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# Modelling the Effects of Corporate Taxation in the Underground Economy

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***Abstract***
This paper develops a two-sector search model of the labour market in which firms in one sector (the informal sector) evade profit taxes (underground economy). A comparative static analysis is employed to analyze the impact of corporate taxation on unemployment, occupational choice of individuals, mix of jobs, welfare of agents and the size of informal sector. The findings suggest that profit, firing and payroll taxation have the same effects on the above economic variables. However, if a

the tax rate on the reported income in the static case is ambiguous. However, Yitzhaki (1974) showed that if the fine for tax evaders is imposed on the evaded tax and not on

(ii) The flow of firms out of the taxed sector is not equal to the flow of firms into the tax evading sector and therefore the total number and the arrival rate of vacant jobs decrease. In a Nash bargaining process the former effect increases the outside option (threatening point) of individuals who search only for jobs in the underground sector whereas the latter effect decreases it. If the impact of (i) is greater than that of (ii) on the outside option of those individuals then their welfare unambiguously increases.

Apart from welfare effects, we investigate the impact of an increase of profit and firing taxes on total unemployment and relative sectoral employment (thus measuring the size of the underground sector). We also examine how profit and severance taxes influence the occupational choice of individuals and the mix of vacancies. We find that payroll taxation in Albrecht et al (2006) work has the same impact on the above economic variables with the corporate and the firing tax in our analysis. However, our assumption about the endogeneity of the arrival rate of informal sector jobs is the driving force behind the result, that less people accept only informal sector jobs as corporate income tax or severance tax increase, when the parameter which captures the “technological” advances in the matching process is high enough. This result is the opposite from that of the firing and payroll tax in Albrecht et al (2006). Such a result cannot be obtained in the case of payroll taxation in the Albrecht et al paper, since the arrival rate of informal sector jobs is exogenous. This result also leads us to important policy implications. Active labour market policies favouring technological advances in the matching process between employers and employees (technological advances in the matching process include reforms such as the computerization of employment offices, job advertising on the internet, job-search assistance policies, governmental subsidies into policies helping the matching process etc.), will ‘moderate’ the expansion of the underground sector caused by an increase in profit/firing taxes. Moreover, the adoption of such policies, will limit the reduction of wages induced by higher taxes (corporate/firing).

The model which is closest to ours is that of Albrecht, Navarro and Vroman (2006). In their paper, they extended the standard Mortensen and Pissarides (1994) model of the

$$rW_i(a) = w_i(a) + \delta[U(a) - W_i(a)] \qquad i = 1,2 \tag{2}$$

where $w_i(a)$ is the wage received by a worker with skill vector $a$, employed to sector $i$. Equation (2), determines the flow value of employment as the sum of the flow return to employment (the wage) plus the instantaneous capital loss.

**Firms**

*a) Vacant*

The Bellman equation for vacancies is

$$rV_i = -c + \frac{m(\theta)}{\theta} E_a[\rho_i(a) \max\{J_i(a) - V_i, 0\}] \tag{3}$$

Equation (3) incorporates the assumption that $a$ is unknown to vacancies before they contact workers and it is only realized when the meeting is taking place. However, firms know the distribution of $a$ s'. Thus they form expectations about their capital gain from becoming filled.

*b) Filled*

The flow value to a firm in sector $i$ filled by a worker of type $a$ is

$$rJ_i(a) = [a_i - w_i(a)](1 - \omega_i p_i \tau) + \delta[V_i - J_i(a)] \tag{4}$$

where $\omega_1 p_1 = 1$ and $\omega_2 p_2 = \omega p$.

**Wage Formation and Reservation Ability**

Define $\hat{W}_i(a,w)$ as the value of employment in sector $i$ on wage $w$. If there is positive surplus, then

$$r\hat{W}_i = w + \delta[U(a) - \hat{W}_i] \Rightarrow \hat{W}_i = \frac{w + \delta U(a)}{r + \delta}$$

Define $\hat{J}_i(a,w)$ as the value of a filled vacancy in sector $i$ on wage $w$. If there is positive surplus, then

$$r\hat{J}_i = (1 - \omega_i p_i \tau)[a_i - w] + \delta[V_i - \hat{J}_i] \Rightarrow \hat{J}_i = \frac{(1 - \omega_i p_i \tau)[a_i - w] + \delta V_i}{r + \delta}$$

