“Needed: A Theory of Total Factor Productivity”

1. Introduction

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1. Introduction, *continued*

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2. Index Number Approach

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- ! y_t , p_t
- ! $x_{K,t}$, $w_{K,t}$
- ! $x_{L,t}$, $w_{L,t}$

2. Index Number Approach, *continued*

A state

2. Index Number Approach, *continued*

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3. Production function approach

- TFP can also be defined with reference to a production function
- This actually leads to for four interpretations of TFP

3. Production function approach, *continued*

3. Production function approach, *continued*

Let $\mu_t \; ! \quad \ln y_t / \; t$ be the instantaneous rate of technological change; we then have:

(8) $$\frac{\partial f(\cdot)}{\partial t} = \mu_t y_t$$

Following Diewert and Morrison (1986), we define the following index of TFP:

(10) $$_{, \,!1} \qquad \frac{(\quad _{, \,!1}, \quad _{, \,!1}, \;)}{(\quad _{, \,!1}, \quad _{, \,!1}, \; ! \, 1)} \qquad (\quad _{,} , \quad _{,} ,$$

3. Production function approach,

3. Production function approach, *continued*

3. Production function approach, *continued*

(20)

$= \ln I_{t,t-1}$

3. Production function approach, *continued*

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t t

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t t

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3. Production function approach, *continued*

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Table 1

imates Parameter

4. Impact of TFP on factor rental prices

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! ϕ_{KT}

4. Impact of TFP on factor rental prices, *continued*

! ϕ_{KT}

4. Impact of TFP on factor rental prices, *continued*"

4. Impact of TFP on factor rental prices, *continued*"

$$\frac{\phi_{KT}}{s_{L,t}}$$

$$(23) \qquad \hat{w}_{L,t} = \mu_t$$

where the hat (^) indicates a relative ch

4. Impact of TFP on factor rental prices, continued!

¥! As long as the technology is progressing, the first term on the right hand side is positive

¥! If $!_{KT}$ is positive, technological change is anti-labor biased

¥! It might even be that $!_{KT}/s_{L,t} > \mu_t$, in which case technological change would be ultra anti-labor biased: technological change would then lead to an actual fall in the wage rateÉ

¥! É even though technological progress would unambiguously increase average labor productivity

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4. Impact of TFP on factor rental prices,continued

¥!

5. Disembodied factor augmenting technological change

5. Disembodied factor augmenting technological change, *continued*

5. Disembodied factor augmenting technological change, *continued*"

5. Disembodied factor augmenting technological change, *continued*

…, then (1) is again valid. Furthermore, if the true production function is given by (…

Indeed, one can show that:

$$+\frac{1}{2}\phi_{xx}(u_x - u_t)[(\ln x_{\cdot} - \ln x_{\cdot}) + (\ln x_{\cdot} - \ln x_{\cdot})] \qquad (41)$$

$$= \ln T$$

5. Disembodied factor augmenting technological change,

Table 1

Parameter estimates

	(11)	*(31)*
α_0	8.38851	8.39259
	(3522.9)	(4874.6)
[illegible]	[illegible]	[illegible]
LL	226.70	224.09
[illegible]	0.01158	0.01008
[illegible]	[illegible]	[illegible]

