

November 10, 2016

Asse Pr c ng hen r d ng s En er n en

Jiang Luo* and Avaniidhar Subrahmanyam**

*Nanyang Business School, Nanyang Technological University, Singapore.

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1. Introduction

The goal of our paper is to consider a financial market equilibrium where agents derive utility from the act of trading. In much of our work, information is symmetric and there are multiple assets, each traded by agents who possess the standard exponential-normal utility function over wealth but some of whom get additional utility from trading. We do not model the origins of this utility; it could possibly emanate from the thrill of seeing position values fluctuate. The notion that agents may gamble for pleasure is well-

equilibrium declines as the direct utility from trading rises.

It is reasonable to suppose that agents may derive greater utility from some stocks relative to others. The two main characteristics proposed by Kumar (2009) for stocks that are attractive to individual investors are high (positive) skewness, and high volatility. Since our model has normally distributed payoffs it unfortunately cannot speak to skewness preference. We instead consider the assumption that agents obtain more utility from stocks with more volatile payoffs. We find evidence that under reasonable conditions, such as

with agents desiring greater utility from trading that stock. However, stocks from which

should exhibit greater trading volume and lower volatility, and less evidence of covariance risk pricing. These stocks should also exhibit nonlinear responses to positive news. Our analysis also suggests that covariance risk pricing should be less visible in countries or economies where retail investors form a bigger fraction of the trading population.

The idea that agents may trade for purposes of deriving enjoyment from trading is not new; but explicit theoretical modeling of this notion does not yet appear in the literature.

how agents who derive direct utility from trading affect price informativeness. Section 5 presents a dynamic extension, and Section 6 concludes. All proofs of propositions and corollaries, unless otherwise stated, appear in Appendix A, while Appendix B presents some ancillary derivations.

The Model

There are two dates, 0 and 1, and $N + 1$ risky securities. At date 1, these securities pay liquidating dividends of $d = (d_1, \dots, d_{N+1})'$

The utility function of the i 'th regular (non-) trader is the standard exponential one:

$$u_i(x_i) = -\exp(-\lambda x_i)$$

with $\lambda > 0$. Based on the normality assumption of our model, he chooses X_i^* to maximize

$$E[u_i(x_i)] = -\exp\left[-\lambda \left(\delta + [X_i^* (\mu - \delta) - 0.5 X_i^{*2} \text{Var}(\mu)]\right)\right]$$

The first order condition (f.o.c.) with respect to X_i^* (w.r.t.)

take more aggressive positions relative to traditional utility maximizers. As the utility of trading increases, the position vector explodes, and beyond a certain level of λ , there is no interior optimum. The scale of the position taken per unit expected price appreciation increases in λ , which governs how much additional utility is derived from trading.

1.2 Security Payoffs and Factor Structure

We now explicitly model security payoffs as a factor structure to analyze how volume, volatility, and the pricing of covariance risk are affected by the presence of λ traders. In what follows, unless otherwise specified, a generic random variable, $\tilde{\epsilon}$, follows a normal distribution with mean zero and variance σ^2 .

The payoff of the i -th risky security takes a factor expression:

$$s_i = \bar{s}_i + \sum_{j=1}^K (\beta_{ij} \tilde{\epsilon}_j) + \tilde{\epsilon}_i \quad (1)$$

All $\tilde{\epsilon}_j$'s and $\tilde{\epsilon}_i$'s follow independent normal distributions.

2. An Equilibrium Model of Market Prices

As in Daniel, Hirshleifer, and Subrahmanyam (2001), we use the risky securities to construct portfolios mimicking the K factors and $N-K$ residuals. We refer to these portfolios as the K factor portfolios and $N-K$ residual portfolios.

price volatility). To simplify our analysis, we assume that the variance σ_j^2 is sufficiently small. Specifically, we assume that

$$\sigma_j^2 \leq \frac{1}{2} \min(1, 4\sigma_j^2) \quad (2)$$

This condition facilitates the derivation of the results because it ensures that the traders' penchant for aggressively "buying low and selling high," which drives many of our results, is not too adversely affected by excessive supply noise. The assumption is reasonable because we would not expect uncertainty in stock issuance and buyback activity to be unduly large in general.

We denote the utility from trading the j 'th basic security as u_j . This implies that the

The market clearing condition requires

$$\bar{j} + \tilde{z}j = X_N j(j) + (1 -)X j(j)$$

from which we derive the prices and returns (i.e., price changes) as presented in the following proposition.

Proposition 1 The price and the return of the j 'th basic security are given by

$$j = -\lambda j(j)(\bar{j} + \tilde{z}j)$$

$$\tilde{j} = \tilde{j} - j = \tilde{j} + \lambda j(j)(\bar{j} + \tilde{z}j)$$

where $\lambda j(j) =$ 1

basic security is given by

$$\begin{aligned}
 N_j &= [X_{N_j}(\tilde{j})] = \left[\frac{\lambda_j(\bar{j} + \tilde{z}_j)}{j} (\tilde{j} + \lambda_j(\bar{j} + \tilde{z}_j)) \right] \\
 &= \frac{\lambda_j(\bar{j})^2}{j} (\bar{j} + z_j) \\
 j &= [X_j(\tilde{j})] = \left[\frac{\lambda_j(\bar{j} + \tilde{z}_j)}{(j - \lambda_j/2)} (\tilde{j} + \lambda_j(\bar{j} + \tilde{z}_j)) \right] \\
 &= \frac{\lambda_j(\bar{j})^2}{j - \lambda_j/2} (\bar{j} + z_j) \tag{5}
 \end{aligned}$$

Note that $j > N_j$. Thus, j traders earn a greater expected profit from trading the basic security than do non- j traders. This simply emanates from the notion that, in

σ_y

It follows from Proposition 1 that the price and return volatilities of the i 'th basic security are

Proposition (i) Consider two basic securities, s and s' , with $\beta_j = \beta_{j'}$, $z_j = z_{j'}$, but $\sigma_j > \sigma_{j'}$. Then, $\lambda_j < \lambda_{j'}$.

(ii) The basic security with very large σ_j (i.e., $\sigma_j \rightarrow \infty$) has $\lambda_j \rightarrow 0$.

