

MODL5113

=bhYfdfYh]b['5\WYYX]hUh]cb 'DfYdUfUh]cb

GYa YghYf'C bY''##'&\$%

\ci fg"

⊒Uʻgh XYbhYl dYf]YbWrgʻ]`bYggža]gUXj Ybh fYʻcfʻch\YfʻcWW ffYbWrʻh\Uha U_YgʻUVgYbWrʻZfca ʻU WUgg#UWlj]hmi bUj c]XUV`YžcfʻYl dYWgʻhcʻVYʻUVgYbhZfca ʻUʻZcfh\Wca]b[ʻWUgg#UWlj]hmžh\Ymg\ci `XʻgYY_ dYfa]gg]cbʻZfca ʻh\Yʻ7 ci fgYʻ5i h\cf]hmžʻUbXʻk \YfYʻUdd`]WUV`Yžh\Y]fʻfYei Yghg\ci `XʻVYʻUVWca dUb]YXʻVmUb cf][]bUʻcfʻVWff]ZJYX`WcdmcZUʻa YX]WU``WYff]ZJWUhYʻcfʻch\YfʻZcfa `cZUddfcdf]UhYʻYj]XYbWY"

5.7 ci fgY '5 i h\cf]hma UmYI W gY 'U'ghi XYbhZica 'WUggYg'cf'UWjj]hJYg'Zcf'i d'hc'cbY'a cbh\"'< ck Yj Yfž'h\Ym a UmUgg][b'UXX]hJcbU``UbX#cf'U'hYfbUhjj Y'hUg_g'hc'Ybgi fY 'Wca d`]UbW''' 5.7 ci fgY '5 i h\cf]hmWcbg]XYf]b['h\Y [fUbh]b['cZUVgYbW'a i ghVY'gUhJgZJYX'U'ghi XYbhk]``'gh]``VY'UV'Y'hc'a YYhh\Y'Wci fgY'g'`YUfb]b['ci hWca Yg UbX#cf'j c'i a Y'cZ`YUfb]b["'5 'ghi XYbhgYY_]b['Uddfcj U`'hc'VY'UVgYbhZcf'a cfY'h\Ub'cbY'a cbh\'a i ghiUdd`m]b'k f]hJb['hc'h\Y'8 YUb'UbX'dfcj]XY'U``cf][]bU`'cf'WYfhJZYX'gi ddcfh]b['XcW a YbhUh]cb"

: cfa cfY`]bZcfa Uhjcb`UVci hih\Y`UthYbXUbWY`dfchcWc`g`]b`h\Y`: UW\`hmcZ5fhg`UbX`GcWJU` GWJYbWYg.`<u>\hndg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbHghi XYbhg#UWUXYa]W]bZcfa Uhjcb#dfchcWc`g![i]XY`]bYg#</u>

5 WUXYa]W = b Zcfa Uh]cb

: cf`YggYbh]U``ghi XYbh`]bZcfa Uh]cb`fY`Uh]b[`hc.`fYei Yghg`Zcf`Yl hYbg]cb/fYj]Yk `cZa Uf_g/cWW dUh]cbU``\YU`h\UbX`gUZ/hm'Yl Ua]bUh]cb`dfcWXi fYg/gdYW]U``Wcbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/ghi XYbhYei]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbh]U``UWUXYa]W]bZcfa Uh]cbž qYY`\hdg.#k k k "Ufhq"i bak "YXi "Ui #W ffYbhqhi XYbhq#UWUXYa]W]bZcfa Uh]cb#dfchcWc`a![i]XY`]bYq#

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y 7ci fgY

H\]g`UXj UbWX`]bhYfdfYhJb[`Wti fgY`]g`]bhYbXYX`Zcf`ghi XYbhg`Ybfc``YX`]b`h\Y`Z]bU``gYa YghYf`cZh\Y`A UghYf`cZ \DYfdfYhJb[`f] &\$&t`UbX`A UghYf`cZ\DhYfdfYhJb[`UbX`HfUbg`UhJcb`Ghi X]Yg`f] &\$(t"`6i]`X]b[`cb`g_]``g XYj Y`cdYX`]b`dfYj]ci g`Wti fgYgžmci `k]```Z ff\Yf`XYj Y`cd`mci f`]bhYfdfYhJb[`Wta dYhYbWJYg`hc`fYUW\`h\Y ghUbXUfXg`fYei]fYX`Vmh\Y`5i ghfU`]Ub`Wta a i b]hmUbX``Y[U`#\tilde{\text{minimize}}\dfcZ\fg]cb"`H\Y`\tilde{\text{minimize}}\dfcZ\fg]cb"`H\Y`\tilde{\text{minimize}}\dfcZ\fg]cb"`H\Y`\tilde{\text{minimize}}\dfcZ\fg]cb\dfc\ff\dfYhYfg`UbX HfUbg`Uncfgt:\DhYfdfYhJb[`U\WYYX]hUhJcb`YI Ua]bUhJcb"

