

LING5031

=ggi Ygʻ]bʻGYWcbXʻ@Ub[i U[Yʻ5Wei]g]hjcb

GYa YghYf'Hk c''#'&\$%

7 ci fgY Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`q

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
>Ua Yg [·] @YY	ʻUa Yg"`YY4 i bgk "YXi "Ui	,	&&* A cfj Yb 6fck b	dfYZYf [·] Ya U]`

GW.cc`7cbhJWh =bZcfa Uhlcb

GWXcc`cZ<ia Ub]hYg'UbX'@Ub[iU[Yg

@cWUhjcb. 'GW\cc`'C ZZj\\xizA cfj Yb'6fck b'6i]`X]b[z'@Yj Y``&z\&),

CdYb]b[< ci fg. A cbXUm!: f]XUmž-Ua !!) da

D\cbY. Ž*%'&'-',) '%*, %

: UI . Ž*%'&'-',) ', +\$)

9a U]`. \<u>U`4 i bgk "YXi "Ui</u>

5 h Yb X Ub WY F Yei] f Ya Yb h g

5 qh XYbh]q Yl dYWYX hc UhYbX U``WUqq WcbhUWi\ci fq Zcf U ZUW!hc! ZUW! fl & L'cf V YbXYX Wci fqY UbX WeadYhY'U``'UWAj]hYq'Zef'U'VYbXYX'cf'Z ``micb`]bY'WeifqY"

5 gh XYbhk\c'Uff]j Ygʻa cfY h\Ub′%) a]bi hYgʻ`UhYʻa UmVY dYbU`]gYX Zcf'bcb!UthYbXUbVW" Zgi VX U'dYbU`hm]g^]a dcgYXz*h\Y`ghi XYbha i ghVY`]bZcfa YX`j YfVU``mUhh\Y`YbX`cZWUgg^UbX`UXj]gYX`]b`k f]h|b['k]h\]b`&(\ci fg"

ZUʻgh XYbhYl dYf]YbW'gʻ]`bYggža]gUXj Ybh fYʻcfʻch\YfʻcVW ffYbW h\Uha U_YgʻUVgYbW Zfca ʻU WUgg#JWNj jmi buj cjXUV`Yžcf`YI dYWmg`hc`VY`UVgYbhZfca`U`Zcfh\Wcajb[`WUgg#JWNj jmz`h\Ymg\ci`X`gYY_ dYfa]qq]cb Zfca 'h\Y'7 ci fqY'5i h\cf]|mz'UbX'k\YfY'Udd`]WUV`Yz'h\Y]f`fYei Yqhq\ci `X'VY'UW\can dUb]YX'VmUb cf][]bU`cf`WfflZYX`WcdmcZU'a YX]WU`WfflZWJHY`cf`ch\Yf`Zcfa 'cZUddfcdf]UHY`Yj]XYbWY"

5 7 ci fqY 5 i h\cf|lma UmYl W qY U qhi XYbh2fca WUqqYq cf UWlij |h|Yq Zcf i d hc cbY a cbh\" < ck Y j Yfz h\Ym a UmUgg][b'UXX]hcbU`UbX#cf'U`hYfbUh]j Y'hUg_g'hc'Ybgi fY'Wca d`]UbWY"'5'7 ci fgY'5i h\cf]hmWcbg]XYf]b['h\Y [fubhb[cZuvqYbWraighvYrguhqZ]YXrurgh XYbhk]```qh]```VYruv`Yrbc'a YYhrh\YrWcifqY'q``Yufb]b[cihWcaYq UbX#cfjciaYcZYUfb]b["5 gh XYbhgYY_]b['Uddfcj U`hc VY UVgYbhZcfa cfY h\Ub cbY a cbh\ a i ghUdd`m]b k f]h]b['hc h\Y 8 YUb UbX dfcj]XY 'U```cf][]bU``cf`WYfh]Z[YX gi ddcfh]b['XcW a YbHUh]cb"

cfacfy]bzcfaUt]cbUVcihthYUthYbXUbWYdfchcVc`g]bthYY:UW`tmcZ5fhgUbX'GcWJU` GWYbWrg. \htdg.#k k k "Uffig"i bak "YXi "Ui #W ffYbhah XYbha#UWJXYa \\mathbb{W}\delta Zcfa Uhcb#dfchcWc\a![i \]XY\\delta Ya#

5 WUXYa W = DZcfa Uhlcb

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb['hc.`fYei Yghg`Zcf`Yl hYbg]cb/fYj]Yk 'cZa Uf_g/cWW dUhJcbU``\YU`h\ UbX`gUZYhm'Yl Ua]bUhJcb`dfcWlXi fYg/gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/

7 ci fgY 8YHJ] g 7 fYX]hDc]bhg * Gi a a UfmcZh\Y 7 ci fgY

.