This proposition suggests that high σ_j can lead to low λ_j and, therefore, attenuate the predictive power of s 's. Particularly, $\lambda_j \rightarrow 0$ for stocks with very large σ_j (i.e., $\sigma_j \rightarrow \infty$). In this extreme case, s 's lose power in explaining stock return completely. The basic intuition is that s -traders, via their tendency to "buy low and sell high" attenuate the pricing of risk (they reduce the equilibrium risk premium

net supply, a high average price denotes a high risk premium on average, and prices on average decrease as they converge to fundamental values. Thus “overpricing” is a natural feature of markets with positive net supply even without α -traders. However, since traders get direct utility from trading, when they absorb risky supplies, they are willing on average to pay more than rational investors for absorbing a given amount of supply, leading to greater “overpricing” than that naturally induced by risk premia.

$$r = d + \alpha \sigma^2 \frac{V}{e}$$

We now examine trading volume within our model. We aim to ascertain how trading volume is influenced by the presence of agents who derive direct utility from trading, and to investigate how volume might be associated with required returns on risky assets.

Let us assume that the initial endowment of the i 'th basic security possessed by each agent equals the per capita mean supply $\bar{s}_i + \tilde{z}_i$. It follows from Eq. (3) and Proposition 1 that the i 'th non- α trader's trade equals

$$X_{N,i} = \frac{r_i - (r - \alpha \sigma^2 \frac{V}{e})}{\sigma^2} (\bar{s}_i + \tilde{z}_i) \quad \left[(\right.$$

Eqs. (9) and (10), we can express the total expected trading volume in the basic security as

$$j \equiv 0.5 \left[|X_N(j) - (\bar{j} + \tilde{z}j)| \right] + 0.5(1 - \alpha) \left[|X(j) - (\bar{j} + \tilde{z}j)| \right] \quad (11)$$

We then have the following result.

Corollary The expected trading volume, j , increases in α .

Corollary 3 indicates that stocks in which agents have a greater level of utility from trading exhibit greater trading volume, which is an intuitive result. Since the α -adjusted expected return is more negative, the greater is j (Proposition 3), our analysis indicates that, ceteris paribus, stocks with high volume (i.e., high j stocks) will earn low average returns on a risk-adjusted basis. This is consistent with the negative relation between trading volume and required returns documented, for example, in Datar, Naik, and Radcliffe (1998) and Brennan, Chordia, and Subrahmanyam (1998).⁸ Based on Merton (1987) who argues that some (possibly, retail) investors might invest only in the most visible stocks, visibility (as measured by analyst following and brand visibility) might be a reasonable proxy for j . Our analysis suggests that such proxies will be associated with high volume and low average returns. In the next subsection, we consider another proxy for j , the volatility of the underlying asset's cash flows.

more attracted to volatile companies.⁹ We thus assume that $\beta_j = \beta_j^2$ where $\beta_j > 0$ (the assumption on β_j is needed to obtain an interior optimum). We show below that under reasonable conditions, our analysis accords with Ang, Hodrick, Xing, and Zhang (2006), who demonstrate a negative cross-sectional relation betwe

Lemma 1 and Proposition 4 imply that for typical basic securities, there is a negative relation (induced by β_j) between IVOL and the β_j -adjusted expected return. This is broadly consistent with Ang, Hodrick, Xing, and Zhang (2006), where stocks with high idiosyncratic volatility earn lower average returns. Proxying for total volatility by β_j , our analysis also accords with Baker and Haugen (2012) who show that low risk stocks outperform high risk stocks in the vast majority of international equity markets.

The above analysis indicates that total volatility is negatively priced in the cross-section. However, in aggregate, risk is positively priced. To see this, note from Proposition 1 that the return of the market portfolio (over the risk free interest rate which is normalized to be zero) is given by

$$\tilde{r}_M = \sum_{j=1}^{+N} (\bar{r}_j + \tilde{z}_j) \tilde{r}_j = \sum_{j=1}^{+N} [(\bar{r}_j + \tilde{z}_j)(\tilde{r}_j + \beta_j(\bar{r}_j - \tilde{r}_j))]$$
 (13)

The following proposition can readily be derived.

Proposition The market risk premium, (\tilde{r}_M) , is positive.

Thus, our model is consistent with the negative pricing of volatility in the cross-section, but a positive pricing of risk in the aggregate (Haugen and Baker (2010), Ang, Hodrick, Xing, and Zhang (2006), and Mehra and Prescott (1985)).

B c o h e O r g n e c r e s

The previous analysis focused on the basic securities for tractability. We now show that our main results carry over to the original securities. We can use Eq. (1) to reconstruct

the return of the original risky asset can be expressed as:

$$\tilde{r}_j = \sum_{i=1}^n \beta_{ij} [\tilde{r}_i + \lambda_i (r_f - r_f)] + \tilde{z}_j$$

Proposition 2 indicates that $\beta_j = \frac{\lambda_j(\beta_j)\text{Var}(\tilde{M})}{\lambda_j + 2\lambda_j(\beta_j)^2}$, the slope of the relation between (\tilde{r}_j) and $r_{M,t}$, decreases in β_j and can be as low as zero.

One can estimate IVOL_j by regressing \tilde{r}_j on factor mimicking portfolios' and the market portfolio's returns. From Eq. (14) we see that after a

Comparing the Economy with No or Partial Presence of Graders

We now compare the equilibria with (i) complete absence of the graders and (ii) presence of the graders in some, but not all, securities. For simplicity, the analysis in this section is focused on the basic securities.

1 Comparing the Economy with No Graders

Consider two economies. In the first economy, all agents are non-traders, while in the second, all are traders. For a variable in the basic economy, we use A and A^N to indicate its counterpart in the all- and all-non- economies.

The first economy, the all-non- economy, is equivalent to the basic economy with $\alpha_j = 1$ and $\beta_j(\alpha_j) = \beta_j$. Using a similar derivation as that for Proposition 1, we can show that the price and return of the j 'th basic security are given by

$$p_j^{A^N} = \frac{1}{R} \frac{dV_j}{dz_j} \quad \text{and} \quad r_j^{A^N} = \frac{dV_j}{V_j} \frac{dz_j}{dz_j}$$

where $\beta_M^{AN} = \frac{\text{Cov}(\tilde{r}_M^{AN}, \tilde{r}_M^{AN})}{\text{Var}(\tilde{r}_M^{AN})}$.

Let $\beta_j^{AN} \equiv \frac{\beta_j \text{Var}(\tilde{r}_M^{AN})}{\beta_j + 2\sigma_j^2}$ denote the slope of the relation between (\tilde{r}_j^{AN}) and \tilde{r}_M^{AN} . An obvious observation is that $\beta_j^{AN} > 0$. Therefore, \tilde{r}_j^{AN} 's still have power to predict stock returns. If $\sigma_j = 0$, then $\beta_j^{AN} = \beta_j$ is identical across all assets. In this case, \tilde{r}_j^{AN} 's are the only predictive variable for expected returns.