*Symmetric Nash Bargaining*

$$w = \arg\max_w [\hat{W}_i(a,w) - U(a)][\hat{J}_i(a,w) - V_i]$$

$$\Rightarrow \frac{1}{r + \delta}[\hat{J}_i(a,w) - V_i] = \frac{1 - \omega_i p_i \tau}{r + \delta}[\hat{W}_i(a,w) - U(a)]$$

Then at the equilibrium wage $w^*$, $J_i = \hat{J}_i$, $W_i = \hat{W}_i$ satisfing

$$\frac{\quad _i \qquad\quad _i}{\omega_i \quad _i\tau}$$

$$_i \qquad\quad _i$$

Proof: Suppose that $a_1 \geq a_2$. This implies that $W_1(a) \geq W_2(a)$. It can be easily shown that there is an $a_2 = a_2^R$, such that

$$W_2(a_1, a_2^R) = U(a_1, a_2^R) \Rightarrow rU(a_1, a_2^R) = a_2^R$$

From equation (1) we get:

$$rU(a_1, a_2^R) = m(\theta)\varphi[W_1(a_1, a_2^R) - U(a_1, a_2^R)]$$

By using equation (2) and (7), we can show that $a_2^R(a_1) = \dfrac{m\varphi a_1}{2(r+\delta)+m\varphi}$. Hence if $a_1 \geq a_2 \geq a_2^R(a_1)$, then $W_2(a) \geq U(a)$ and individuals accept jobs in both sectors. However if $a_2 < a_2^R \Rightarrow a_2 < rU(a)$

Lemmas 1, 2 and 3 are illustrated in Diagram 1. The blue line is the 45$^o$ degree line. On the horizontal axis is the productive capability ($a_1$) of each individual in sector 1 and on the vertical axis is the corresponding capability ($a_2$) in sector 2. The green (red) line is the 'frontier' above (below) which individuals accept jobs only in sector 2 (1).

Following from Lemmas 1, 2 and 3 when the arrival rate of job offers increases and the job's destruction rate and/or the interest rate decreases, the red (green) line shifts upwards (downwards). Moreover, when the arrival rate of sector 1 (2) employment opportunities goes up, the red (green) line shifts upwards (downwards).

By using Lemmas 1, 2 and 3 and equations (1), (2) and (7) we get the following flow values of unemployment:

$$rU(a) = \frac{m(.)\phi a_1}{2(r+\delta)+m(.)\phi} \quad \text{if} a_2 \le a_2^R \tag{8}$$

$$rU(a) = \frac{m(.)(1-\phi)a_2}{2(r+\delta)+m(.)(1-\phi)} \quad \text{if} a_1 \le a_1^R \tag{9}$$

$$rU(a) = \frac{m(.)[\phi a_1 + (1-\phi)a_2]}{2(r+\delta)+m(.)} \quad \text{otherwise} \tag{10}$$

The flow value of unemployment for workers with $a$'s below the reservation values

Let $\lambda_t(a)$ and $g_t(a)$ denote the densities of unemployed and employed individuals respectively with skill vector, $a$, at time $t$. The above densities are related by the restriction that $f(a) = \lambda_t(a) + g_t(a)$ where $f(a) = 1$ is the density of the total population which does not depend on time. During any infinitely small interval of time, $dt$, unemployed individuals with $a_2 \leq a_2^R(a_1)$ and $0 \leq a_1 \leq 1$ become employed at rate $m(\theta)\varphi dt$ whereas a fraction $\delta dt$ of them lose their job. Hence, the evolution of employed individuals with $0 \leq a_2 \leq a_2^R(a_1)$ and $0 \leq a_1 \leq 1$ will be equal to $m(\theta)\varphi\lambda_t(a)dt - \delta g_t(a)dt$. Similarly, we can define the evolution of employed individuals with $0 \leq a_1 \leq a_1^R$, $1 \geq a_2 \geq 0$, and $\tilde{a}_1 \leq a_1 \leq 1$, $a_2^R < a_2 \leq 1$ and $a_2^R(a_1) < a_2 < a_1^{R^{-1}}$, $0 \leq a_1 \leq \tilde{a}_1$ (where $a_1^{R^{-1}}(\tilde{a}_1) = 1$ and $a_1^{R^{-1}}$ is the inverse function of $a_1^R$). In steady-state the evolution of employed individuals is equal to zero (i.e., the flow of workers out of unemployment sh8(u)6.9 Tc 8(30al)-6(s)6.9( 8(u138.42 56826 23.008 0 TD(2)Tj-3.5