Mci 'k]```Yl d`cfY'h fYY'cj YfUfW\]b[']ggi Yg']b'gYWcbX'`Ub[i U[Y'UWei]g]h]cb'fYgYUfW. 'h\Y'YZZYWg'cZ]bghfi W]cbz'ZcW g'cb Zcfa z'UbX'h\Y']bhYfUW]cb'UddfcUW."``H\Y']ggi Yg'Yl d`cfYX'i bXYf'h\Y'YZZYWg'cZ]bghfi W]cbz'UbZh\Y'cj YfU``fU[[fY[UhYŁYZZYWg'cZ]bghfi W]cbz'WcbghfU]b]b['ZUWcfg'h\Uh`]a]hcf'ch\Yfk]gY'Yl d`U]b'h\Y YZZYWg'cZ]bghfi W]cbz'WcbghfU]b]b['ZUWcfg'h\Uh`]a]hcf'ch\Yfk]gY'Yl d`U]b'h\Y YZZYWg'cZ]bghfi W]cbz'WcbghfU]b]b['ZUWcfg'h\Uh`]a]hcf'ch\Yfk]gY'Yl d`U]b'h\Y YZZYWg'cZ]bghfi W]cbz'WcbghfU]b]b['fY"["z'WUggfcca 'j Yfgi g'ghi Xm UVfcUXŁ"'H\Y']ggi Yg'Yl d`cfYX'i bXYf'ZcW g'cb'Zcfa 'UfY'h\Y'fc'Yg'cZdfUW]WzYl d`UbUh]cb'UbX'ZYYXVUW_]b gYWcbX'`Ub[i U[Y'UWei]g]h]cb"'H\Y']ggi Yg'Yl d`cfYX'i bXYf'h\Y'=bhYfUW]cb'5ddfcUW\'UfY'k\UhiWcbgh]hi hYg Yj]XYbWY'h\UhiYufb]b['\Ug'hU_Yb'd'UWY'Ug'U'fYgi 'hcZ]bhYfUW]cbz'h\Y'ci hWca Yg'cZU'ZcW g'cb'Zcfa 'Xi f]b[']bhYfUW]cbz'ZYXVUW_Xi f]b[']bhYfUW]cbz'ZYXVUW_Xi f]b[']bhYfUW]cbz'ZYXVUW_Xi f]b[']bhYfUW]cbz'ZYXVUW_Xi f]b[']bhYfUW]cbz'ZYXVUW_Xi f]b[']bhYfUW]cbz'UbX'UhYbhJcbz'UbX'UhYbhJcbz'UbX'UhYbhJcbz'UbX'UhYbhJcbz'UbX'UhYbhJcbz'UbX'Uk'UfYbYgg"

5 hm Y WcbWi g]cb cZh]g Wci fgY h Y ghi XYbhk] ``VY UV Y hc

- %" `Wf]h]WU``mYI Ua]bY`UbX`Yj U`i UhY`Ya d]f]WU``fYgYUfW\`]bWi X]b[`]ggi Yg`cZfYgYUfW\`XYg][b`UbX`XUHU UbU`mg]g
- &" YI d`]\\\\Y'a U]b'dc]bhg`cb'j Uf]ci g`]ggi Yg`]b`gY\\\cbX``Ub[i U[Y'U\\\ei]g]h]cb'fYgYUf\\\
- ' "Webglfi Wiufyj]Yk 'cZ`]hYfUhi fY'h\Uh]XYbh]Z]Yg'h\Y``]a]hUh]cbg'cZh\UhVcXmcZk cf_'UbX'WebWi XYg k]h\UWe\YfYbhgYhcZZ hi fY!'cc_]b['fYgYUfW\ei Ygh]cbg'UbX'\mdch\Yg]gYg'dchYbh]U`'ci hWea Yg
- ("dfcdcgY`fXYg][bŁ`Ub`Yad]f]WU``gh Xmh\UhUXXfYggYg`U`gYhcZfYgYUfW\`ei Ygh]cbg`UbX`hYghg`U`gYhcZ\mdch\YgYg

HYUW]b['GhfUhY[]Yg'