The second economy, the all- σ economy, is equivalent to the basic economy with $\sigma = 0$ and $\beta_j(\sigma) = \beta_j - \beta_j \sigma^2$. Using a similar derivation as that for Proposition 1, we can show that the price and return of the j 'th basic security is given by

$$\begin{aligned} \beta_j^A &= -(\beta_j - \beta_j \sigma^2)(\bar{r}_j + \tilde{z}_j) \\ \tilde{r}_j^A &= \bar{r}_j + (\beta_j - \beta_j \sigma^2)(\bar{r}_j + \tilde{z}_j) \end{aligned} \quad (18)$$

The return of the market portfolio is $\tilde{r}_M^A = \sum_{j=1}^{+N} (\bar{r}_j + \tilde{z}_j) \tilde{r}_j^A$.

Similar to Eq. (7), the covariance between the returns of the basic security and the market portfolio is given by

$$\text{Cov}(\tilde{r}_j^A, \tilde{r}_M^A) = \text{Cov}(\tilde{r}_j^A (\bar{r}_j + \tilde{z}_j), \tilde{r}_j^A) = \beta_j \bar{r}_j + 2\sigma_j^2 (\beta_j - \beta_j \sigma^2)^2 \tilde{z}_j$$

Then, the expected return of the basic security can be expressed as:

$$E(\tilde{r}_j^A) = (\beta_j - \beta_j \sigma^2) \bar{r}_j = \frac{(\beta_j - \beta_j \sigma^2) \text{Var}(\tilde{r}_M^A)}{\beta_j + 2\sigma_j^2 (\beta_j - \beta_j \sigma^2)^2} \beta_j^A$$

where $\beta_M^A = \frac{\text{Cov}(\tilde{r}_M^A, \tilde{r}_M^A)}{\text{Var}(\tilde{r}_M^A)}$.

We compare the two economies in the following proposition.

Proposition 1 (i) $E(\tilde{r}_j^{AN}) > E(\tilde{r}_j^A)$ and $\text{Var}(\tilde{r}_j^{AN}) > \text{Var}(\tilde{r}_j^A)$. Thus, the j 'th basic security has higher expected return and volatility in the all- σ economy than in the all- $\sigma=0$ economy.

(ii) $(\tilde{r}_M^{AN}) > (\tilde{r}_M^A)$ and $\text{Var}(\tilde{r}_M^{AN}) > \text{Var}(\tilde{r}_M^A)$. Thus, the market portfolio has higher expected return and volatility in the all-non- economy than in the all- economy.

(iii) $\beta^{AN} > \beta^A$. Thus, 's have more predictive power in the all-non- economy than in the all- economy.

In general, within the all-

The above proposition implies that beta pricing will be less evident in securities that are traded relatively more by μ traders. Again, the notion is simply that μ traders, via their more aggressive trading in securities where they are present, attenuate the pricing of risk. The above proposition indicates cross-sectional variation in risk pricing according to whether μ traders are more or less likely to be present. Thus, if retail investors are more likely to be present in visible, brand name stocks (Frieder and Subrahmanyam (2005)), then covariance risk pricing will be less evident in these stocks.

Graders and heron for on E cency of oc Pr ces

We now consider a model with information asymmetry which allows us to examine how traders or5wdotrrtostibne10.0-83(r)2ctradwd.

endowments of riskfree assets and preferences are unchanged relative to the basic model and the direct utility parameter is denoted by λ (without the subscript to denote the single asset). We modify the basic model by postulating that each trader can observe \tilde{z} by spending a positive and constant cost c . In equilibrium, a mass $(1 - \alpha)$ of traders choose to become informed by paying the cost c ; a mass $(1 - \alpha)(1 - \alpha)$ of traders choose to remain uninformed. $\alpha \in [0, 1]$ is determined in equilibrium. The following proposition describes the pricing function in this setting:

Proposition In equilibrium, the price function takes a linear form

$$p = \lambda - \alpha + (\tilde{z} - \tilde{z}_0) \quad (19)$$

where $(\tilde{z} - \tilde{z}_0) = \tilde{z} - \tilde{z}_0$ (or simply \tilde{z}) has a variance $\sigma^2 = \alpha + \alpha^2 \sigma_z^2$. The parameters λ , α , and \tilde{z}_0 are given by

$$\begin{aligned} \lambda &= \frac{\alpha}{1 + \alpha + \alpha^2} \\ &= \frac{\alpha}{1 + \alpha + \alpha^2} \\ &= \frac{1}{1 + \alpha} \end{aligned}$$

where

$$\begin{aligned} \alpha &\equiv \frac{1}{1 + \alpha} \\ \alpha &\equiv \frac{(1 - \alpha)}{(1 - \alpha)^2} \\ \alpha &\equiv \frac{(1 - \alpha)(1 - \alpha)}{(1 - \alpha) + (1 - \alpha)^2} \end{aligned}$$

Using standard Grossman and Stiglitz (1980)-type arguments, we can derive the following lemma:

Lemma 1 If $\Delta(c) \equiv \exp(2c) \cdot \frac{\text{Var}(\tilde{v}) - \frac{\sigma^2}{2}}{\text{Var}(\tilde{v}) - \frac{\sigma^2}{2}} - 1$ is negative (positive), then the trader prefers to become informed by spending c (remain uninformed). If $\Delta(c) = 0$, then he is indifferent between becoming informed and remaining uninformed.

In the above lemma, $\Delta(c)$ is a function of c because according to Proposition 8, \tilde{v} and therefore $\text{Var}(\tilde{v})$ are functions of c .

Proposition [0 1]

obtained from Proposition 8) and therefore the informativeness of $\sigma = \tilde{\sigma} - \tilde{z}$, $\text{Var}(\sigma | \cdot)$. Second, σ may increase or decrease, the mass of informed traders who choose to become informed, and therefore increase or decrease the informativeness of $\sigma = \tilde{\sigma} - \tilde{z}$, $\text{Var}(\sigma | \cdot)$. The appendix shows that the first effect dominates. Therefore, taken together, $\text{Var}(\sigma | \cdot)$ decreases in σ .

Part (ii) states that $\text{Var}(\sigma | \cdot)$ does not depend on σ . The reason for this is that as Eq. (20) indicates, an increase in σ , the mass of informed non-traders, reduces $(1 - \sigma)$, the mass of informed traders. Thus, $\sigma = \frac{1}{\sigma + \frac{(1 - \sigma)^2}{\sigma}}$ and therefore the informativeness of $\sigma = \tilde{\sigma} - \tilde{z}$, $\text{Var}(\sigma | \cdot)$, remain unchanged.