5 hh Y WcbWi g]cb cZh]g Wci fgY h Y gh XYbhk] ``VY UV Y hc

- %" C fU``m'Wca dfY\YbX'Wca d`Yl 'gci fWY'a YggU[Yg'UbX'hYl hg']b'9b[`]g\'UbX'h\Y'ch\Yf'fY`Yj Ubh `Ub[i U[Y
- &" 7 f]h]\\U``mUbU`mgY`cfU``gdYY\\\`]b`9b[`]g\`UbX`h\Y`ch\Yf`fY`Yj Ubh`Ub[i U[Y`
- ' " 5 WW fuh'm Webj Yfhih. Y gei fw' a Yggu[Y he h. Y huf[Yh`ub[i U[Y efu`mi g]b[X]ZZYfYbh']bh'fdfYh]b[a cXYq
- ("8Y`]j Yf'h\Y']bhYfdfYhYX'i HhYfUbWY']b'h\Y'Uddfcdf]UhY'fY[]ghYf'UbX'ghmY'
-) " A UbU[Y 'UbX 'WccfX]bUHY 'hf]UX]WV] `]b[i U` `]bHYfUWI]cbq
- *" 5dd`m_bck `YX[Y`cZh\Y`7cXY`cZ9h\]Wg`UbX`h\Ycf]Yg`cZ+bhYfdfYh]b[`]b`Ub`YI Ua]bUh]cb`g]h Uh]cb

HYUWX]b[GhfUhY[]Yg

H\]g`Wci fgY`Wca V]bYg`k YY_`ma i `h]`]b[i U```YWh fYg`UbX``Ub[i U[Y!gdYV]\\\]Wh hcf]U`g"

+b'th\Y'a i 'th']b[i U```YWfi fYžmci 'k]``fYj]gY'UbX'Vi]`X'cb'mci f'Yl]gth|b['_bck`YX[Y'cZ]btYfdfYth|b['th\YcfmUbX'

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[CihWca Ygʻ5ggYggYX
91 Ua]bUh]cb	* \$1	91 Ua]bUh]cbʻdYf]cX	%ž ž(ž) ž*
±b!WUgg'UggYgga YbhcZ a cbc`]b[i U`'g_]``g'UbX'gi V! g_]``g'UggYgga Ybh	%\$I	K YY_gʻ(ž, ʻUbXʻ‰	%ž&Ž Ž(Ž*
±b!WUgg'UggYgga YbhcZ V]`]b[i U``]bhYfdfYh]b['g_]``g'	' \$ı	Bch5dd`]WUV`Y	%Ž Ž(Ž)Ž*

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% '9l Ua]bUh]cb

GHJfh:XUhY.

8YHJ]`g. ʿJ]j U`]bhYfdfYhJb['YI Ua]bUhJcb" ⊕X]j]Xi U``ZYYXVUW_`UbX`a Uf_]b["H\]gʻ]gʻh\YʻZJbU``UggYgga YbhZcf UHYbXUbWY`di fdcgYg"

5XX]h]cbU`XYhJ]`g.

cfa U``UWWYYX]hUhjcb`YIUa]bUhjcb`WcbXiWYX`Xifjb[`h\Y`YIUa]bUhjcb`dYfjcX"

5ggYgga Ybh& ±9!WUgg UggYgga YbhcZa cbc`]b[i U`g_]``g UbX gi V!g_]``g UggYgga Ybh

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. 'C b[c]b[']b!WUgg`UggYgga YbhcZa cbc`]b[i U``g_]``g`UbX`gi V!g_]``g`Xi f]b['a i 'h]`]b[i U```YWh fYg fWca dfY\Ybg]cbž'bchY`hU_]b[ž'9b[`]g\``Ub[i U[Y]bhYfdfYh]b['XY`]j YfmŁ"Gh XYbhg`UfY'bch]Z]YX`]b`UXj UbW\`cZh\Y`UggYgga YbhWf]hYf]U`UbX`YI dYWHJbcbg"'H\YmfYWY]j Y`Zcfa Uh]j Y`ZYYXVUW_`fY[UfX]b['h\Y`dfcW\gg`UbX dfcXi Whh\fci [\ci hh\Y`gYa YghYf'k]h\`fY[UfXg`hc'j Uf]ci g`g_]``g`UbX`a cXYg`cZ]bhYfdfYh]b[": YYXVUW_`]g dfcj]XYX`]bX]j]Xi U``mUbX`hc'h\Y`WUgg"

5XX]h]cbU`'XYhU]`g.

