

GWYbWrg. \http://dcfa_Uhcb#dfchcWc\g![i]XY\]bYg#

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5WUXYa]W±bZcfaUh]cb

: cf`YggYbhJU``ghi XYbh`]bZcfa UhJcb`fY`UhJb['hc.`fYei Yghg`Zcf`YI hYbg]cb/fYj]Yk 'cZa Uf_g/cWW dUhJcbU``\YU`h\
UbX`gUZYhm'YI Ua]bUhJcb'dfcWYXi fYg/gdYWJU``Wcbg]XYfUhJcb`]b'h\Y'Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
ghi XYbhYei]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbhJU``UWUXYa]W]bZcfa UhJcbž
qYY`\htdg.#k k k "Ufhq"i bak "YXi "Ui #W ffYbhqhi XYbhq#UWUXYa]W]bZcfa UhJcb#dfchcWc`a![i]XY`]bYa#

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7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

± 'h\]g'Wci fgY'mci 'k]```YUfb'\ck 'hc'UbU`mgY'hYI hg'Zcf'h\Y'di fdcgY'cZhfUbg`Uh]cb"'6UgYX'cb'`GmghYa]W
: i bWf]cbU`'@|b[i]gh]WgžU'a cXY``h\UhfY[UfXg``Ub[i U[Y'Ug'U'a YUb]b[!a U_]b['fYgci fWYž'h\Y'Wci fgY
]bhfcXi Wrg'mci 'hc'U'gYhcZhcc`g'h\Uhk]```YbUV`Y'mci 'hc'i bXYfghUbX``]b[i]gh]WwXc]Wrg'a UXY']b'U'hYI hik]h\]b
]hg'WcbhYI higc'h\Uhmci 'WUb'a U_Y']bZcfa YX'XYW[g]cbg'k\Yb'hfUbg`Uh]b["'

5 hh Y WcbWi g]cb cZh]g Wci fgY h Y gh XYbhk] ` VY UV Y hc

- %" 5bU`mgY`[fUa a Uh]WU``UbX``YI]WU``WXc]WYg`a UXY`]b`hYI hg`UbX`]bhYfdfYhih\Y`UbU`mg]g`]b`WcbbYWIjcb`hc a YUb]b[`UbX`WcbhYI h```
- &" 5dd`mhYl hUbU`mg]g`hc`a U_Y`]bZcfa YX`XYV\]g]cbg`]b`hfUbg`Uh]cb````
- ' " >i gh/ZmirfUbg`Uh/cb`VX.c]\Wg`]b`dfcZYgg]cbU``WcbhYI hg```
- ("FYUX hYI hg Wf]hJWU``mZcf'h\Y'di fdcgY'cZhfUbg`UhJcb

HYUWX]b['GhfUhY[]Yq'

A C 8 @) %** `]bffcXi W'g`\ck 'hc`UbU`mgY`hYI hg`Zcf`h\Y`di fdcgY`cZhfUbg`Uh]cb`UbX`]bhYfdfYh]b[`XfUk]b[`cb GmghYa]W: i bWljcbU``@]b[i]gh]Wg`fG: @!"`H\]g``]b[i]gh]Wh\Ycfm]g`\][\`mfY`Yj Ubh'hc`hfUbg`Uh]cb`UbX]bhYfdfYh]b[`VYWUi gY`]h'dfcj]XYg`i gYZ\``hcc`g`bYYXYX`hc`UbU`mgY`a YUb]b[`]b`WcbhYI hz`k \]W\`]g`U`WcfY Y`Ya Ybh]b`hfUbg`Uh]b[`UbX`]bhYfdfYh]b["'