Overall, we find an increase in utility derived from trading leads to increased price informativeness in equilibrium but the mass of informed traders does not affect this informativeness.

A Dynamic Equilibrium where trading volume depends on past profits

We now consider a dynamic extension of our setting where the utility from trading depends on past profits. Specifically, we model the notion that if an agent earns positive profits,

cause an overreaction to mildly positive information.

We assume that a single risky security is traded at Dates $t = 0, 1, 2,$ and 3 and revert to the case of symmetric information.¹¹

where X_2 is the quantity of risky security he has bought at date 2 and continues to hold until the end of the game, and \bar{X}_2 is a positive constant.

The i 'th trader is endowed with \bar{X}_i units of risky securities. For convenience, we let $\tilde{\sigma}_i$'s, $i = 1, 2$, and 3 , have the same variance σ^2 . Let the price and return of the risk free asset be 1. We then have the following result:

Proposition 1 There is an equilibrium characterized by the following prices:

- P_0 is given by

$$P_0 = \bar{P}_0 + \bar{X}_0 - 2 \bar{X}_1$$

where $\lambda(\theta) = \frac{1}{1 - \frac{\theta}{\sigma^2}}$. The variable θ is uniquely determined by

$$0 = \int_{\theta}^{\infty} \frac{\tilde{X}_1 - \lambda(\theta) \bar{X}_1}{\exp(-\tilde{X}_1)} \cdot \left[\exp(-0.5 \frac{(\lambda(\theta))^2}{\sigma^2}) + (1 - \lambda(\theta)) \exp(-0.5 \frac{(\lambda(\theta))^2}{\sigma^2}) \right] d\left(\frac{\tilde{X}_1}{\sigma^2}\right) + \int_{-\infty}^{\theta} \frac{\tilde{X}_1 - \lambda(\theta) \bar{X}_1}{\exp\left[-\tilde{X}_1 - \lambda(\theta) \bar{X}_1\right]} \exp(-0.5 \frac{(\lambda(\theta))^2}{\sigma^2}) d\left(\frac{\tilde{X}_1}{\sigma^2}\right)$$

and $\Phi(\cdot)$ is the cumulative density function of standard normal distribution.

- If $\tilde{X}_1 > \bar{X}_1$, then

$$P_1 = \bar{P}_0 + \tilde{X}_1 - \lambda(\theta) \bar{X}_1$$

$$P_2 = \bar{P}_0 + \tilde{X}_1 + \tilde{X}_2 - \lambda(\theta) \bar{X}_1$$

Because $P_1 > P_0$, a mass $1 - \lambda(\theta)$ of traders convert to \tilde{X}_1 traders immediately following Date 1.

- If $\tilde{e}_1 > \bar{e}_1$, then

$$p_1 = \bar{p} + \tilde{e}_1 - 2\bar{e}_1$$

$$p_2 = \bar{p} + \tilde{e}_1 + \tilde{e}_2 - \bar{e}_2$$

Because $p_1 > 0$, all traders remain non-traders throughout the timeline.

Here is a sketch of the proof of this proposition (the formal proof is in the Appendix). We use backward induction. There are three steps. In the first step, we study the equilibrium demands and prices at Dates 1 and 2 conditional on the event that at Date 1, traders make money because $\tilde{e}_1 > \bar{e}_1$ so $p_1 > 0$ (call this Regime 1). Note that in this regime, at Date 2, there is a mass $(1 - \alpha)$ of non-traders (α traders). At Date 1, an agent knows that he will be a non-trader (α trader) with probability $(1 - \alpha)$. In the second step, we study the equilibrium demands and prices at dates 1 and 2 conditional on the event that at Date 1, traders do not make money because $\tilde{e}_1 < \bar{e}_1$ so $p_1 = 0$ (call this Regime 2). This step is simpler than the first step because all traders are non-traders. In the third step, we focus on Date 0, and derive the expressions for p_0 and the threshold \bar{e}_1 .

There are two interesting results. The first relates to the price reaction for \tilde{e}_1 around the threshold \bar{e}_1 :

$$p_1(\tilde{e}_1 > \bar{e}_1) = \bar{p} + \tilde{e}_1 - 2\bar{e}_1 - \alpha(\bar{e}_2 - \tilde{e}_2)$$

$$p_1(\tilde{e}_1 < \bar{e}_1) = \bar{p} + \tilde{e}_1 - 2\bar{e}_1$$

It is easy to show that

$$p_1(\tilde{e}_1 > \bar{e}_1) - p_1(\tilde{e}_1 < \bar{e}_1) = \alpha(\bar{e}_2 - \tilde{e}_2) > 0$$

because $\alpha(\bar{e}_2 - \tilde{e}_2) > 0$. This suggests a small \tilde{e}_1 (e.g., earnings) can induce a significant price movement. Another interpretation of this observation is that a relatively minor piece

The second result relates to long-run performance. If $\tilde{\pi}_1 > \pi^*$, then the subsequent returns are

$$r_{2-t} - r_{1-t} = \tilde{\pi}_2 + \pi^* \quad \text{and} \quad r_{-2} - r_{-1} = \tilde{\pi}_3 + \lambda(\pi^*)^2$$

If $\tilde{\pi}_1 < \pi^*$, then the subsequent returns are

$$r_{2-t} - r_{1-t} = \tilde{\pi}_2 + \pi^* \quad \text{and} \quad r_{-2} - r_{-1} = \tilde{\pi}_3 + \lambda(\pi^*)^2$$

A comparison between these two cases suggests that if $\tilde{\pi}_1 < \pi^*$, then there is a long-run underperformance because $\lambda(\pi^*)^2 < 0$. Thus, a minor piece of good news can cause securities to become dramatically overpriced and thus exhibit subpar returns in the long run.

Figure 1 plots the price paths conditional on the public announcement $\tilde{\pi}_1$. We assume the parameter values $\lambda = 5$, $\alpha = 1$, $\beta = 1$, $\gamma = 0.5$, $\delta = 0.5$, and $\theta = 0.2$. The realizations of $\tilde{\pi}_2$ and $\tilde{\pi}_3$ are assumed to be zero, i.e., their mean. This implies that the threshold $\pi^* = -0.388$. Moving from the bottom to the top, each path in the figure represents a realization of $\tilde{\pi}_1$ from -1 to 0 (step size=0.025). $\tilde{\pi}_1 < \pi^*$ for the paths indicated by 's. $\tilde{\pi}_1 > \pi^*$ for the paths indicated by *'s. We see that if $\tilde{\pi}_1$ is below the threshold $\pi^* = -0.388$, the price reaction to $\tilde{\pi}_1$ is non-positive. Once $\tilde{\pi}_1$ has surpassed the threshold $\pi^* = -0.388$, the price reaction becomes positive.

