



*“Needed: A Theory of Total Factor Productivity”*

# 1. Introduction

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# 1. Introduction, *continued*

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## 2. Index Number Approach

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!  $y_t, p_t$

!  $x_{K,t}, w_{K,t}$

!  $x_{L,t}, w_{L,t}$

## **2. Index Number Approach, *continued***

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## 2. Index Number Approach, *continued*

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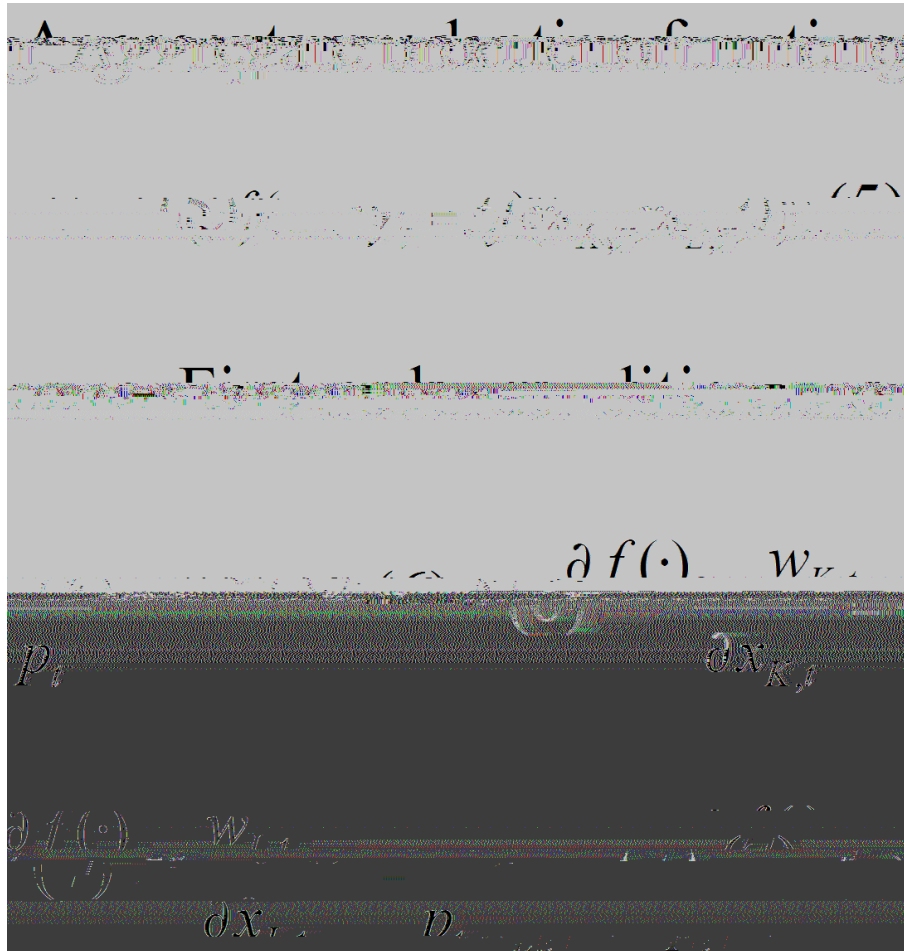
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### 3. Production function approach

- ⚠ TFP can also be defined with reference to a production function
- ⚠ This actually leads to four interpretations of TFP



### 3. Production function approach, *continued*



### 3. Production function approach, *continued*

Let  $\mu_t = \ln y_t / t$  be the instantaneous rate of technological change; we then have:

$$(8) \quad \frac{\partial f(\cdot)}{\partial t} = \mu_t y_t$$

Following Diewert and Morrison (1986), we define the following index of TFP:

$$(10) \quad TFP_{t,t_0} = \frac{f(x_{t,t_0}, 1)}{f(x_{t_0,t_0}, 1)}$$

### **3. Production function approach,**

### 3. Production function approach, *continued*

$$\frac{\partial \ln f(\cdot)}{\partial t} = \frac{\partial \ln f(\cdot)}{\partial \ln Y} \frac{\partial \ln Y}{\partial t} = \beta + \theta (\ln Y - \ln Y^*)$$