5```fYUX]b[g'Zcf'h\]g'Wci fgY'UfY'Ya d]f]WD``ghi X]Yg'Yl Ua]b]b['h\Y'gYWcbX``Ub[i U[Y'UWei]g]h]cb'cZ`]b[i]gh]Wgfi Wi fYg"H\Y'cf]YbhUh]cb'cZh\Y'fYgYUfW\']g``Uf[Y`mei Ubh]hUh]j Y"'K Y'UddfcUW\'YUW\'fYUX]b[a Yh\cX]WD``m'':]fghz'k Y'Yl Ua]bY'h\Y'Ui h\cffg'ghUhYa YbhcZh\Y'dfcV'Ya 'hc'V'fYgYUfW\YX''K Y'UbU`mmY'h\YfYj]Yk 'cZ`]hYfUh fY'Ug'U'XYj]WY'hc'a chjj UhY'fYgYUfW\'ei Ygh]cbg'UbX'\mdch\YgYg''=a dcffUbhmz'k Y'Yl Ua]bY b'XYhU]`'h\Y'fYgYUfW\'XYg][b'UbX'\ck']h'UWWca d`]g\Yg'h\Y'ghUhYX'[cU'`cf'U]a 'cZh\Y'ghi Xm''K Y'Wcbg]XYf XUhU'UbU`mg]g'UbX'h\Y'fYgi 'hg'fcZhYb'VUgYX']b'ghUhJhJhJhWD`'UbU`mg]gz''

Ghi XYbhgʻa i ghifYUX`WcgY`m'UbX`k]h\`XYhU]``hc`UWWca d`]g\`h\Y`UVcj Y`gc`h\Uhh\Y`Wci fgY`WUb`VY`Ub]bhYfUWNjj Y`gYa]bUf!ghm`Y``YUfb]b[`YI dYf]YbWY"

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
F YgYUfVX 'dfcdcgU`'	(\$1	\$, #%#&\$% `\$' .\$\$`DA	% <u>*</u> &. <u>Z</u>
7`Ugg'hYgh'	(\$1	&\$#\$, #&\$% `%&.\$\$`DA	%ž&
DfYgYblUljcb ⁻	&\$ı	Bch5dd`]WUV`Y	%

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% F YgYUfW dfcdcgU`

GHJfh:XUhY.

@Yb[h\.'r')\$\$ k cfXg

8YHJ]`g. `Gh` XYbhg`k]```dfcdcgY`cf][]bU`žYa d]f]\W\`fYgYUf\\\"'H\Y`dfcdcgU``k]```\\b\b\Jb`U`gh\h\a YbhcZh\Y`dfcV`Ya ž'U`fYj]\k `cZ`]h\Y`dfcdcgYX gh`Xm'`k f]hh\b`ZYYXVU_

Hi fb]h]b gYh]b[. `H\]g Ugg][ba Ybh]g gi Va]hhYX h\fci [\`Hi fb]h]b UbX gh XYbhg Xc bchgYY Hi fb]h]b g]a]`Uf]hmfYdcfhg"

5ggYgga Ybh'& '7`Ugg'hYgh

GhUfh XUhY.

@Yb[h\.^]b^WUgg^hYgh

8YHJ]`g.

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

- K cf_'gi Va]hhyx'%5'hc'% 'XUmg'UZhYf'h\Y'gh]di `Uhyx'XYUX`]bY'k]``'VY'UggYggYX'UbX'ZYYXVUW_dfcj]XYX'Vi h'U'a Uf_'cZnYfc'k]``'VY'fYWcfXYX'''=Zh\Y'k cf_'k ci `X'\Uj Y'fYWf]j YX'U'dUgg'a Uf_'Vi h'Zcf h\Y'`'UhYbYgg'UbX'h\Y'k cf_']g'U'Wca di `gcfm'Wci fgY'Wca dcbYbh'f\i fX`Y'fYei]fYa Ybhz'U'gh XYbh'k]`` VY'XYYA YX'hc'\Uj Y'a Yhh\Uh'fYei]fYa Ybh/
- K cf_'gi Va]hhYX`&\$`cf`a cfY`XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```bchVY`UWWdhYX`Zcf UggYgga YbhUbX`k]```fYWY]j Y`bc`ZYYXVUW_ža Uf_'cf`[fUXY"'=Zh\Y`UggYgga YbhHUg_']g`U`Wca di `gcfm Wca dcbYbhcZh\Y`Wci fgY`U`gh XYbhk]```fYWY]j Y`Ub`l bgUh]gZJWcfm: U]``fl : Ł'[fUXY`Ug`U`fYgi `hcZ i bgUh]gZJWcfmdYfZcfa UbWY`]b`Ub`YggYbh]U``Wca dcbYbhcZh\Y`Wci fgY"