Particularly, look at the two paths bordering the hollow area. The south path is for $\tilde{\pi}_1 = -0.4$. The north path is for $\tilde{\pi}_1 = -0.375$. Although $\tilde{\pi}_1$ differs by only 0.025 across the two path groups, the price reactions are very different. On both paths, $r_0 = 3.612$. However, on the south path, $r_1 = 3.6$ so $r_1 - r_0 = -0.012$; on the north path, $r_1 = 3.9583$ so $r_1 - r_0 = 0.3463$. The difference in the price reaction, $r_1 - r_0$, equals 0.3583, which is more than fourteen times the difference in $\tilde{\pi}_1$ (0.025).

The immediate return subsequent to the release of $\tilde{\pi}_1$, $r_2 - r_1 = 0.5$, is identical

across all $\tilde{\omega}_1$ paths. But the long-run performance for the paths with $\tilde{\omega}_1$ indicated by ω_1^* 's, $\omega_1 - \omega_2 = 0.1667$, is lower than that for the paths with $\tilde{\omega}_1$ indicated by ω_1 's, $\omega_1 - \omega_2 = 0.5$. This indicates long-run underperformance following a good public announcement. The underperformance is characteristic of bubble-like episodes in the stock market (such as the technology bubble of the 1990s, viz. Brunnermeier and Nagel (2004)), whereas the positive event (that creates the bubble) could be something as simple as good initial sales or earnings figures for the relevant sector.

More generally, the preceding analysis suggests a testable implication. Specifically, for stocks that are popular amongst retail investors, we predict a nonlinear response to positive news, that is a small reaction to modest news announcement, but a disproportionately larger reaction to major (positive) announcements. Following the large positive announcements, these stocks should exhibit long-run reversals (conditional on the news).

Conclusion

In this paper, we present a model where agents derive direct utility from trading. We show that the presence of such agents causes assets to be overpriced, attenuates beta pricing and volatility, and raises trading volume in financial markets. Assets with high trading volume earn lower expected returns. Assuming that agents derive greater utility from trading more volatile stocks, our model accords with a set of intriguing empirical findings: Volatility is priced negatively in the cross-section, but positively in the aggregate (viz. Haugen and Baker (2010)). Agents with greater utility from trading exploit private information more aggressively, thus raising pricing efficiency. Further, the presence of agents who receive direct utility from trading causes “bubbles,” i.e., overreactions in asset prices if agents’ utility from trading depends on past profit outcomes in financial markets.

Untested implications of our analysis are that stocks that are popular amongst retail investors should exhibit weaker evidence of covariance risk pricing and lower volatility, with greater trading activity. These stocks should also exhibit disproportionate price reactions to moderately positive news announcements. The analysis, under reasonable additional assumptions, also accords with a variety of documented stylized facts: the negative relation between average returns and volume as well as idiosyncratic (or total) volatility (Datar, Naik, and Radcliffe (1998), Ang, Hodrick, Xing, and Zhang (2006), Baker and Haugen (2012)), the lack of evidence consistent with covariance risk pricing (Fama and French (1992)), the pricing of covariance risk conditional on sentiment (Antoniu, Doukas, and Subrahmanyam (2016)), and the rise of volatility in conjunction with the rise in institutional holdings (Campbell, Lettau, Malkiel, and Xu (2001) and Malkiel and Xu (1999)).

Our work raises many issues. First, it would be interesting to examine a fully dynamic model with exits and entry by such agents. Second, it may be interesting to

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Appendix A

Proof of Corollary 1. From Proposition 1, $\lambda_j(\beta) = \frac{1}{\frac{1}{z_j} + \frac{1}{j - \beta^2}}$. It is easy to show after taking derivatives that $\lambda_j(\beta)$ decreases in β , and increases in z_j . Finally, $\lambda_j(0) = \frac{1}{\frac{1}{z_j} + \frac{1}{j}} = \frac{z_j}{j + z_j}$.

Q.E.D.

Proof of Proposition 1(i) From Corollary 1, $\lambda_j(\beta) > \lambda_{j'}(\beta)$ because $z_j > z_{j'}$. Note that $\lambda_j(\beta) = \frac{\lambda_j(\beta) \text{Var}(\tilde{M}_j)}{z_j + 2\lambda_j(\beta)^2 z_j}$. For $\lambda_j(\beta) > \lambda_{j'}(\beta)$, it suffices that

$$\begin{aligned} & \frac{\lambda_j(\beta)}{z_j + 2\lambda_j(\beta)^2 z_j} > \frac{\lambda_{j'}(\beta)}{z_{j'} + 2\lambda_{j'}(\beta)^2 z_{j'}} \\ & \lambda_j(\beta) [z_j + 2\lambda_j(\beta)^2 z_j] > \lambda_{j'}(\beta) [z_{j'} + 2\lambda_{j'}(\beta)^2 z_{j'}] \\ & = [\lambda_j(\beta) - \lambda_{j'}(\beta)] [z_j - 2\lambda_j(\beta)\lambda_{j'}(\beta) z_j] \\ & \quad + 2\lambda_j(\beta)\lambda_{j'}(\beta) z_j - \lambda_{j'}(\beta) z_{j'} \\ & \quad + 2\lambda_{j'}(\beta)^2 z_{j'} - \lambda_j(\beta) z_j \\ & \quad > 0 \end{aligned}$$

where the second " $>$ " follows from $\lambda_j(\beta) > \lambda_{j'}(\beta)$, the first inequality follows from $\lambda_j(\beta) > \lambda_{j'}(\beta)$ (see Corollary 1), and the last inequality obtains under the assumption $z_j > \frac{1}{2} \min(1, 4 - 2)$ (see Condition (2)).

(ii) If $\beta^2 > z_j$, then $\lambda_j(\beta) = \frac{1}{\frac{1}{z_j} + \frac{1}{j - \beta^2}} = 0$ so that $\lambda_j(\beta) = 0$.

Q.E.D.

Proof of Proposition 1(ii) From Corollary 1, $\lambda_j(\beta) = \frac{z_j}{j + z_j}$. Therefore, the β -adjusted expected return of the j th basic security $= \left[\frac{z_j}{j + 2\lambda_j(\beta)^2 z_j} - \lambda_j(\beta) \right] \bar{r}_j > 0$.