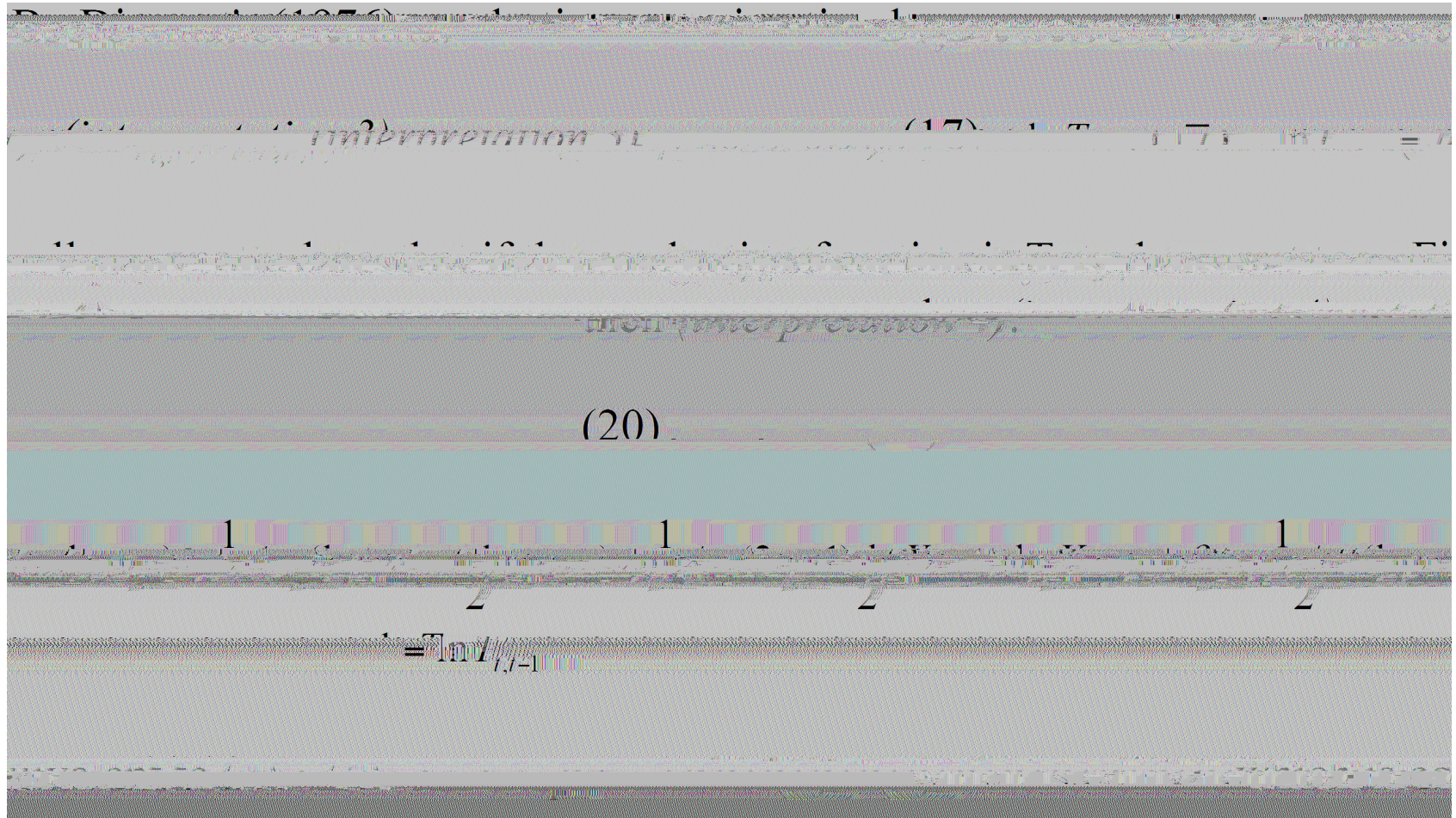
$$\frac{\partial \ln f(\cdot)}{\partial t} = \frac{1}{2} \frac{\partial \ln f(\cdot)}{\partial \ln Y} \frac{\partial \ln Y}{\partial t} + \frac{1}{2} \frac{\partial \ln f(\cdot)}{\partial \ln Y} \frac{\partial \ln Y}{\partial t}$$

$$\frac{\partial \ln f(\cdot)}{\partial t} = \frac{1}{2} \frac{\partial \ln f(\cdot)}{\partial \ln Y} \frac{\partial \ln Y}{\partial t} + \frac{1}{2} \frac{\partial \ln f(\cdot)}{\partial \ln Y} \frac{\partial \ln Y}{\partial t}$$

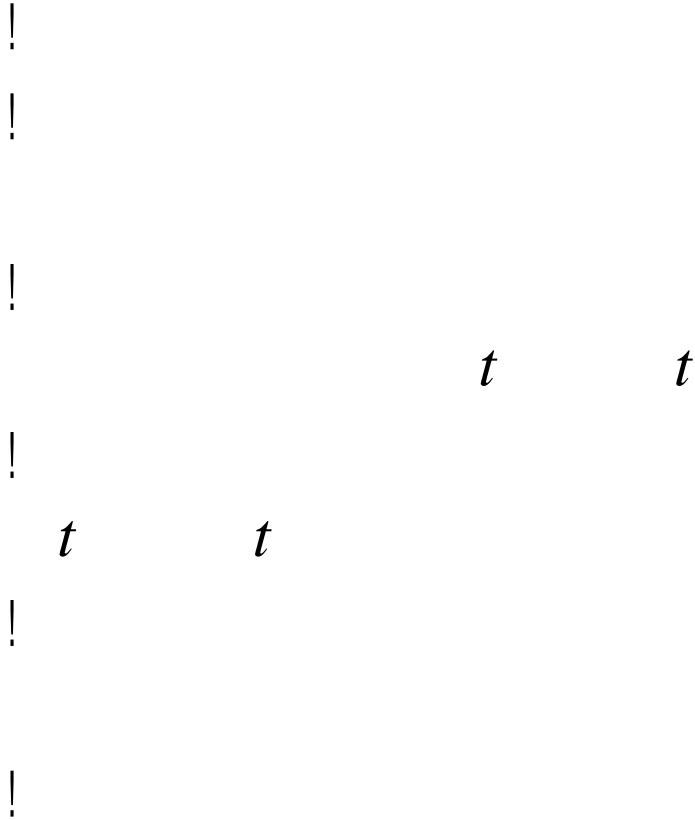
$$\frac{\partial \ln f(\cdot)}{\partial t} = \frac{1}{2} \frac{\partial \ln f(\cdot)}{\partial \ln Y} \frac{\partial \ln Y}{\partial t} + \frac{1}{2} \frac{\partial \ln f(\cdot)}{\partial \ln Y} \frac{\partial \ln Y}{\partial t}$$



### 3. Production function approach, *continued*



### 3. Production function approach, *continued*



### 3. Production function approach, *continued*

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imates Parameter

Parameter	Estimate
$\beta_1$	0.0000
$\beta_2$	0.0000
$\beta_3$	0.0000
$\beta_4$	0.0000
$\beta_5$	0.0000
$\beta_6$	0.0000
$\beta_7$	0.0000
$\beta_8$	0.0000
$\beta_9$	0.0000
$\beta_{10}$	0.0000
$\beta_{11}$	0.0000
$\beta_{12}$	0.0000
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$\beta_{98}$	0.0000
$\beta_{99}$	0.0000
$\beta_{100}$	0.0000