$$u = \int_{a_2=0}^{1}\int_{a_1=0}^{1} \lambda(a) da_1 da_2$$

By doing the calculations, we obtain

$$u(\theta,\varphi) = \frac{\delta m}{2}\{\frac{(1-\varphi)}{[2(r+\delta)+m(1-\varphi)][m(1-\varphi)+\delta]} + \frac{\varphi}{[2(r+\delta)+m\varphi](m\varphi+\delta)}\} + (r+\delta)\psi\{\frac{4(r+\delta)+m}{[2(r+\delta)+m\varphi][2(r+\delta)+m(1-\varphi)]}\} \quad (17)$$

The steady-state employment can be defined as $1-u$.

**Definition 1** *A steady-state equilibrium is a five tuple* $a_2^R$, $a_1^R$, $\theta$, $u$, $\varphi$ *that satisfy: (i) 'Free' entry, i.e.,* $V_i = 0, \ i = 1,2$, *(ii) 'Balanced flows,' i.e., the flow of workers out of unemployment equals to the flow of workers into unemployment (equation (17)) and (iii) the reservation properties in Lemmas 2 and 3.*

Let $F(a_1, a_2)$ denote the cumulative distribution function (c.d.f.) describing the distribution of $a$ across unemployed workers. Then:

$$\frac{\partial^2 F}{\partial a_1 \partial a_2} = \frac{\lambda(a)}{u}$$

The free entry conditions can be written as:

$$c = \frac{m(\ )}{\ } \int_{a_1=0}^{1}\int_{a_2=a_2^R}^{1} J_1(a) \frac{\partial^2 F}{\partial a_1 \partial a_2} da_2 da_1$$

$$= \frac{(1\quad)}{(\ ,\ )}\{\ \frac{[\ (1\quad)\ ]}{[2(\quad)\quad(1\quad)]\ [\ (1\quad)\quad]}$$

$$\frac{(1)\ {}_{2}{}^{1}(\ ,\ )}{{}_{1}(\ ,\ )}\qquad {}_{1}\ {}_{2}\qquad \frac{1\qquad 1}{{}_{2}(1)\ {}_{1}(\ ,\ )}\qquad {}_{1}\ {}_{2}\} \tag{19}$$

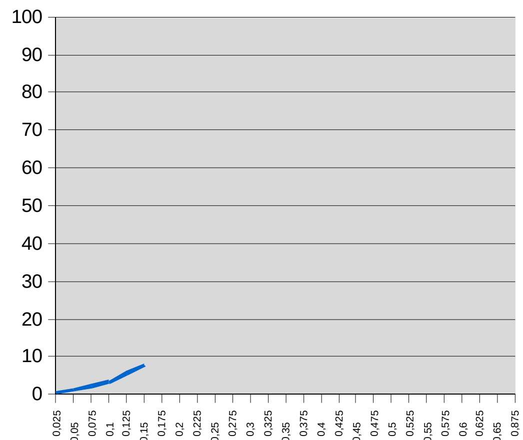
where $= \frac{[2(\quad)\ {}_{2}\quad(\ {}_{2}\quad {}_{1})]}{2(\quad)[2(\quad)\quad]}$ and ${}_{2}(1) = \frac{\qquad}{2(\quad)}$.

equations (18), (19) are equal if $\varphi = 0.5$ )[8], and if we assume that the elasticity of $m(.)$ with respect to $\theta$ is less than or equal to 0.5 (in the case of a Cobb–Douglas matching function characterized by constant returns to scale, $m(\theta) = A\theta^{\gamma}$ where $\gamma$ is the elasticity of ( )

as a result of the lower vacancy supply in sector 1 will give an incentive in sector 2 to increase its vacancy creation. However, this effect will be completely offset by the negative effect caused by the change in the mix of vacancies in favour of the shadow sector[10] and the increase in tax rates. At the end the steady-state measure of labour market tightness and the fraction of sector 1 vacancies will diminish. A number of individuals previously searching only for sector 1 jobs will now accept offers in both sectors as $a_2^R$ unambiguously decreases.[11] The impact that the taxation of profits has on $a_1^R$ is ambiguous. The reason for that is the existence of two opposing effects; the 'tightness' effect (reduction of $\theta$) which decreases $a_1^R$