(ii) From Corollary 1, $\lambda_j(j) \leq \lambda_j(j')$ because $j > j'$. The difference between the j -adjusted expected returns of the j 'th and j' 'th basic securities is

$$\begin{aligned}
 & - \left[\lambda_j(j) + 2 \lambda_j(j)^2 z_j - \lambda_j(j) \right] \bar{j} + \left[\lambda_j(j') + 2 \lambda_j(j')^2 z_j - \lambda_j(j') \right] \bar{j} \\
 = & \left[\lambda_j(j) - \lambda_j(j') - 2 (\lambda_j(j)^2 - \lambda_j(j')^2) z_j \right] \bar{j} \\
 & 2 (\lambda_j(j) + \lambda_j(j')) z_j - 1 \\
 & 4 \lambda_j(j) z_j - 1 \\
 & 0
 \end{aligned}$$

where the " $>$ " follows from $\lambda_j(j) \leq \lambda_j(j')$, the first inequality follows from $\lambda_j(j) \leq \lambda_j(j')$ (see Corollary 1), and the last inequality obtains under the assumption $z_j \geq \frac{1}{2} \min(1, 4 - 2)$ (see Condition (2)).

Q.E.D

Proof of Corollary 1. Write Eqs. (9) and (10) as

$$\begin{aligned}
 X_N(j) &= (\bar{j} + \tilde{z}j) & (A_N \bar{j} A_N^2 z_j) \\
 X(j) &= (\bar{j} + \tilde{z}j) & (A \bar{j} A^2 z_j)
 \end{aligned}$$

where

$$A_N \equiv \frac{\lambda_j(j)}{j} - 1 \quad \text{and} \quad A \equiv \frac{\lambda_j(j)}{j - \frac{j}{2}} - 1$$

Here are some intermediate results we will use in the proof of this corollary. First, $A_N \geq 0$ and decreases in j from Corollary 1. Second, $A = \frac{\lambda_j(j)}{j - \frac{j}{2}} - 1 =$

of $X_N(j) - (\bar{j} + \tilde{z}j)$ and $X(j) - (\bar{j} + \tilde{z}j)$,¹³ we can express the total expected trading volume in the j 'th basic security (Eq. (11)) as

$$\begin{aligned} j &= 0.5 \left[|X_N(j) - (\bar{j} + \tilde{z}j)| \right] + 0.5(1 - \alpha) \left[|X(j) - (\bar{j} + \tilde{z}j)| \right] \\ &= 0.5 \left(-A_N \frac{\bar{j}}{z_j} \right) \left[2 \left(-\frac{\bar{j}}{z_j} \right) - \frac{\bar{j}}{z_j} (1 - 2 \left(-\frac{\bar{j}}{z_j} \right)) \right] \\ &\quad + 0.5(1 - \alpha) A \frac{\bar{j}}{z_j} \left[2 \left(-\frac{\bar{j}}{z_j} \right) + \frac{\bar{j}}{z_j} (1 - 2 \left(-\frac{\bar{j}}{z_j} \right)) \right] \end{aligned}$$

Footnote 13 indicates that the values in the brackets are positive. From the above analysis, A_N decreases in j , and A increases in j . Therefore, j increases in j .

Q.E.D.

Proof of Lemma 1. From Proposition 1, $\frac{\text{Var}(\tilde{j})}{\text{Var}(\tilde{M})} = \frac{j + \alpha j(j)^2 z_j}{\text{Var}(\tilde{M})}$. This implies that typical basic securities with small $\frac{\text{Var}(\tilde{j})}{\text{Var}(\tilde{M})}$ also have small $\frac{j}{\text{Var}(\tilde{M})}$ and $\frac{\alpha j(j)^2 z_j}{\text{Var}(\tilde{M})}$. We will use this property in the proof of this Lemma.

From Eq. (12), Proposition 1, and our computation of $\text{Cov}(\tilde{j}, \tilde{M})$ in Appendix B,

$$\begin{aligned} \text{IVOL}_j &= \text{Var}(\tilde{j}) - \frac{\text{Cov}(\tilde{j}, \tilde{M})^2}{\text{Var}(\tilde{M})} \\ &= \frac{j + \alpha j(j)^2 z_j}{\text{Var}(\tilde{M})} - \frac{(j + 2\alpha j(j)^2 z_j)^2}{\text{Var}(\tilde{M})} \end{aligned}$$

Denote $(j) = j$. It follows that for the j 'th and j' 'th typical basic securities,

$$\begin{aligned} \text{IVOL}_j - \text{IVOL}_{j'} &= \left[\frac{j + \alpha j(j)^2 z_j}{\text{Var}(\tilde{M})} \right] - \left[\frac{j' + \alpha j'(j')^2 z_j}{\text{Var}(\tilde{M})} \right] \\ &\quad - \frac{j + 2\alpha j(j)^2 z_j + j' + 2\alpha j'(j')^2 z_j}{\text{Var}(\tilde{M})} \\ &= \left[\frac{j + 2\alpha j(j)^2 z_j}{\text{Var}(\tilde{M})} \right] - \left[\frac{j' + 2\alpha j'(j')^2 z_j}{\text{Var}(\tilde{M})} \right] \end{aligned}$$

¹³ If $y \sim N(\bar{y})$,

$$\begin{aligned}
&= \left[\frac{1}{z_j} + 2 \frac{z_j}{z_j} \left(\frac{z_j}{z_j} \right)^2 \right] - \left[\frac{1}{z_{j'}} + 2 \frac{z_{j'}}{z_{j'}} \left(\frac{z_{j'}}{z_{j'}} \right)^2 \right] \\
&\quad - \delta \left[\left[\frac{1}{z_j} + 2 \frac{z_j}{z_j} \left(\frac{z_j}{z_j} \right)^2 \right] - \left[\frac{1}{z_{j'}} + 2 \frac{z_{j'}}{z_{j'}} \left(\frac{z_{j'}}{z_{j'}} \right)^2 \right] \right] \\
&= (1 - \delta) \left(\frac{1}{z_j} - \frac{1}{z_{j'}} \right) + (1 - 2\delta) \frac{z_j}{z_j} \left[\left(\frac{z_j}{z_j} \right)^2 - \left(\frac{z_{j'}}{z_{j'}} \right)^2 \right]
\end{aligned}$$

where $\delta = \frac{\frac{1}{z_j} + 2 \frac{z_j}{z_j} \left(\frac{z_j}{z_j} \right)^2 + \frac{1}{z_{j'}} + 2 \frac{z_{j'}}{z_{j'}} \left(\frac{z_{j'}}{z_{j'}} \right)^2}{\text{Var}(\tilde{M})}$. It follows from the above derived property that for the j 'th and j' 'th typical basic securities, δ must be small. Thus, $1 - \delta \approx 1 - 2\delta > 0$. Note that $\frac{z_j}{z_j} > \frac{z_{j'}}{z_{j'}}$. For $\text{IVOL}^j > \text{IVOL}^{j'}$, it suffices that $\frac{z_j}{z_j} > \frac{z_{j'}}{z_{j'}}$, which holds because

$$\frac{d \left(\frac{z_j}{z_j} \right)}{d z_j} = \frac{z_j}{z_j^2} > \frac{z_{j'}}{z_{j'}^2}$$