6. The decomposition of TFP between labor and capital

6. The decomposition of TFP between labor and capital, *continued*

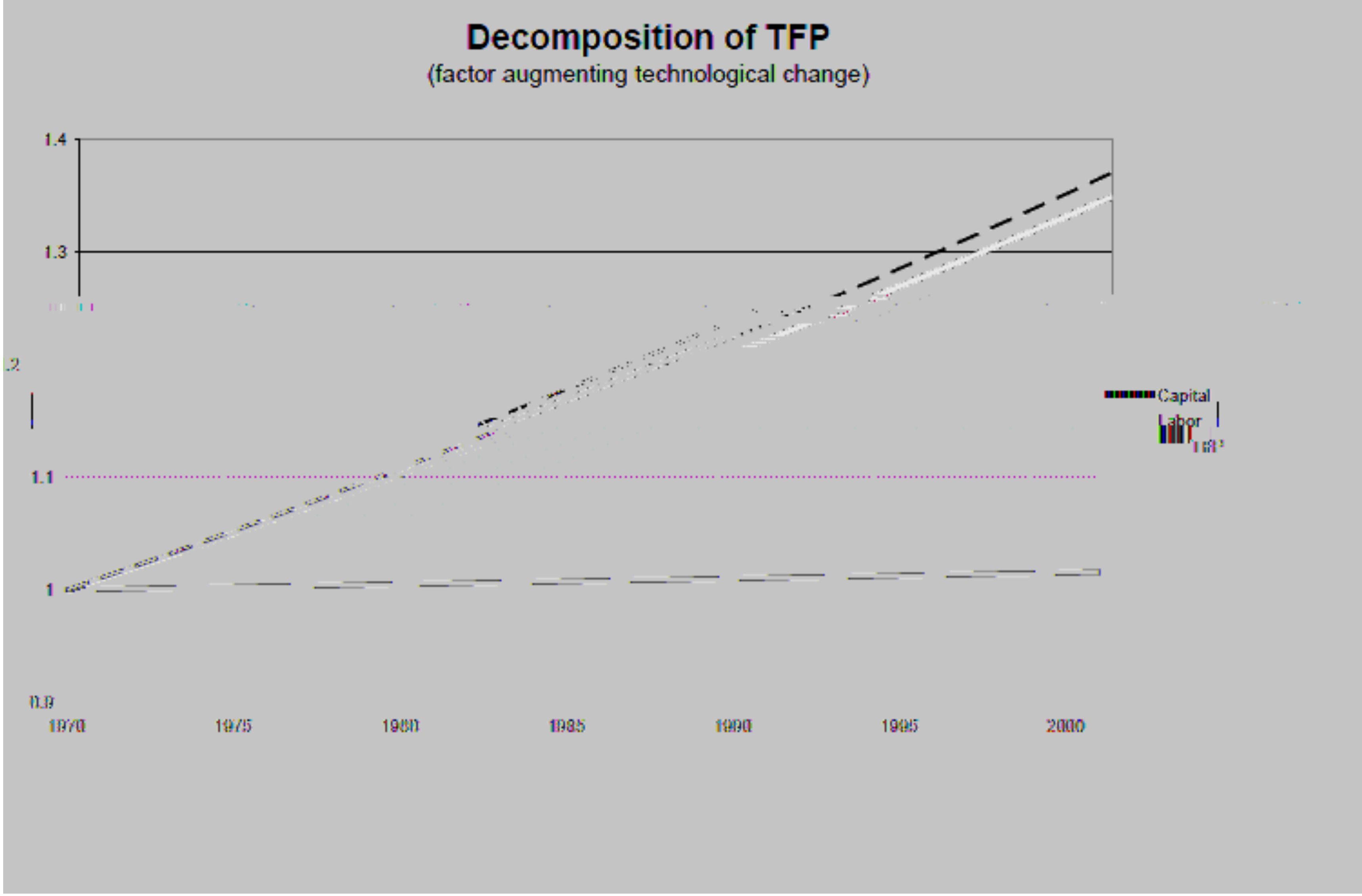
$$= \frac{1}{2}(s_{K,t} + s_{K,t-1})\mu_K$$

$$= \frac{1}{2}(s_{L,t} + s_{L,t-1})\mu_L$$

so that, in view of (38):

$$= T^K_{t,t-1} T^L_{t,t-1} \quad (46)$$

Figure 1


Decomposition of TFP
(factor augmenting technological change)
1.4
1.3
1.1
1
0.9
1970
1975
1980
1985
1990
1995
2000
Capital
Labor

7. Factor augmenting technological change and TP flexibility

Thus:

$$(49) \qquad \widetilde{x}_{K,t} \equiv x_{K,t}\, e^{\mu_K t + \frac{1}{2}\lambda_K t^2}$$

…tion (τ… The instantaneous rates of factor augment… $_{K,t}$

…unctions of time: … and τ…) are now f…

$$\tau_{K,t} = \frac{\partial \ln x_{K,t}(x_{K,t}, t)}{\partial t} = \mu_K + \lambda_K t \qquad (51)$$

Introducing (49) and (50) into (30), we get:

$$\ln y_t = \alpha_0 + \beta_K \ln x_{K,t} + (1-\beta_K)\ln x_{L,t} + \beta_K \mu_K t + (1-\beta_K)\mu_L t$$

[illegible]