H\]g']bZcfa Uh]cb']g'U`gc'Uj U]`UV`Y'Uh

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uhcb#dfchcWc`g![i]XY`]bYg#

GdYWJU`7cbg]XYfUhJcb5dd`JWUhJcbg

 $\label{thm:condition} $$Mci `WUb`Udd`m'Zcf'gdYW]U``Wcbg]XYfUh]cb`k \Yb`]``bYgg`cf`ch\Yf`W]fW a ghUbWfg`]bhYfZYfY`k]h\`mci f UggYgga Ybh'dYfZcfa UbWf''$

G]W_bYggža]gUXj Ybhi fY'cf'ch\Yf'W]fW a ghUbWrg'VYmcbX'mci f'Wcbhfc``a Um

ł DfYj Ybhmci Zfca Wca d`Yhjb['U'Wci fgY'fYei]fYa Ybhž

ł? YYd mci Zfca 'UttYbX]b['Ub'UggYggUV'Y'UWt]]tmž

ł Ghod moi igi Va]hh]b[iUggYggUV Y k cf_ Zcf U Wci fgYž

ł G][b]ZJWUbhmiUZZYWimci fidYfZcfa UbWY]biUggYggUVYkcf_žVYjhUiZcfa U`YbX!cZgYa YghYfiYl Ua]bUhjcbžU WUggihYghžU`UVcfUhcfmhYghžU`gYa]bUfidfYgYbhUhjcbicfiUbmch\YfiZcfa icZUggYgga Ybh'

cfilia fth Yfixyfuljgi]bify`uhjcbihciGdYWju`i7 cbg]XYfuhjcbi]bWi X]b[iiK \Ybihci5dd`mižii<ckihci5dd`mi`ubXiiGiddcfhjb[i8cWia Ybhuhjcbiid`YugYifyZyfihcih\YiGdYWju`i7 cbg]XYfuhjcbik YVghjY.ii\hdg.#ghiXYbhibgk "YXii"ui #gdYWju`iWcbg]XYfuhjcb

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]Uf]ga]bhch\yzc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]Y 'a cghmfYhU]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'UbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk'k \c`Yžk]h\ci h'Uddfcdf]UhY'fYZYfYbW]b['"

7 c``i g]cb. k cf_jb[k]h ch\Yfg'Vi hdUgg]b[cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['Uh'Ubmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_žghYU`]b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fWYg'k\]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[`h\Y`″gYWcbXUfm″gci fWY`Zfca 'k\]WX`_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWhfYZYfYbWb[dfUWhWyg.

- i bXYfglUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbhUhi BGK
- VY 'Uk UfY 'cZd`U[]Uf]ga ž Wcdmf][\hzi BGK 'Ghi XYbhi 7 cXY 'cZ 7 cbXi WhUbX '5 WW dhUV 'Y 'I gY 'cZ I BGK '=7 H'F Ygci fW g'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg'cZVY\Uj]ci f'Yl dYWhYX 'cZYj YfmcbY ']b'h\Y 'l BGK 'Wca a i b]hm
- `cWUhY gYfj]WYg 'UbX]bZcfa Uhjcb 'UVci hI BGK 'UbX I BGK '@]VfUfm

7ci fgY GWX YXi `Y

<u>J]Yk "WUgg"hja YHUV"Y</u>

H]a YHJV`Y

8 UhY	Hmd Y	7 cbhYbh
K_YY_"%"&" ">i `m!"&- ">i `m	GYa]bUf	∌lfcXi Wjcb
		L // I/ J/P[: C // P()/
I/	C//- 11-11E	F YUX]b['GYh'%"
K YY_'&.'' \$'>i`m!') 5i[igh	GYa]bUf	FYUX]b['GYh'&"
K YY_" . "* "5i [i gh! "%	GYa]bUf	FYUX]b['GYh' "
5i [i gh]	- 1-1 -
K YY_'(.'% '5i[igh!'% 5i[igh		

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

 $\label{eq:hamiltonian} \text{H\YfY']} g'bc'hYl'hVcc_'Zcf'h\] g'Wci fgY'''5```fYUX]b[g'UfY'`ci fbU``Ufh]WYg'h\Uh'WUb'VY'UWWYggYX'h\fci [\'h\Y''@]VfUfm'$

FYWca a YbXYX FYgci fWfg

BchUj U]`UV`Y

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

 $\label{eq:continuous} Ghi XYbhiZYXVUW_`]g`[Uh\YfYX`h\fci[\abel{eq:continuous}]b`h\YfYei]YbWY"`DfYj]cig`ZYYXVUW_`\Ug``YX`hc`W\Ub[Yg`]b`h\YfYei]fYei]fYX`fYUX]b[g"$

a U[Y'7fYX]h

GmbYf[]Yg^]b^GcibX^&\$%*

7F = 7CG

7 F = 7 C G 'Dfcj]XYf'7 cXY. '\$\$\$-,;