Proof of Proposition 1. It follows from Eq. (13) that

$$\begin{aligned}
 (\tilde{M}) &= \sum_{j=1}^{+N} \left[\tilde{j} \tilde{j} + \tilde{z} \tilde{j} + \sqrt{j(j+1)} (\tilde{f} + 2 \tilde{j} \tilde{z} + \tilde{z}^2) \right] \\
 &= \sum_{j=1}^{+N} \left[\right]
 \end{aligned}$$

It follows immediately that

$$\left(\tilde{M}^A \right) = \sum_{j=1}^{+N} \left[\left(\bar{j} + \tilde{z}_j \right) \left(\tilde{j} + \left(j - j^2 \right) \left(\bar{j} + \tilde{z}_j \right) \right) \right] = \sum_{j=1}^{+N} \left[\left(j - j^2 \right) \left(\bar{j} + \tilde{z}_j \right) \right]$$

$$\text{Var} \left(\tilde{M}^A \right) = \sum_{j=1}^{+N} \left[\left(j + 4 \left(j - j^2 \right)^2 \right) \tilde{z}_j^2 + j \tilde{z}_j + 2 \left(j - j^2 \right)^2 \tilde{z}_j \right]$$

where the last equality follows from Eq. (6) because the all- economy is the case in

which, $\left(j \right) = j - j^2$.

A direct comparison indicates that $\left(\tilde{M}^{AN} \right) = \left(\tilde{M}^A \right)$ and $\text{Var} \left(\tilde{M}^{AN} \right) = \text{Var} \left(\tilde{M}^A \right)$

$$2 \leq j(z_j) -$$

$$2 \leq z_j -$$

$$0$$

where the second " \leq " and the first inequality follow from $j(z_j) \leq z_j$ (see Corollary 1), and the last inequality obtains under the assumption $z_j \geq \frac{1}{2} \min(1, 4 - 2)$ (see Condition (2)).

Q.E.D.

Proof of Proposition 1. Note that

$$| \tilde{z}_j | \leq z_j$$

Conjecture that the stock price takes the linear form given in Eq. (19). An i 'th trader, who chooses to remain uninformed, can infer \tilde{v}_i from the stock price. Note that

$$| \tilde{v}_i = (\tilde{v} + \tilde{z}_i - (1 - \alpha) \tilde{v}) + \tilde{z}_i$$

The uninformed trader has the following expected utility conditional on \tilde{v}_i :

$$E[u_i(\tilde{v}_i)] = -\exp\left[-\left[\tilde{v}_i \delta + X_i(\tilde{v}_i - \tilde{v}) - 0.5 X_i^2(\text{Var}(\tilde{v}_i) - \sigma^2)\right]\right] \quad (24)$$

The f.o.c. w.r.t. X_i implies that his demand can be expressed as:

$$X_i(\tilde{v}_i) = \frac{(\tilde{v}_i - \tilde{v})}{(\text{Var}(\tilde{v}_i) - \sigma^2)} = \frac{\tilde{v}_i + \tilde{z}_i - \tilde{v}}{(\alpha(1 - \alpha) + \sigma^2)} \quad (25)$$

The market clearing condition requires that

$$\tilde{v} + \tilde{z} = \alpha \cdot X_N(\tilde{v}) + (1 - \alpha) \cdot X(\tilde{v}) + (1 - \alpha)(1 - \alpha) \cdot X(\tilde{v})$$

Proof of Lemma 1. Consider an informed trader's expected utility, given by Eq. (22).

Plugging in the optimal demand for the stock in Eq. (23) yields

$$\begin{aligned}
 E[u(\tilde{v}|\tilde{y})] &= -\exp[-(\delta - c)] \exp\left[-0.5 \frac{[(\tilde{v}|\tilde{y}) - \tilde{v}]^2}{\text{Var}(\tilde{v}|\tilde{y})}\right] \\
 &= -\exp[-(\delta - c)] \exp\left[-0.5 \frac{(\tilde{v} + \tilde{y} - \tilde{v})^2}{\text{Var}(\tilde{v}|\tilde{y})}\right] \\
 &= -\exp[-(\delta - c)] \exp\left[-0.5 \frac{\text{Var}(\tilde{v}|\tilde{y})}{\text{Var}(\tilde{v}|\tilde{y})} Y^2\right]
 \end{aligned}$$

where $Y \equiv \frac{\tilde{y} - \tilde{v}}{\sqrt{\text{Var}(\tilde{v}|\tilde{y})}}$ and $Y| \tilde{v} \equiv \frac{(\tilde{v} + \tilde{y}|\tilde{v}) - \tilde{v}}{\sqrt{\text{Var}(\tilde{v}|\tilde{y})}}$. Thus,

$$\begin{aligned}
 E[u(\tilde{v}|\tilde{y})] &= E\left[E[u(\tilde{v}|\tilde{y})|\tilde{v}]\right] \\
 &= -\exp[-(\delta - c)] \left[\exp\left[-0.5 \frac{\text{Var}(\tilde{v}|\tilde{y})}{\text{Var}(\tilde{v}|\tilde{y})} Y^2\right] \right] \\
 &= -\frac{\exp[-(\delta - c)]}{\sqrt{1 + \frac{\text{Var}(\tilde{v}|\tilde{y})}{\text{Var}(\tilde{v}|\tilde{y})}}} \exp\left[-0.5 \frac{\text{Var}(\tilde{v}|\tilde{y})}{\text{Var}(\tilde{v}|\tilde{y})} \left[\frac{\tilde{y} + \tilde{v} - \tilde{v}}{\sqrt{\text{Var}(\tilde{v}|\tilde{y})}}\right]^2\right]
 \end{aligned}$$

It follows immediately that

$$\begin{aligned} & \left[\frac{\partial \ln(\cdot)}{\partial \ln(\cdot)} \right] - \left[\frac{\partial \ln(\cdot)}{\partial \ln(\cdot)} \right] \\ &= \left[\exp(\rho) \sqrt{\frac{\text{Var}(\tilde{\cdot}) - \frac{\rho^2}{2}}{\text{Var}(\cdot) - \frac{\rho^2}{2}} - 1} \right] \left[\frac{\partial \ln(\cdot)}{\partial \ln(\cdot)} \right] \end{aligned}$$

Taking the ex ante expectation yields

$$\left[\frac{\partial \ln(\cdot)}{\partial \ln(\cdot)} \right] - \left[\frac{\partial \ln(\cdot)}{\partial \ln(\cdot)} \right] = \left[\exp(\rho) \sqrt{\frac{\text{Var}(\tilde{\cdot}) - \frac{\rho^2}{2}}{\text{Var}(\cdot) - \frac{\rho^2}{2}} - 1} \right] \left[\frac{\partial \ln(\cdot)}{\partial \ln(\cdot)} \right]$$

$\text{Var}(w_1)$ decreases in α .