parameters are: $A = 0.5$, $c = 0.3$, $\delta = 0.1$, $r = 0.05$, $\omega = 0.3$, $p = 1.5$ and $m(\theta) = A\sqrt{\theta}$ (where $A$ is the parameter capturing the technological advances in the matching process). Finally, Table 2 presents a case where (23) does not hold ( $A = 1.5$, $c = 0.3$, $\delta = 0.1$, $r = 0.05$, $\omega = 0.3$, results[12] regarding informal sector as a firing or a payroll tax increases, as long as matching technology is relatively high (high matching technology can be obtained through active labour market policies). This effect will mitigate the expansion of the underground sector. In other words, active labour market policies assisting the matching process will limit the negative effects of taxation. Moreover, the average wage decreases with corporate income tax regardless of inequality (23) but when (23) does not hold –the constant of matching is high– then the decrease is smoother. This result is illustrated in Diagrams 2 and 3.

**Diagram 2**



Our welfare analysis will focus on the individuals who accept jobs only in the underground sector. This is because under certain conditions their welfare increases with profit tax. More specifically, according to our previous analysis, if (23) holds then $a_1^R$ and the arrival rate of sector 2 vacancies both increase with profit taxation. An immediate result from the increase of $a_1^R$ is the increase of the wage received by the individuals who are employed in sector 2 and their $a_2 \le a_1^R$ (for those individuals $a_1^R$ represents their reservation wage). Moreover, an increase in the arrival rate of jobs in the underground sector decreases the period of unemployment for those searching for sector 2 jobs. Hence, the welfare of individuals with $a_2 \le a_1^R$ (accept jobs only in sector 2) after the increase of profit tax unambiguously increases.

## 4.1 FIRING TAXES

Under a firing tax and without corporate taxes equation (4) becomes

$$rJ_i(a) = [a_i - w_i(a)] + \delta[V_i - J_i(a) - s_i] \tag{4b}$$

where $s_2 = \omega ps$, $s_1 = s$ and $s$ is the firing tax. Mathematical calculations yield

$$J_1(a|\ a_2 \le a_2^R) = \frac{a_1}{2(r+\delta) + m(.)\varphi} - \frac{\delta s}{r+\delta} \tag{11b}$$

$$J_2(a|\ a_1 \le a_1^R) = \frac{a_2}{2(r+\delta) + m(.)(1-\varphi)} - \frac{\delta \omega ps}{r+\delta} \tag{12b}$$

$$J_1(a|\ a_2 \ge a_1 > a_1^R \,\&\, a_1 \ge a_2 > a_2^R) = \frac{[2(r+\delta)a_1 + m(1-\varphi)(a_1 - a_2)]}{2(r+\delta)[2(r+\delta)+m]} - \frac{\delta s}{r+\delta} \tag{13b}$$

$$J_2(a|\ a_2 \ge a_1 > a_1^R \,\&\, a_1 \ge a_2 > a_2^R) = \frac{2(r+\delta)a_2 + m\varphi(a_2 - a_1)}{2(r+\delta)[2(r+\delta)+m]} - \frac{\delta \omega ps}{r+\delta} \tag{14b}$$

The flow values of unemployment are the same with these in the analysis of corporate taxation. Hence, under a firing tax the equilibrium values of $\theta$ and $\varphi$ are given from the following equations:

{[ ( , ) [2( ) ] ( )

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$$c = \frac{m}{\theta u(\theta,\varphi)}\{\int_0^1 \frac{a_2[m(1-\varphi)a_2]\delta}{[2(r+\delta)+m(1-\varphi)]^2[m(1-\varphi)+\delta]}da_2 + \int_0^{a_2^R(1)}\int_{a_1^R(\theta,\varphi)}^{a_2^{R^{-1}}(\theta,\varphi)} \zeta da_1 da_2 + \int_{a_2^R(1)}^1\int_{a_1^R(\theta,\varphi)}^1 \zeta da_1 da_2\} - \frac{\delta\omega ps}{(r+\delta)}\{[\int_0^1\int_0^{a_2^R(\theta,\varphi)} \frac{\delta}{m(1-\varphi)+\delta}da_1 da_2 + \int_0^{a_2^R(1)}\int_{a_1^R(\theta,\varphi)}^{a_2^{R^{-1}}(\theta,\varphi)} \frac{\delta}{m+\delta}da_1 da_2 + \int_{a_2^R(1)}^1\int_{a_1^R(\theta,\varphi)}^1 \frac{\delta}{m+\delta}da_1 da_2]\} \quad (19b)$$