## 4. Impact of TFP on factor rental prices

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$$\phi_{KT}$$

## 4. Impact of TFP on factor rental prices, *continued*

!  $\phi_{KT}$

## 4. Impact of TFP on factor rental prices, *continued*"



#### 4. Impact of TFP on factor rental prices, *continued*"

$$\frac{\hat{\phi}_{KT}}{S_{L,t}} \quad (23) \quad \hat{w}_{L,t} = \mu_t$$

where the hat ( $\hat{\cdot}$ ) indicates a relative change

#### 4. Impact of TFP on factor rental prices, continued

- ⌘ As long as the technology is progressing, the first term on the right hand side is positive
- ⌘ If  $\dot{\theta}_{KT}$  is positive, technological change is anti-labor biased
- ⌘ It might even be that  $\dot{\theta}_{KT}/s_{L,t} > \mu_t$ , in which case technological change would be ultra anti-labor biased: technological change would then lead to an actual fall in the wage rate  $\dot{w}$
- ⌘  $\dot{w}$  even though technological progress would unambiguously increase average labor productivity

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## 4. Impact of TFP on factor rental prices,continued

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## **5. Disembodied factor augmenting technological change, *continued***



## 5. Disembodied factor augmenting technological change, *continued*"

The result could be obtained in a similar manner with only a few modifications to the model. In particular, we assume that the production function is given by (22)

$$Y = A K^\alpha L^{1-\alpha} \quad (22)$$

where  $A$  is a technology parameter that grows at rate  $g$ . For the rate of technological change ( $g = d \ln A / dt$ ) we get:

$$g = \frac{1}{\alpha} \left( \frac{dY}{dt} \right) - \frac{1}{1-\alpha} \left( \frac{dL}{dt} \right) - \frac{1}{\alpha} \left( \frac{dK}{dt} \right) \quad (23)$$

For the rate of technological change ( $g = d \ln A / dt$ ) we get:

$$g = \frac{1}{\alpha} \left( \frac{dY}{dt} \right) - \frac{1}{1-\alpha} \left( \frac{dL}{dt} \right) - \frac{1}{\alpha} \left( \frac{dK}{dt} \right) \quad (24)$$





## 5. Disembodied factor augmenting technological change, *continued*<sup>11</sup>

Monotonic increasing  $\phi(u)$  can not be a simple representation of (11):

$$\frac{1}{\sum_{t=0}^{\infty} \beta^t} \left[ \frac{1}{\sum_{t=0}^{\infty} \beta^t} \left( \frac{1}{\sum_{t=0}^{\infty} \beta^t} \right) \right]$$

then (1) is again valid. Furthermore, if the true production function is given by (11) then (1) is again valid. Indeed, one can show that:

$$\frac{1}{\sum_{t=0}^{\infty} \beta^t} \left[ \frac{1}{\sum_{t=0}^{\infty} \beta^t} \left( \frac{1}{\sum_{t=0}^{\infty} \beta^t} \right) \right]$$

$$+ \frac{1}{\sum_{t=0}^{\infty} \beta^t} \phi(u_T - u_t) [(\ln x_{T,t} - \ln x_{t,T}) + (\ln x_{T,t} - \ln x_{t,T})]$$

(41)

$$= \ln T$$

## **5. Disembodied factor augmenting technological change,**



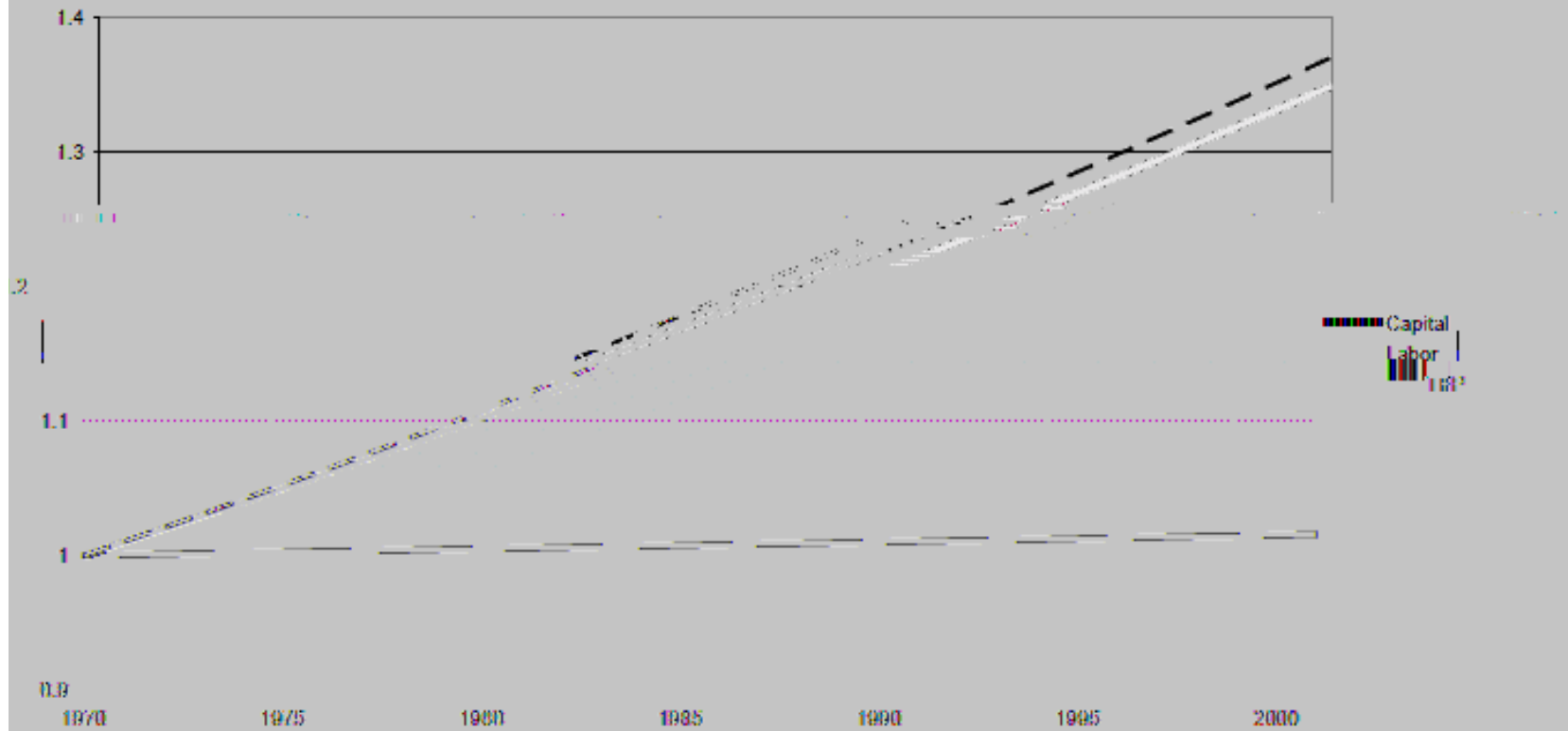
## 6. The decomposition of TFP between labor and capital