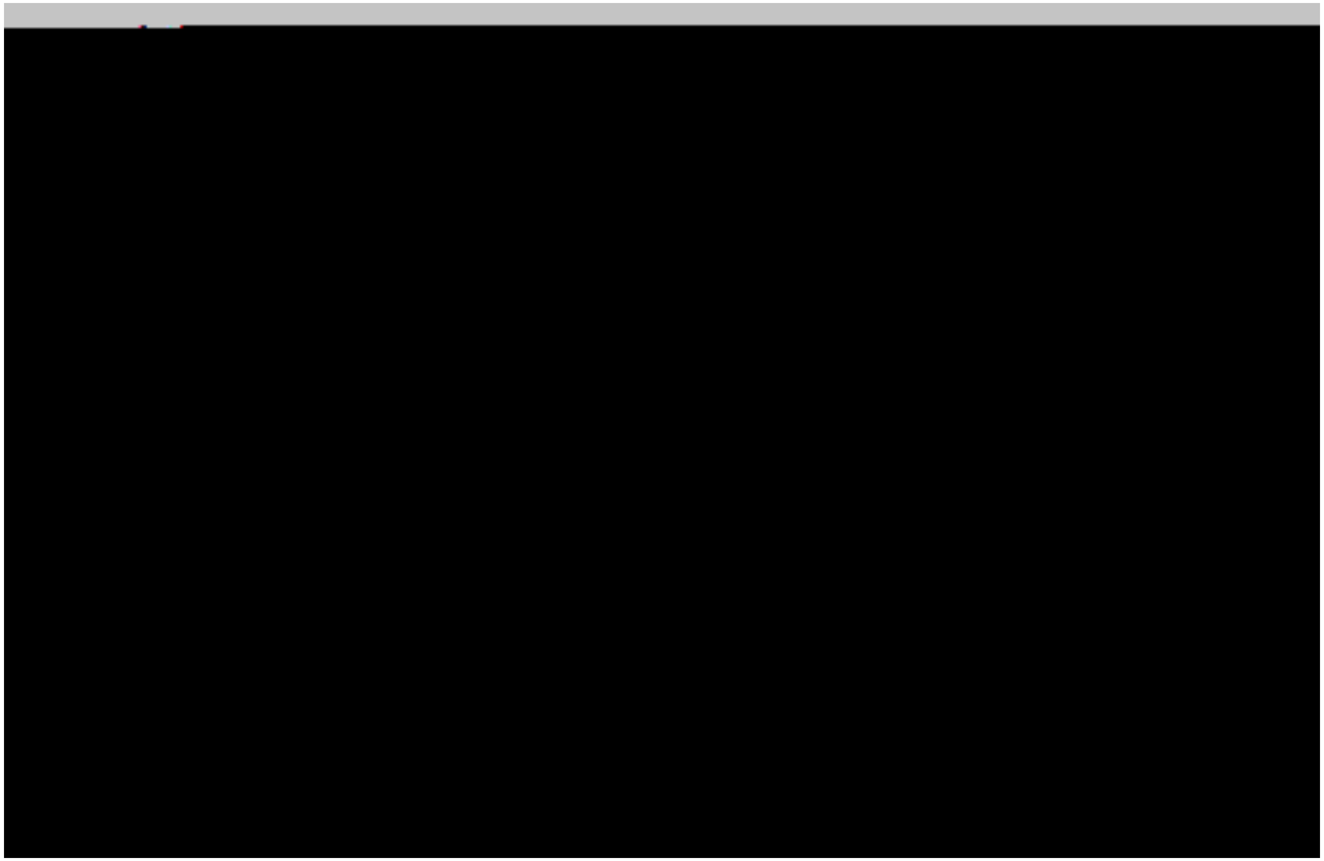
$$+\frac{1}{8}\phi_{KK}(\lambda_K - \lambda_L)^2 t^2$$

7. Factor augmenting technological change and TP flexibility, *continued*

The inverse demand functions are now as follows:

$$+\frac{1}{2}\phi_{KK}(\lambda_K-\lambda_L)t^2$$

7. Factor augmenting technological change and TP flexibility, *continued*



7. Factor augmenting technological change and TP flexibility, *continued*

[redacted]