The existence of equilibrium can be easily proven (see the Appendix). Following the same procedure with the above subsection, we get $\frac{d\theta}{ds}|_{\varphi=0.5}, \frac{d\varphi}{ds}|_{\varphi=0.5} < 0$ (see the Appendix). Hence, an increase in firing tax (when corporate tax is zero) has the same effects as the increase in corporate taxation.

Table 4: *Simulation of the model when (23b) does not hold*

[illegible]

The last two columns of Tables 3 and 4 present the fraction of sector 1 and sector 2 employed (where inside the brackets is the absolute number of employed). The fourth and the fifth column present the range of the reservation abilities.

## 5. CONCLUSION

In this paper, we examined how profit and firing taxation influence the size of the underground economy. We conclude that the impact of wage and payroll taxation on the size of the underground sector (a subject which is widely examined by the literature) is the same with that of profit and firing taxation. More specifically as profit tax or severance tax increases, the size of the underground sector increases too. Moreover, we showed that the adoption of active labour market policies which assist unemployed individuals to find more easily the 'whereabouts' of vacant jobs, will 'mitigate' the expansion of underground sector and the reduction of wages caused by taxation. Finally, active labour market policies can increase the welfare of a subgroup of individuals as taxation (corporate or firing) increases.

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As $\theta \to 0$, $\lim_{\theta \to 0}\left(\frac{2\delta[2\delta(r+\delta)+m(r+\delta)+m0.5(m+\delta)]}{(m+\delta)(m+2\delta)[m0.5+2(r+\delta)]}\right)=1$,

$\lim_{\theta \to 0}\frac{[2(r+\delta)a_2+m0.5(a_2-a_1)]\psi}{2(r+\delta)[2(r+\delta)+m]}=\frac{a_2}{2(r+\delta)}$,

$\lim_{\theta \to 0}\frac{a_2[m0.5a_2]\delta}{[2(r+\delta)+m0.5]^2[m0.5+\delta]}=0$,

$\lim_{\theta \to 0} a_2^R(1)=0$,

$\lim_{\theta \to 0} a_1^R(\theta,\varphi)=0$,

$\lim_{\theta \to 0} a_2^R(\theta,0.5)=0$,

$\lim_{\theta \to 0} a_1^R(\theta,0.5)=0$ (since the reservation values are equal to zero as $\theta \to 0$, the first two double integrals of the above equation are equal to zero). From our assumptions $\lim_{\theta \to 0}\frac{m}{\theta}=\infty$. Hence as $\theta \to 0$ the r.h.s of (24) approaches infinity.

As $\theta \to \infty$, the reservation productivities are equal to the 45$^o$ line. Hence the last two integrals of (24) are equal to zero. We get that $\lim_{\theta \to \infty}\frac{m}{\theta u(\theta,0.5)}\int_0^1\frac{a_2[m0.5a_2]\delta}{[2(r+\delta)+m0.5]^2[m0.5+\delta]}da_2=$

$$\lim_{\theta \to \infty}\frac{m}{\theta}\lim_{\theta \to \infty}\frac{2[m0.5]}{3[2(r+\delta)+m0.5]}\frac{(m+\delta)}{2[2\delta(r+\delta)+m(r+\delta)+m0.5(m+\delta)]}\overset{\text{by applying del Hospital rule}}{=}0$$

Hence as $\theta \to \infty$ the r.h.s. of (24) approaches zero. Moreover as we have shown the r.h.s. of (24) decreases in $\theta$. The above analysis implies that a unique equilibrium exists for $\tau=0$. The same result is derived in the case of severance tax since $\frac{\quad}{(\quad)}$

$$c = \frac{\delta(1-\tau)\{6(r+\delta)[m+2(r+\delta)]+m^2\}}{12[m+2(r+\delta)]^2(r+\delta)(m+\delta)}\frac{m}{\theta u(\theta,0)} \Rightarrow$$

$$c = \frac{\delta(1-\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,0)} + \frac{\delta(1-\tau)m^2}{12[m+2(r+\delta)]^2(r+\delta)(m+\delta)}\frac{m}{\theta u(\theta,0)} = \Lambda(\theta)$$