(ii) Consider Eq. (20) again. It is obvious that $\text{Var}(w_1)$ depends on variables such as c_1 , α , β , and $\text{Var}(w_2)$, which do not involve α .

Q.E.D.

Proof of Proposition 1. We solve for the equilibrium and prove the proposition using backward induction, in three steps.

Step 1: In this step, suppose $w_1 = 0$ and therefore $w_1 = 0$ (which we will show below). An agent remains a non-trader (becomes a trader) with probability $(1 - \alpha)$. Thus, there is a mass α of non-traders and a mass $1 - \alpha$ of traders at Date 2.

Focus on Date 2 for the moment. Write an i 'th non-trader's wealth at Date 3 as $w_3 = w_2 + X_2(w_1 - w_2)$. His expected utility at Date 2 can be expressed as:

$$E[u_N(w_3) | w_1, w_2] = -\exp\left[-\frac{w_2}{\sigma^2} - \left[X_2(w_1 - w_2) - 0.5 X_2^2 \text{Var}(w_1 - w_2)\right]\right] \quad (26)$$

He needs to choose X_2 to maximize this expected utility. The f.o.c. implies that his demand can be expressed as:

$$X_{N,2}(w_1 - w_2) = \frac{(w_1 - w_2) - \sigma^2}{\text{Var}(w_1 - w_2)} = \frac{w_1 - w_2 - \sigma^2}{\sigma^2} \quad (27)$$

An i 'th trader's expected utility at Date 2 can be expressed as:

$$E[u_T(w_3 + X_2(w_1 - w_2)) | w_1, w_2] = -\exp\left[-\frac{w_2}{\sigma^2} - \left[X_2(w_1 - w_2) - 0.5 X_2^2 \text{Var}(w_1 - w_2) + 0.5 X_2^2 \sigma^2\right]\right] \quad (28)$$

He needs to choose X_2 to maximize this expected utility. The f.o.c. implies that his demand can be expressed as:

$$X_{T,2}(w_1 - w_2) = \frac{(w_1 - w_2) - \sigma^2}{\text{Var}(w_1 - w_2) - \sigma^2} = \frac{w_1 - w_2 - \sigma^2}{\sigma^2(1 - \alpha)} \quad (29)$$

It follows from Eqs. (27) and (29) that the market clearing condition requires

$$\begin{aligned} \bar{X} &= \theta \cdot X_{N-2}(\tilde{x}_1, \tilde{x}_2) + (1 - \theta) \cdot X_2(\tilde{x}_1, \tilde{x}_2) \\ &= \end{aligned}$$

He needs to choose X_1 to maximize this expected utility. The f.o.c. implies that his demand can be expressed as:

$$X_1(\tilde{p}_1) = \frac{(E_2[\tilde{p}_1] - p_1)}{\text{Var}(E_2[\tilde{p}_1])} = \frac{\bar{p}_1 + \tilde{p}_1 - \sigma(\tilde{p}_1) - p_1}{\text{Var}(E_2[\tilde{p}_1])}$$

The market clearing condition, $X_1(\tilde{p}_1) = \bar{x}_1$, implies

$$\bar{x}_1 = \frac{\bar{p}_1 + \tilde{p}_1 - \sigma(\tilde{p}_1) - p_1}{\text{Var}(E_2[\tilde{p}_1])} \quad (33)$$

Plugging the derived $X_1(\tilde{p}_1)$ and \bar{x}_1

characterized by Eqs. (33) and (34). If $\tilde{\pi}_1$ so $\pi_1 > 0$ and he does not make money, he will remain a non-trader for sure. He will be in the regime characterized by Eqs. (35), and (36). Accounting for both cases, we can write his expected utility at date 0 as

It is straightforward to show that the right hand side of this equation decreases in θ , and is positive (negative) if $\theta < \theta^*$ ($\theta > \theta^*$). Therefore, θ^* is uniquely determined by this equation.

Q.E.D.

Appendix B

Construction of the Portfolio Return by the Definition
Section

From Proposition 1, the returns of the

Computation of the Covariance between the Basic Securities and Market Portfolio returns From the Definition

From the expressions of \tilde{r}_j and \tilde{r}_M ,

$$\begin{aligned} \text{Cov}(\tilde{r}_j, \tilde{r}_M) &= \text{Cov}\left(\tilde{r}_j, \sum_{j=1}^{+N} (\tilde{r}_j + \tilde{z}_j) \tilde{r}_j\right) \\ &= \text{Cov}(\tilde{r}_j, (\tilde{r}_j + \tilde{z}_j) \tilde{r}_j) = \text{Cov}(\tilde{r}_j, \tilde{r}_j \tilde{r}_j) + \text{Cov}(\tilde{r}_j, \tilde{z}_j \tilde{r}_j) \end{aligned}$$

where the second equality obtains because basic securities have independent random supplies and payoffs.

$$\begin{aligned} \text{Cov}(\tilde{r}_j, \tilde{r}_j \tilde{r}_j) &= \tilde{r}_j \text{Var}(\tilde{r}_j) = \tilde{r}_j \left(\tilde{r}_j + \sum_j \tilde{r}_j + \sum_j \tilde{r}_j^2 \right) \tilde{r}_j \\ \text{Cov}(\tilde{r}_j, \tilde{z}_j \tilde{r}_j) &= (\tilde{z}_j \tilde{r}_j) - (\tilde{r}_j) (\tilde{z}_j \tilde{r}_j) \\ &= \left[\tilde{z}_j (\tilde{r}_j + \sum_j \tilde{r}_j) (\tilde{r}_j + \tilde{z}_j) \right]^2 \\ &\quad - \left[\tilde{r}_j + \sum_j \tilde{r}_j (\tilde{r}_j + \tilde{z}_j) \right] \left[\tilde{z}_j (\tilde{r}_j + \sum_j \tilde{r}_j) (\tilde{r}_j + \tilde{z}_j) \right] \\ &= 0 \end{aligned}$$

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