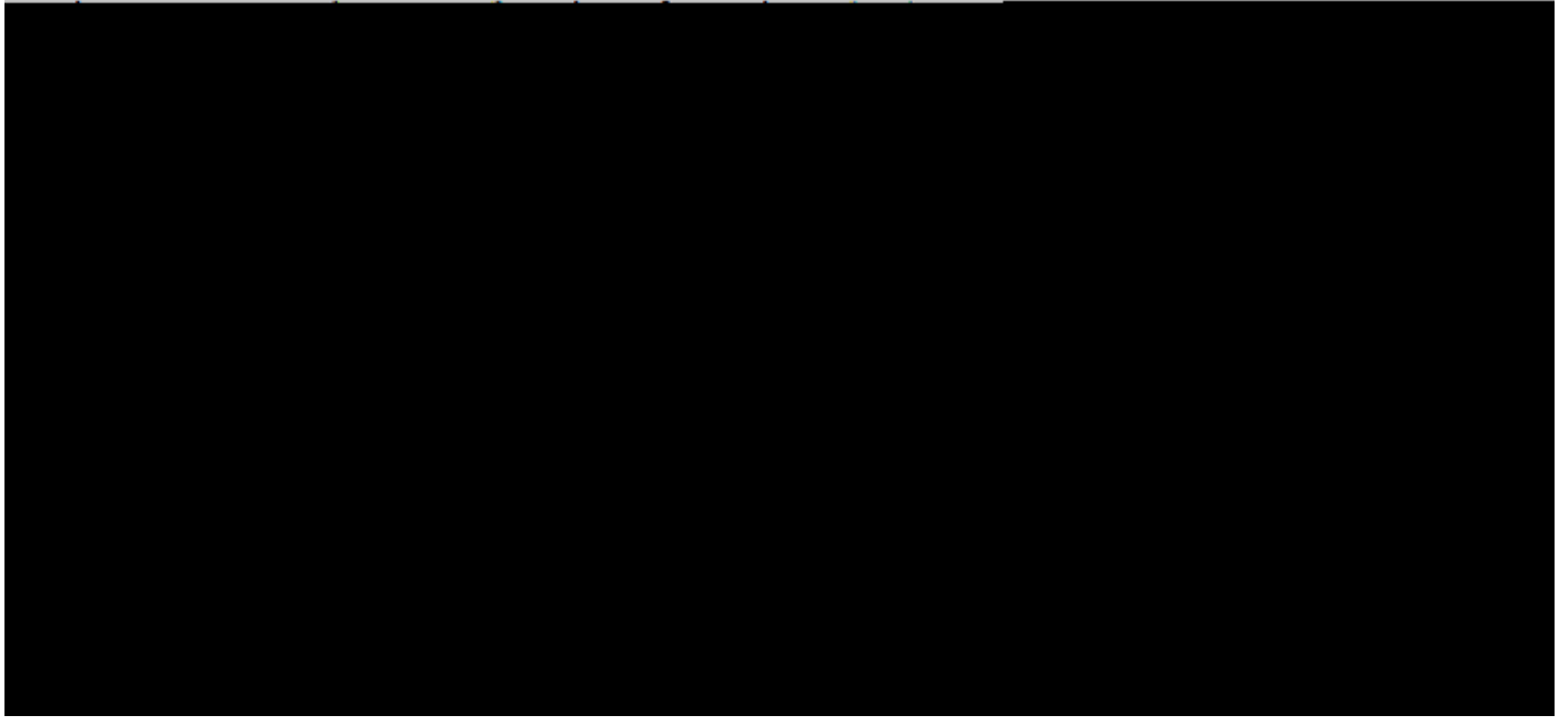
Figure 1

Decomposition of TFP  
(factor augmenting technological change)





## 7. Factor augmenting technological change and TP flexibility



## 7. Factor augmenting technological change and TP flexibility, *continued*

Thus:

$$(49) \quad \tilde{x}_{K,t} \equiv x_{K,t} e^{\mu_K t + \frac{1}{2} \lambda_K t^2}$$

The instantaneous rates of factor augmentation ( $\tau_{K,t}$ ) are now functions of time:  $\tau_{K,t} = \mu_K + \lambda_K t$  and  $\tau_{L,t}$  are now functions of time:  $\tau_{L,t} = \mu_L + \lambda_L t$

$$\tau_{K,t} = \frac{\partial \ln \tilde{x}_{K,t}(x_{K,t}, t)}{\partial t} = \mu_K + \lambda_K t \quad (51)$$