 $H\fYY']bghU'a\ Ybhg'cZ]bhYfdfYh]b[']bhc'9b[']g\'hc'XYa\ cbghfUhY'g_]``g']b'X]ZYfYbh]bhYfdfYh]b['a cXYg']bhYfdfYh]b['a cXYg']bhYfdfYh]bhYfdfYh]b['a cXYg']$

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' Hi & fb \end{tabular} $$ h' \end{tabular}$

5ggYgga Ybh' . : 1. WUgg UggYgga Ybhc ZV]`]b[i U`]bhYfdfYh]b['g_]``g

GhUfhiXUhY. Bch5dd`]WUV`Y

 $8YHU]^*g. `Cb[c]b[`]b!WUgg^*UggYgga YbhcZ]bH/fdfYh]b[`g_]^*g^*]b'X]ZZYfYbha cXYg^*Xif]b[`h\Y^*Ub[iU[Y!gdYW]Z]Whhcf]U^*g^*Ghi XYbhg^*UfY^bch]Z]YX^]b^*UXjUbWFcZh\Y^*UggYgga YbhWf]HYf]U^*UbX^YIdYWfUh]cbg^*'H\Ym$

 $fYWY]jY'ZcfaUHjY'YXVUW_fY[UfX]b['H\Y'dfcWYgg'UbX'dfcXiWhhfci[\cih\Y'gYaYgHYf'k]h\'fY[UfXg'hcjuf]cig'g_]``g'UbX'acXYg'cZ]bHYfdfYHjb['': YYXVUW_]g'dfcj]XYX']bX]j]XiU``mUbX'hc'H\Y'WUgg''$

5XX]h]cbU`XYhJ]`g.

H\]g^]g^]b^cb[c]b[~WUggfcca~UggYgga YbH

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

 $Gh\ XYbhg\ UfY\ YI\ dYWMYX\ hc\ di\ hih\ Y]f\ bUa\ Yg\ UbX\ gh\ XYbhbi\ a\ VYfg\ cb\ Yj\ YfmdU[\ Y\ cZh\ Y]f\ Ugg][\ ba\ Ybhg\ bug\ Ybhg\ bug\ Yhhg\ Yhhg\ bug\ Yhhg\ Yhhg\ bug\ Yhhg\ bug\ Yhhg\ bug\ Yhhg\ bug\ Yhhg\ bug\ Yhhg\ Yhhg\ bug\ Yhhg\ Yhhg\ Yhhg\ bug\ Yhhg\ Yhhg\$

Hi fb]h]b Gi Va]gg]cb

⊋mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UhhYa dhjb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hjbž'd'YUgY hY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-' ,) '' ' '%cf'Ya U]'hXYa 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui</u> ł `G][b]Z]\WbhmUZZY\Wimci f`dYfZcfa Ub\Wr`]b`UggYggUV`Y`k cf_ž`VY`]h'U`Zcfa U``YbX!cZgYa YghYf`Yl Ua]bUh]cbž`U \WUgg`hYghž`U``UVcfUhcfmhYghž`U`gYa]bUf`dfYgYbhUh]cb`cf`Ubmch\Yf`Zcfa 'cZUggYgga Ybh'

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

±DUddfcdf]UhY`dUfUd\fUg]b[.'W\Ub[]b['U'ZYk'kcfXg'UbX'd\fUgYg'k\]`Y`acghmfYhJ]b]b['h\Y`cf][]bU`ghfi WhifY`UbX`]bZcfaUh]cb`k]h\ci hUW_bck`YX[YaYbh'H\]g'U`gc'Udd`]Yg`]b'dfYgYbhUh]cbg'k\YfY`gcaYcbYdUfUd\fUgYg`Ubch\Yf'g`]XYUg`cf'kcfXg'k]h\ci hWfYX]h''±iU`gc'Udd`]Yg`hc'd]YW]b['hc[Yh\Yf'ei chYg'UbXdUfUd\fUgYg`]bhc'U'bYk'k\c`Yžk]h\ci hfYZ/fYbW]b['UbX'U'ghi XYbh'g`ck b'UbU`mg]g`hc'Vf]b['h\Y`aUhYf]U`hc[Yh\Yf''

7 c``i g]cb. 'k cf_]b['k]h\ 'ch\Yfg'Vi h'dUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['Uh'Ubmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU`]b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d`YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d`Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[ˈh\Y`″gYWcbXUfm″gci fW/Zfca k \]WX`_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ff`gY`Zld`U[]\Uf]ga "Ł`gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk\YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\\$f`UqqYqqaYYbhcf\\&f`UhI BGK `cf`Ubch\Yf`]\m' '

7 cffy Wrify ZyfybWjb['dfUWf)Wyg.

• DUfUd\fug]b[ž'gi a a Uf]g]b[ž'YggUmk f]r]b['UbX'r]a Y'a UbU[Ya Ybh

‰

• VYHYfa UbU[Ymcifh]a Y

1	I	l

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		Dc`]WY`UbX``Y[U``hYfa]bc`c[mei]n"	
		7 cbgYW hjj Y`]bhYfdfYhUhjcb`]bhc`9b[`]g\`hc`VY gi Va]hhYX`]b`Hi fb]hjb"	
K YY_'% . '&, 'A Um'! '' >i bY	Hcd]W	K fUd`i d.`@Y[U``UbX`Wci fhi]bHYfdfYH]b[`dfUW¶WY"	

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

>" @YYž5" 6i nc " 5a_ g` [fk 'S` YgSYW[` fWdbdWf[` Yž3 I ad] Taa] ž'H\Y': YXYfUh]cb dfYggž'&\$\$-"