]f bUa Yg UbX gh XYbhbi a VYfg cb Yj YfmdU[Y cZh\Y]f Ugg][ba Ybhg"

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u>"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'=\maintention
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a Um'Udd'm\Zcf'Ub'YI hYbg]cb\ZVi hmci
a i ghfYhJ]b'mci f'h]\W_Yhbi a VYf\Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW'a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WJh]cb'''\=\maintention
Uh\WJ'`mfYWY']j Y'U'h]\W_Yhbi a VYf\ZVi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]\Z\WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU`mž'UggYgga YbhitUg_g'a i ghVY'gi Va]hhYX'Y`YWfcb]WU``mj]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bghUbWfg'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'ghUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYhU]`g"

@UhY 5ggYgga Ybh'DYbU`h]Yg

5b'UggYggYX'HUg_']g'XYYa YX``UhY']Z]h']g'gi Va]hhYX'UZhYf'h\Y'gdYW]Z[YX'h]a Y'UbX'XUhY'Ug'gYhci h']b'h\Y Wci fqY'@YUfb]b['A UbU[Ya YbhGmqhYa 'f@A GŁ"

 $\label{thm:condition} $$HY^UHY^dYbU^m]g^hY^cgg^cZ)_L^cZ^hY^hcHU^dcgg]VY^aUf_g^Zcf^hYY^HUg_Z^cf^YUW^XUmcf^dUfhhYfYcZhY^kcf_]g^UHY^@UhYbYgg^k]^*]bW^XY^kYY_YbXg^UbX^di_V^]WYc^]XUmg^"HY]g^XcYg^bchUdd^mhc^U'HUg_'hWh]g^UqqYqqYX^Vi_hbc^aUf_]g^Uk_UfXYX"$$$

K cf_'gi Va]hhYX'Zci fhYYb'fl%(Ł'XUmg'UZhYf'h\Y'Xi Y'XUhY'k]```VY'a Uf_YX'UbX'ZYYXVUW_'dfcj]XYX'Vi hbc'a Uf_ k]```VY'fYWcfXYX"'=Zh\Y'k cf_'k ci `X'\Uj Y'fYWl]j YX'U'dUgg'a Uf_'Vi hZcf'h\Y'`UhYbYgg'UbX'h\Y'k cf_']g'U Wca di `gcfmWci fgY'Wca dcbYbhz'U'gh XYbhk]```VY'XYYa YX'hc'\Uj Y'a Yhh\UhfYei]fYa Ybh''H\]g'XcYg'bch Udd`mhc'U'hUq_'h\Uh]g'UggYggYX'Vi hbc'a Uf_']g'Uk UfXYX"

K cf_'gi Va]hh/X'hk YbhnlcbY'f&&XUmg'UZhYf'h\Y'Xi Y'XUhY'k]```bchVY'UWWdhYX'Zcf'a Uf_]b['cf'ZYYXVUW_UbX'k]```fYWY]j Y'bc'a Uf_'cf'[fUXY"'=Zh\Y'UggYgga YbhhUg_']g'U'Wca di `gcfmWca dcbYbhcZh\Y'Wci fgY'Ugh XYbhk]```Ui hca Uh]WU``mZU]`'h\Y'Wci fgY"

GdYWJU`7cbg]XYfUhJcb'5dd`]WUhJcbg

 $\label{eq:linear_continuity} $$Mc_i = W_0 - W_$

G]W_bYggža]gUXj Ybhi fY'cf'ch\Yf'W]fW a ghUbWrg'VYmcbX'mci f'Wcbhfc``a Um

ł DfYj Ybhimci Zfca Wca d`Yhlb[U.Wci fqY fYei]fYa Ybhž

ł? YYd mci Zfca 'UthYbX]b['Ub 'UggYggUV`Y 'UWhj]tmž

ł Ghod moi gi Va]hh]b['UggYggUV'Y k cf_ Zcf'U Wci fgYž

5 WUXYa | W < cbYglmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`'h\I hcf']XYU'k]h\ci hUW_bck`YX[]b['h\Y gs:bfW*rcfë g]bf'eù shUhog 'aqUf_og't'a\ng b2jc3tt20]'36j'k1]z f7[42g5WD7.048X73j]036tft500vjR.Gg00\rig\BjT 0 30.k17 f742.5'dfYgYbhUhjcbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]h'

± Uddfcdf]UhY 'dUfUd\fUg]b[. 'W\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhU]b]b['h\Y'cf][]bU`ghi Whi fY 'UbX']bZcfa Uh]cb 'k]h\ci h'UW_bck 'YX[Ya Ybh''H\]g'U`gc 'Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY gca YcbYdUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'' ± h'U`gc 'Udd`]Yg'hc'd]YW]b['hc[Yh\Yf'ei chYg'UbXdUfUd\fUgYg']bhc'U'bYk 'k \c'Yž'k]h\ci h'fYZYfYbW]b['UbX'U'ghi XYbhg'ck b'UbU`mg]g'hc'Vf]b['h\Y'a UhYf]U`hc[Yh\Yf''

7 c``i g]cb.'k cf_]b['k]h\ ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'gh XYbhVYZcfY'h\Y'Xi Y'XUhYz'cf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhUbmha Yz'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_z'ghYU`]b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Whqcf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[`h\Y`″gYWcbXUfm″gci fW/Zfca 'k \]WX`_bck`YX[/Y`cZh\Ya`\UgʻVYYb`cVhU]bYX"

8 i d`]\WUh]cb`fl`gY`Zld`U[]\Uf]\da "L`gi/Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h`\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va;\hYX`Zcf;\Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWhfYZYfYbWJb[øfUWJWYg.

• DUfUd\fUgjb[žgi a a Ufjg]b[žYggUmk fjhjb["UbX"hja Y"a UbU[Ya Ybh

• VYHYfa UbU[Ymcifh]a Y