## 7. Factor augmenting technological change and TP flexibility, *continued*

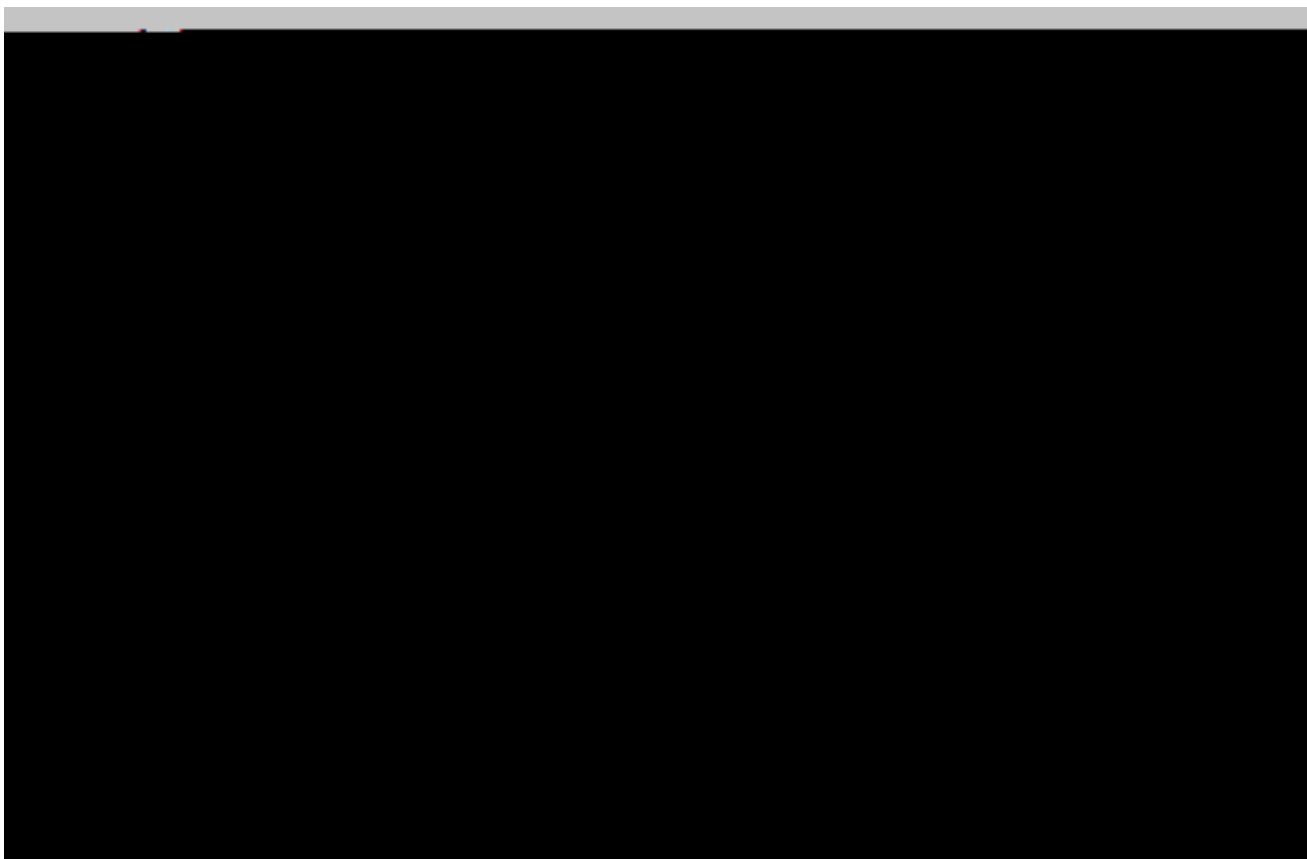
Introducing (49) and (50) into (30) we get:

$$\ln y_t = \alpha_0 + \beta_K \ln x_{K,t} + (1 - \beta_K) \ln x_{L,t} + \beta_K u_{K,t} + (1 - \beta_K) u_{L,t}$$

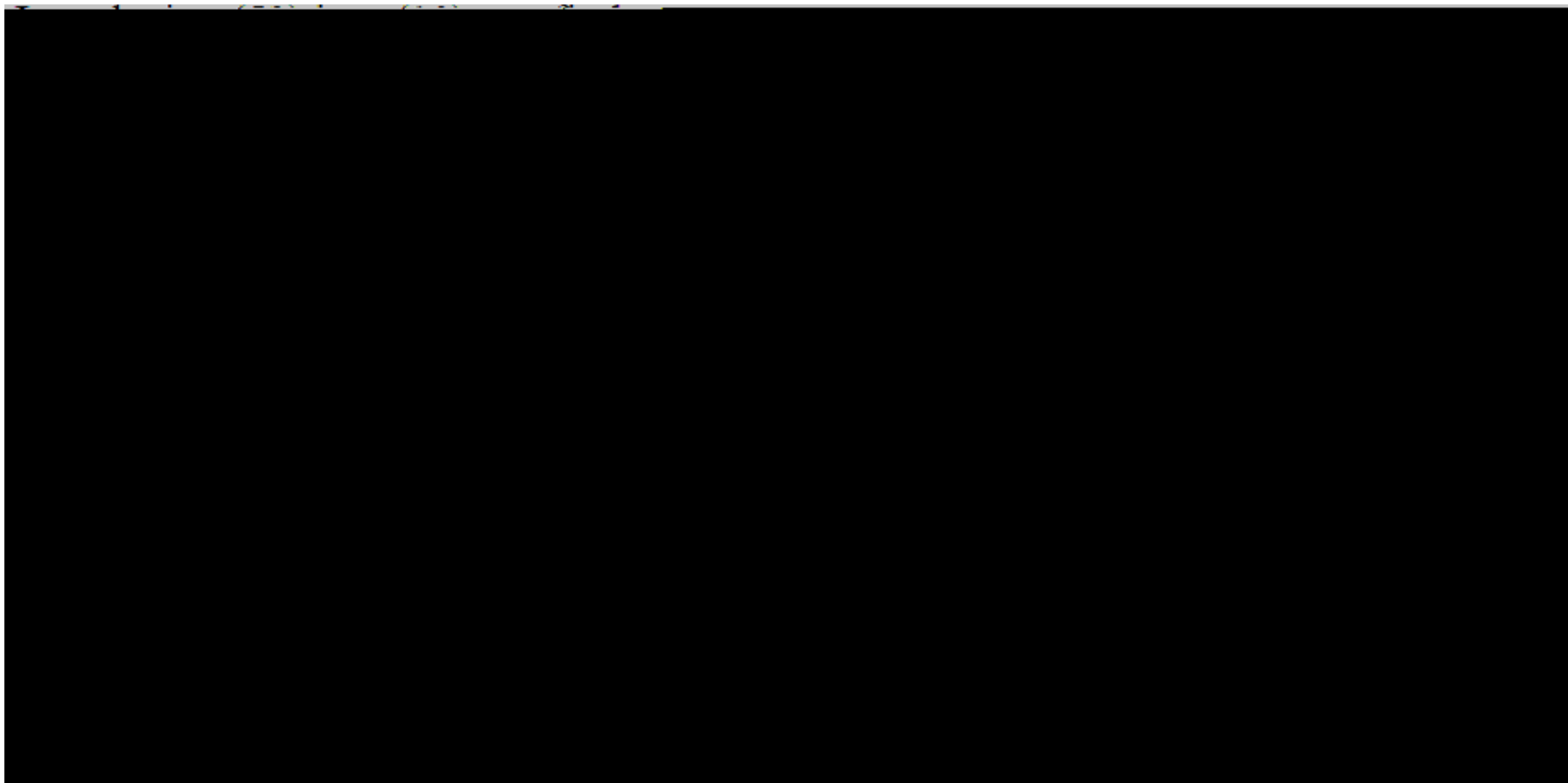
$$-\frac{\phi_{KK}}{\delta} (\lambda_K - \lambda_L) t$$