H\]g`Wci fgY`Wcbg]ghg`cZU`rk c!\ci f``YWi fY`UbX`U`cbY!\ci f`h hcf]U``YUW\kYY_"'=b`h\Y``YWi fYžghi XYbhg`k]``YUfb`h\YcfYh]WU``WcbW'dhg`UVci ha YUb]b[žWcbhYl hUbX`[fUa a Uf"H\Uh]g`\ck `X]ZYfYbh_]bXg`cZa YUb]b[UfY`Wcbghfi YX`rh\fci [\``Yl]\wc!\fua a Uf\Wb\`W\c]\wg`k]h\]b 'hYl h']b 'fY`Uh]cb'hc 'WcbhYl h'=b`h\Y`h hcf]U`ž'h\Ym k]```YUfb`rh\fci [\`Ubi a VYf`cZhfUbg`Uh]cb!fY`UhYX`UWlj]h]Yg`\ck 'hc`UbU`mgY`hYl hg`hc'i bXYfghUbX X]ZYfYbh_]bXg`cZa YUb]b[`UbX`\ck 'hc`bY[ch]UhY`gi W\ a YUb]b[g`]b`h\Y`dfcWgg`cZhfUbg`Uh]cb"`=b`h\]g`k Umžghi XYbhg`k]```VY`Yei]ddYX`k]h\`dfUWf]WU``hcc`g`k]h\`k\]W\`h\YmWUb`i gY`hc'a U_Y`]bZcfa YX`XYVJg]cb`k\Yb h\YmWUffmci hj Uf]ci g`hUg_g`Ug`U`hfUbg`Uhcf`UbX#cf`]bhYfdfYhYf`]b`h\Y`fYU``k cf`X`Ug`k Y```Ug`h\YcfYh]WU`_bck `YX[Y`hc`XYZYbX`h\Y]f`XY`]VYfUhY`XYVJJg]cbgžk\]W\`]g`]bWfYUg]b[`m]a dcfhUbh]b`h\Y`dfcZygg]cbU` Wca a i b]hm]b`h\Y`&ghWb'h fm'``

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

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HYI hiUbU`mg]g'hYghg'	&\$ı	\$' #%\$#&\$% `\$).\$\$`DA	% <u>*</u> (
FYZYW¶jY'YggUm	(\$1	\$+#%%\$\$% `%%\$\$`DA	%ž&Ž Ž(

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% HYI hUbU`mg]g YI Ua

GHJfh:XUhY.

8YHJ]`g. `Gh XYbhg`Wca d`YhY`Ub`Yl Ua]bUh]cb``f%") `\ci fgŁ`hc`XYa cbghfUhY`h\Y]f`UV]`]hmihc`UbU`mgY Wca d`Yl `[fUa a Uh]WU``ZYUh fYg`cZhYl hg"Gh XYbhg`fYWY]j Y`U`gi a a Uh]j Y`[fUXY"

5ggYgga Ybh& HYl hUbU`mg]g hYghg

GHJfh'XUhY. '\$) #\$- #&\$% '\$) . \$\$ 'DA

8 YHJ]`g. `Ghi XYbhg`Wca d`YhY`hk c`hYghg`fl \$`a]bi hYg`YUW\Łhc`XYa cbghfUhY`h\Y]f`UV]`]hmhc`UbU`mgY`VUg]W [fUa a Uh]WJ``ZYUhi fYg`cZhYl hg"Ghi XYbhg`fYWY]j Y`U`gi a a Uh] Y'a Uf_k]h\`]bX]j]Xi U``ZYYXVUW_"`

5ggYgga Ybh' FYZYWNji Y YggUm

GhUfh XUhY.

8YHJ]`g. `Ghi XYbhg`k f]hY`Ub`YggUmfl ž\$\$\$`k cfXgŁ`hc`XYa cbghfUhY`h\Y]f`UV]`]hmhc`Udd`mhYl h\UbU`mg]g _bck`YX[Y`UbX`g_]``g`hc`HfUbg`Uh]cb"Ghi XYbhg`fYWY]j Y`]bX]j]Xi U``Wca a Ybhg`UbX`[fUX]b[`i g]b[`U`fi Vf]WH\]g]g`h\Y`Z]bU``UggYgga YbhZcf`UhHYbXUbWY`di fdcgYg"`

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gh XYbhgʻUfYʻYl dYWYXʻhcʻdi hh\Y]fʻbUa YgʻUbXʻgh XYbhbi a VYfgʻcbʻYj YfmdU[YʻcZh\Y]fʻUgg][ba Ybhgʻ

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV`Ya k \Yb'UhYa dh]b['rc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd`YUgY
hY`Yd\cbY'9I hYfbU`Gi ddcfhcb'-',) "''' %cf'Ya U]`'h\Ya 'cb'<u>YI hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui"</u>"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft!" #mci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi `hk]h\ 'Hi fb]h]b'mci 'a UmUdd`m'Zcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhU]b'mci f'h]W_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgt'hc
]dU XYb