By substituting $\varphi = 0$ into (19) we get:

$$c = \frac{\delta(1-\omega p\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,0)} = \Omega(\theta)$$

The above equations have a solution in $\theta$, since they are decreasing in $\theta$ and $\lim_{\theta\to 0}\Lambda(\theta),\Omega(\theta) = \infty$ and $\lim_{\theta\to\infty}\Lambda(\theta),\Omega(\theta) = 0$.

Let $\theta_1$ be the solution of $c = \Omega(\theta)$ and $\theta_2$ be the solution of $c = \Lambda(\theta)$. In order to prove that the curve described by (18) is above that described by (19) for $\varphi = 0, \tau \neq 0$ (i.e. $\theta_2 > \theta_1$), we have to show that:

$$\Lambda(\theta_1) > \Omega(\theta_1)$$

By solving with respect to $\tau$, we get:

$$\tau < \frac{m^2(\theta_1)}{m^2(\theta_1) + 6(r+\delta)[m(\theta_1)+2(r+\delta)](1-\omega p\tau)}$$

The r.h.s. of the above inequality is positive and less than one. Hence, there will be values of $\tau$, such that $\theta_2 > \theta_1$ for $\varphi = 0$. By substituting $\varphi = 1$ into (18) we get:

$$c = \frac{\delta(1-\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,1)}$$

By substituting $\varphi = 1$ into (19) we get:

$$c = \frac{\delta(1-\omega p\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,1)} + \frac{\delta(1-\omega p\tau)m^2}{12[m+2(r+\delta)]^2(r+\delta)(m+\delta)}\frac{m}{\theta u(\theta,1)}$$

By following the same analysis, it can be easily shown that the value of $\theta$ which satisfies $c = \frac{\delta(1-\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,1)}$, is less than that satisfying $c = \frac{\delta(1-\omega p\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,1)} + \frac{\delta(1-\omega p\tau)m^2}{12[m+2(r+\delta)]^2(r+\delta)(m+\delta)}\frac{m}{\theta u(\theta,1)}$. Hence, the existence of solution for $\tau \neq 0$ is proved.

Starting from $\tau = 0$, $\varphi = 0.5$ an increase in $\tau$ corresponds to a shift of the curve described by (19) downwards. Hence, if we start from the symmetric case where $\tau = 0$ and $\varphi = 0.5$, and increase $\tau$, there will exist an equilibrium characterized by lower $\varphi$ and $\theta$. The same analysis is applied in the case of firing taxes.

A) *Proof that* $\partial Z/\partial\theta < 0$ *(*$\partial\Gamma/\partial\theta < 0$*) when the derivative is calculated for* $\varphi = 0.5$ *and the elasticity of* $m(\theta)$ *w.r.t.* $\theta$ *is less or equal to* $0.5$.

$Z$ consists of two parts: the arrival rate of workers ($m(\theta)/\theta$) divided by the measure of steady-state unemployment and the term inside the braces. The term inside the braces can be alternatively written with the following way:

$$(1 \quad )\{ \int_0^1 \int_0^{a_2^R} \frac{a_1}{[2(r \quad ) \quad m \quad ](m \quad )} da_2 da_1 \quad \int_0^1 \int_{a_2^R}^{a_1} da_2 da_1$$

$$\int_0^{a_1^R(1)} \int_{a_1}^{a_{1R}^1} da_2 da_1 \quad \int_{a_1^R(1)}^1 \int_{a_1}^1 \mu da_2 da_1 \}$$

In the above equation, the derivative of the third integral with respect to $\theta$ is always negative, whereas the derivative of the sum of the first two integrals with respect to labour market tightness is equal to

$$\frac{\delta m'}{3[2(r+\delta)+m\varphi]^3}\{\frac{\varphi[2(r+\delta)\delta-m\varphi\delta-2m^2\varphi^2]}{[\delta+m\varphi]^2}-\frac{\varphi m(r+\delta)}{[\delta+m]^2}-\frac{\varphi 2(r+\delta)}{[\delta+m]}-\frac{2(r+\delta)^2}{[\delta+m]^2}\} \quad (27)$$

But for $\varphi = 0.5$, $\delta/[\delta+m\varphi]^2$ is less than $1/(m+\delta)$. Hence, the derivative of the term inside the braces in equation (18) w.r.t. $\theta$ is always negative if it is evaluated at $\varphi = 0.5$.