## 7. Factor augmenting technological change and TP flexibility, *continued*



## 7. Factor augmenting technological change and TP flexibility, *continued*



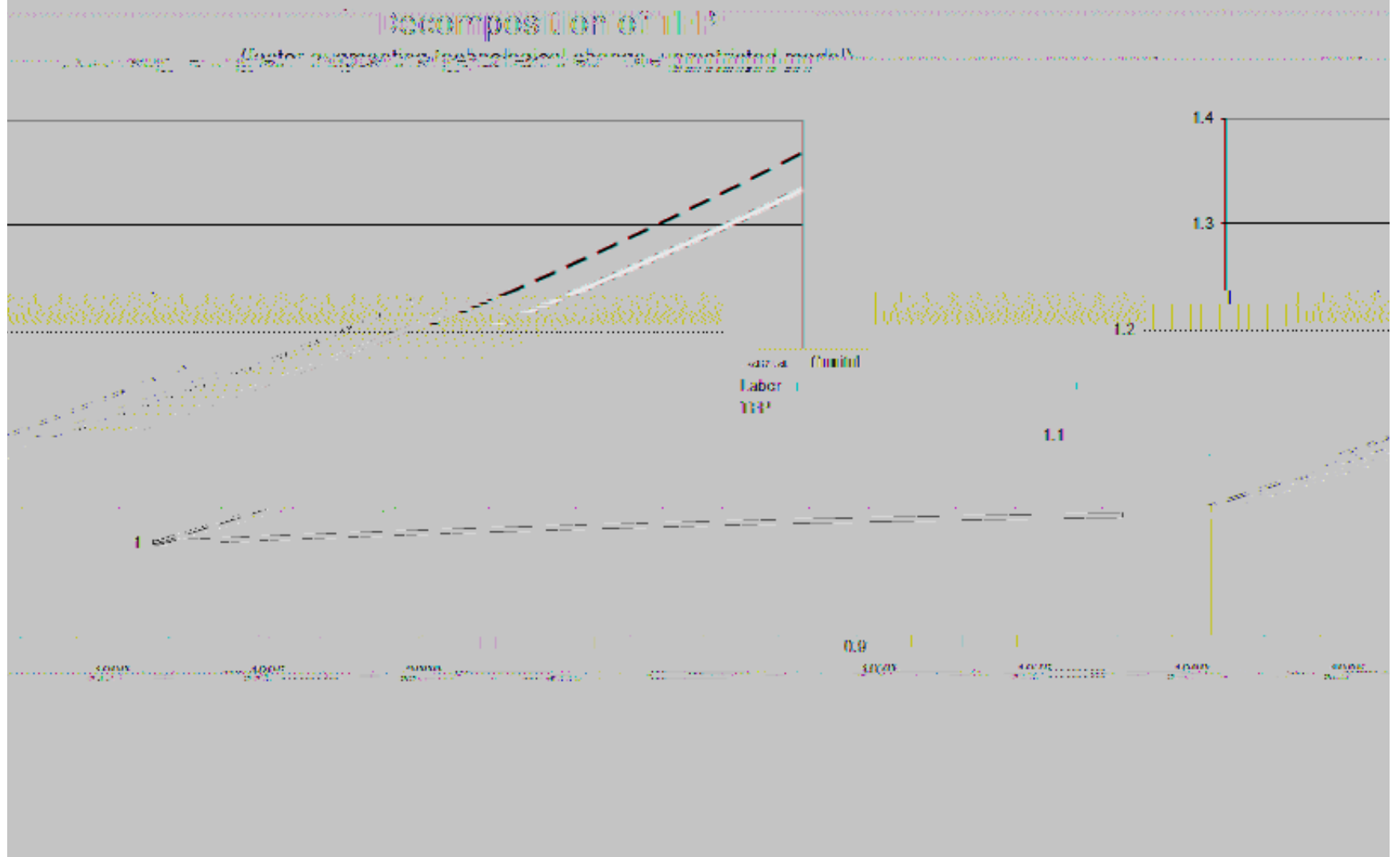
## 7. Factor augmenting technological change and TP flexibility, *continued*