- K cf_'gi Va]hhyx'%5'hc'% 'XUmg'UZhYf'h\Y'gh]di `Uhyx'XYUX`]bY'k]``'VY'UggYggYX'UbX'ZYYXVUW_dfcj]XYX'Vi h'U'a Uf_'cZnYfc'k]``'VY'fYWcfXYX'''=Zh\Y'k cf_'k ci `X'\Uj Y'fYWf]j YX'U'dUgg'a Uf_'Vi h'Zcf h\Y'`'UhYbYgg'UbX'h\Y'k cf_']g'U'Wca di `gcfm'Wci fgY'Wca dcbYbh'f\i fX`Y'fYei]fYa Ybhz'U'gh XYbh'k]`` VY'XYYA YX'hc'\Uj Y'a Yhh\Uh'fYei]fYa Ybh/
- K cf_'gi Va]hhYX`&\$`cf`a cfY`XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```bchVY`UWW\dhYX`Zcf UggYgga YbhUbX`k]```fYWY]j Y`bc`ZYYXVUW_ža Uf_'cf`[fUXY"'=Zh\Y`UggYgga YbhHUg_']g`U`Wca di `gcfm Wca dcbYbhcZh\Y`Wci fgY`U`gh XYbhk]```fYWY]j Y`Ub`l bgUh]gZJWcfm: U]``fl : Ł`[fUXY`Ug`U`fYgi `hcZ i bgUh]gZJWcfmdYfZcfa UbWY`]b`Ub`YggYbh]U``Wca dcbYbhcZh\Y`Wci fgY"

H\]g']bZcfa Uh]cb']g'U`gc'Uj U]`UV`Y'Uh

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uhcb#dfchcWc`g![i]XY`]bYg#

GdYWJU`7cbq]XYfUhcb55dd`]WUhcbq

 $\label{thm:condition} $$Mci `WUb`Udd`m'Zcf'gdYW]U``Wcbg]XYfUh]cb`k \Yb`]``bYgg`cf`ch\Yf'W]fW a ghUbWfg`]bhYfZYfY`k]h\`mci f UggYgga Ybh'dYfZcfa UbWf''$

G]W_bYggža]gUXj Ybhi fY'cf'ch\Yf'W]fW a ghUbWrg'VYmcbX'mci f'Wcbhfc``a Um

ł DfYj Ybhmci Zfca Wca d`Yhjb['U'Wci fgY'fYei]fYa Ybhž

ł "? YYd 'mci 'Zfca 'UthYbX]b['Ub 'UggYggUV'Y 'UWhj]lmž

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ł G][b]Z]WUbhmUZZYWimci fidYfZcfa UbWijbiUggYggUV`Yik cf_žVYi]hUiZcfa U``YbX!cZlgYa YghYfiYl Ua]bUh]cbž'U WUqqihYqhž'Ui`UVcfUhcfmhYqhž'UiqYa]bUfidfYqYbhUh]cbicfiUbmch\YfiZcfa icZiUqqYqga Ybh'

cfilia fth Yfixyfuljgi]bify`uhjcbihciGdYWju`i7 cbg]XYfuhjcbi]bWi X]b[iiK \Ybihci5dd`mižii<ckihci5dd`mi`ubXiiGiddcfhjb[i8cWia Ybhuhjcbiid`YugYifyZyfihcih\YiGdYWju`i7 cbg]XYfuhjcbik YVghjY.ii\hdg.#ghiXYbhibgk "YXii"ui #gdYWju`iWcbg]XYfuhjcb

5 WUXYa | W < cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i\in\WbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiW\YUh]b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWlik]h\ci hiUW_bck iYX[Ya Ybh'i

 $I BGK `[fci dg'd`U[]Uf]ga `]bhc'h\Y'Zc``ck]b[`WUhY[cf]Yg.$

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`h\I hcf']XYU'k]h\ci hUW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfc YWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

7ci fgY GWX YXi `Y

<u>J]Yk "WUgg"hja YHUV"Y</u>

H]a YHJV`Y

8 UhY	Hmd Y	7 cbhYbh
K_YY_"%"&" ">i `m!"&- ">i `m		BC ⁻⁷ @5GG
K YY '&.'' \$'>i `m'!')	@YWh fY	HYI hilmdYgʻUbXʻfY[]ghYf
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FYgci fW/g

DfYgWf]VYX F Ygci fWYg

A Uff]bž'>F ž'A Uff\]YggYbž'7 A =/ 'DU]bhYfž'7 "'f&\$%\$Ł'6 Wb'ak[' Y 8g` Uf[a` S^9 dS_ _ Sd''6Y]']b[.'H\Y 7 ca a YfW]U'`DfYgg"

FYWca a YbXYX FYgci fWfg

FYYj UbhfYgci fWg'UfY'fYWca a YbXYX']b'WUgg'YUW\'k YY_"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

=a U[Y'7 fYX]h

D\chc VmG\UVi '5bck Yf'cb' | bgd`Ug\

7F = 7CG

7F = 7CG Dfc|]XYf 7cXY. \$\$\$-,;