The steady-state unemployment is equal to

$$u(\varphi,\theta) = \int_0^1\int_0^{\frac{m\varphi a_1}{2(r+\delta)+m\varphi}}\frac{\delta}{m\varphi+\delta}da_2da_1+\int_0^1\int_{\frac{m\varphi a_1}{2(r+\delta)+m\varphi}}^{a_1}\frac{\delta}{m+\delta}da_2da_1+$$
$$\int_0^1\int_0^{\frac{m(1-\varphi)a_2}{2(r+\delta)+m(1-\varphi)}}\frac{\delta}{m(1-\varphi)+\delta}da_1da_2+\int_0^1\int_{\frac{m(1-\varphi)a_2}{2(r+\delta)+m(1-\varphi)}}^{a_2}\frac{\delta}{m+\delta}da_1da_2$$

After simplifying the above expression and by multiplying with $\theta/m$, we can show that $u(\varphi,\theta)\theta/m$ is equal to

$$\int_0^1\frac{\delta\theta a_2}{(m+\delta)m}da_2+\int_0^1\frac{\delta\theta a_1}{(m+\delta)m}da_1+\int_0^1\int_0^{\frac{m(1-\varphi)a_2}{2(r+\delta)+m(1-\varphi)}}\frac{\varphi\theta}{[m(1-\varphi)+\delta](m+\delta)}da_1da_2+$$
$$\int_0^1\int_0^{\frac{m\varphi a_1}{2(r+\delta)+m\varphi}}\frac{(1-\varphi)\theta}{(m\varphi+\delta)(m+\delta)}da_2da_1 \quad (28)$$

By differentiating equation (28) w.r.t. $\theta$, we get that $\partial(u\theta/m)/\partial\theta$ is equal to

$$\frac{\delta[\delta(m-\theta m')+m(m-2\theta m')]}{(m+\delta)^2m^2}+\int_0^1\varphi\{\frac{\theta a_1^{R'}}{[m(1-\varphi)+\delta](m+\delta)}+$$
$$\frac{a_1^R[\delta^2+\delta(2-\varphi)(m-\theta m')+(1-\varphi)(m-2\theta m')]}{[m(1-\varphi)+\delta]^2(m+\delta)^2}\}da_2+$$
$$\int_0^1(1-\varphi)\{\frac{\theta a_2^{R'}}{[m\varphi+\delta](m+\delta)}+\frac{a_2^R[\delta^2+\delta(1+\varphi)(m-\theta m')+\varphi(m-2\theta m')]}{[m\varphi+\delta]^2(m+\delta)^2}\}da_1$$

$\partial Z/\partial\theta < 0$ when the derivative is calculated for $\varphi = 0.5$ and the elasticity of $m(\theta)$ w.r.t. $\theta$ is less or equal to $0.5$ (the proof for $\partial\Gamma/\partial\theta$ is similar).

Since $\frac{\delta s}{(r+\delta)}$ does not depend on $\theta$, $\varphi$ the analysis and the results in the case of firing taxes is the same.

*B) Proof that* $\partial Z/\partial\varphi < 0$ *(* $\partial\Gamma/\partial\varphi > 0$ *) when the derivative is calculated for* $\varphi = 0.5$

$Z$ is the product of the the arrival rate of workers ( $m(\theta)/\theta$ ) divided by the measure of steady-state unemployment times the term

But for $\varphi = 0.5$, $\delta/[\delta + m\varphi]^2$ is less than $1/(m+\delta)$ and the last term is equal to zero. Hence, the derivative of the term inside the braces in equation (18) w.r.t. $\varphi$, it is always negative if it is evaluated at $\varphi = 0.5$ (the proof for $\partial\Gamma/\partial\varphi$ is similar).

Since $\dfrac{\delta s}{(r+\delta)}$ does not depend on $\theta$, $\varphi$ the analysis and the results in the case of firing taxes is the same.