$$\begin{aligned}
 & \phi_{LL}(\ln y_{t+1} + \ln y_{t+2} - \ln y_t - \ln y_{t-1}) \lambda_{LL} (2t-1) \\
 & + \frac{1}{2} \phi_{KK}(\mu_K - \mu_L) \mu_K (2t-1) \\
 & + \frac{1}{2} \phi_{LL}(\mu_L - \mu_K) \lambda_{LL} (4t^2 - 4t + 1) + \frac{1}{2} \phi_{LL}(\lambda_{LL} - \lambda_{LL}) \mu_L (2t^2 - 2t + 1) \\
 & + \frac{1}{2} \phi_{LL}(\lambda_{LL} - \lambda_{LL}) \lambda_{LL} (4t^3 - 6t^2 + 4t - 1)
 \end{aligned}$$





Figure 7



# 8. A parsimonious and yet flexible model

case the production function becomes

$$+ \frac{1}{2} \phi_{\mu} (\mu_{\mu} - \mu_{\mu})^2 \sigma^2 + \frac{1}{2} \lambda \sigma^2$$

is identical to (52) and (53) and the restriction is set of ... whereas (54) and (55) be

$$\mu_{\mu} + (1 - \beta_{\mu}) \mu_{\mu} + \phi_{\mu} (\ln v_{\mu} - \ln v_{\mu}) (\mu_{\mu} - \mu_{\mu}) + [\phi_{\mu} (\mu_{\mu} - \mu_{\mu})^2 + \lambda] \sigma^2 \quad (63) \quad \mu_{\mu} = \beta_{\mu} \mu_{\mu}$$

## 8. A parsimonious and yet flexible model, *continued!*

It turns out that the model of equation (62) is equivalent to (11) since there is a one-to-one correspondence between the two sets of parameters:

$$(64) \quad \beta_T = \beta_K \mu_K + (\lambda + \beta_K) \mu_L$$

$$(65) \quad \phi_{KT} = \phi_{KK} (\mu_K - \mu_L)$$

$$(66) \quad \phi_{TT} = \phi_{TT} (\mu_K - \mu_L)^2 + \lambda$$

and expressed in terms of parameters of (11) the coefficient of equation (62) is

$$(67) \quad \phi_{TT} = \phi_{TT} (\mu_K - \mu_L)^2 + \lambda$$

$$(68) \quad \mu_L = \beta_T - \beta_K \frac{\phi_{KT}}{\phi_{KK}}$$

$$(69) \quad \lambda = \phi_{TT} - \frac{\phi_{KT}^2}{\phi_{KK}}$$

## 8. A parsimonious and yet flexible model, *continued*

For TFP we now get:

$$\ln T_{t,t-1} = \beta_K \mu_K + (1 - \beta_K) \mu_L + \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) (\ln x_{K,t} - \ln x_{K,t-1}) \quad (70)$$

TFP is numerically identical to (15)

This esti

$$\ln T_{t,t-1} = (1 - \beta_K) \mu_L + \frac{1}{2} (1 - \beta_K) \phi_{KK} (2t - 1) \mu_K$$

$$+ \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) \mu_L (2t - 1) + \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) \mu_K (2t - 1) + \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) \mu_L (2t - 1) + \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) \mu_K (2t - 1)$$

$$\mu_L \lambda (3t^2 - 3t + 1)$$

$$+ \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) \mu_L (2t - 1) + \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) \mu_K (2t - 1)$$

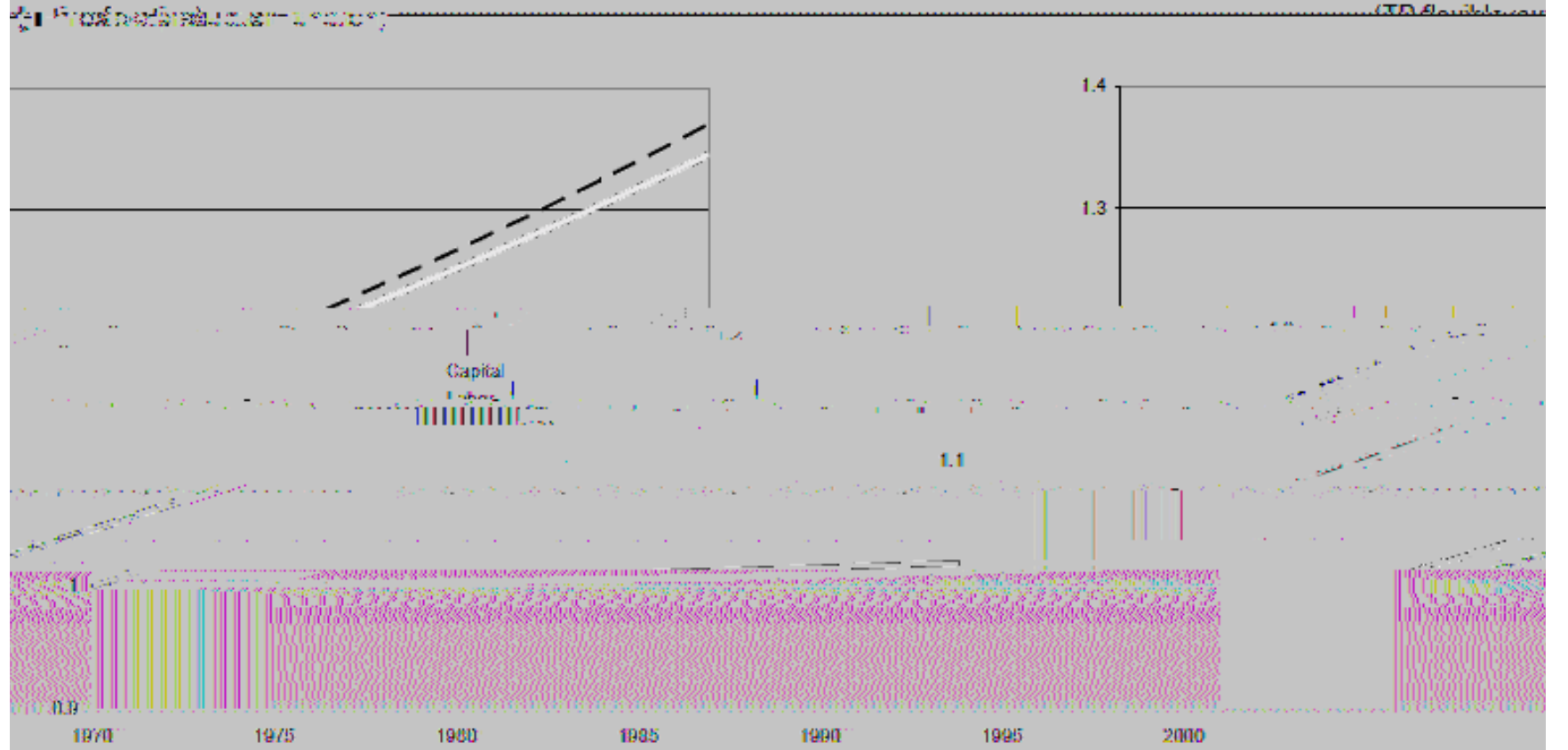




Figure 3

# Decomposition of TFP

# Decomposition



**9. The impact of technological change on factor rental prices reexamined, *continued*"**

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## 9. The impact of technological change on factor rental prices reexamined, *continued*"

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## 9. The impact of technological change on factor rental prices reexamined, *continued*

Let  $\varepsilon_{ij}$  be the inverse price elasticities of factor demands:

$$(73) \quad \varepsilon_{ij} \equiv \frac{\partial \ln \tilde{w}_i(\tilde{x}_K, \tilde{x}_L, p)}{\partial \ln \tilde{x}_j}, \quad i, j \in \{K, L\}$$

$$(74) \quad \varepsilon_{iK} + \varepsilon_{iL} = 0, \quad i \in \{K, L\}$$

It is well known that:

$$(75) \quad \varepsilon_{KL} = \psi_{KL} S_L$$

$$(76) \quad \varepsilon_{LK} = \psi_{KL} S_K$$



## 9. The impact of technological change on factor rental prices reexamined, *continued*"

We thus get for the total change in the rental price of an efficiency unit of capital:

$$\frac{(\hat{R}_K)}{R_K} = \hat{w}_K = \hat{w}_K - \beta_K \hat{K} = \left( \frac{\partial \ln Y}{\partial \ln K} \right) \hat{K} - \beta_K \hat{K}$$

and similarly for labor:

**9. The impact of technological change on factor rental prices reexamined, *continued*"**

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**9. The impact of technological change on factor rental prices reexamined, *continued*"**

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## **10. Generalization to an arbitrary number of inputs, *continued!***



# 10. Generalization to an arbitrary number of inputs, *continued*

We thus find, from (85) and (86), that the eigenvalues  $\lambda$  of  $\mathbf{A}$  are given by

$$(85) \quad \lambda = \sum_{k=1}^{J-1} \beta_k \left( \sum_{l=1}^{J-1} \beta_l \right) \quad (85)$$

$$\mathbf{A}^{-1} \mathbf{A} = \mathbf{I} \Rightarrow \sum_{k=1}^{J-1} \beta_k \left( \sum_{l=1}^{J-1} \beta_l \right) \mathbf{A}^{-1} \mathbf{A} = \mathbf{I} \Rightarrow \sum_{k=1}^{J-1} \beta_k \left( \sum_{l=1}^{J-1} \beta_l \right) \mathbf{A}^{-1} \mathbf{A} = \mathbf{I} \quad (86)$$

$$(87) \quad \lambda + \sum_{k=1}^{J-1} \sum_{l=1}^{J-1} \phi_k (u_k - u_l)(u_l - u_k) = \phi$$

It is not clear from (86) how to proceed. One might try to use the  $(J-1) \times (J-1)$  matrix of the  $\beta_k$ 's, but this is not a square matrix. It is, however, clear that all the  $\beta_k$ 's are bounded by  $\phi$  from (85) and (86).

from (85) and (86)



# 11. Conclusions

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## 11. Conclusions, continued

- ✖ We have shown that in the case of a TP-flexible Translog production function TFP can always be interpreted as the outcome of disembodied, factor augmenting technological change
- ✖ Indeed, we have proposed a convenient way to derive the factor-augmenting rates of technological change from the estimates of such a Translog production function
- ✖ We have found that technological change is almost neutral in the case of the United States, so that TFP is overwhelmingly explained by labor

## 11. Conclusions, *continued*

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***Thank you for your attention!***

## Growth factors 1970-2001

Quantity of capital services:	" #	2.25706
! " # \$ % & ' ( ) * ( + # , ) - ( . / - 0 & 1 / . 2 !	" \$	1.70513
3 - & 1 / ( ) * ( ) " % 4 " % 2 !	%	3.76623
! " # \$ % & ' ( ) * ( ) " % 4 " % 2 !	&	2.52563
3 - & 1 / ( ) * ( + # , ) - ( . / - 0 & 1 / . 2 (	' \$	5.50201
5) % # + ( * # 1 % ) - ( 4 - ) 6 " 1 % & 0 & % ' 2 !	(	1.37071
Capital component of TFP:	$T_K$	1.01850
Labor component of TFP:	$T_L$	1.34581
Capital efficiency:	! #	1.06789
Labor efficiency:	! \$	1.50832
7 # , ) - ( . 8 # - / 2 !	) \$	0.98628
Output per unit of labor:	! ! ! ! ! $L$	1.48119