7. Factor augmenting technological change and TP flexibility, *continued*

$$+\frac{1}{2}\phi_{KK}(\mu_K-\mu_L)\mu_K(2t-1)$$

$$+\frac{1}{2}\phi(\mu-\mu)\lambda(4t^2-4t+1)+\frac{1}{2}\phi(\lambda-\lambda)\mu(2t^2-2t+1)$$

$$+\frac{1}{2}\phi(\lambda-\lambda)\lambda(4t^3-6t^2+4t-1)$$

[redacted]

Table 1

Parameter estimates

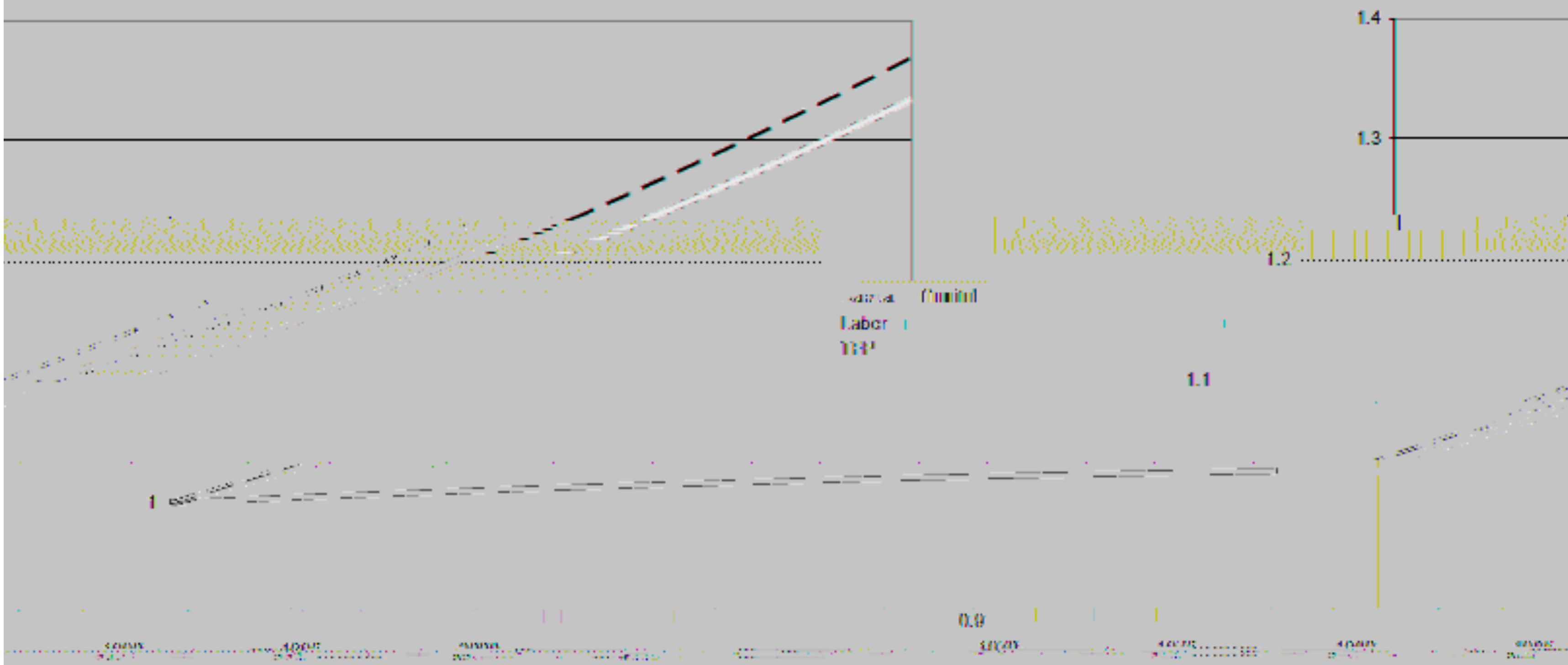
(31)	(53)			(11)
8.30250	8.38044	[illegible]		8.38851
[illegible]	[illegible]	[illegible]		[illegible]
		[illegible]	λ_L	
		[illegible]		
		[illegible]	λ_Σ	
		[illegible]		
226.70	224.09	227.89	*LL*	

0.01116	μ_{2001}	0.01158	0.01008	[illegible]
0.28007	$s_{K,2001}$	0.28452	0.28454	[illegible]
2.01263	$\psi_{KL,2001}$	1.75267	1.66797	[illegible]

$+"

Figure 2

Decomposition of TFP



8. A parsimonious and yet flexible model

case the production function becomes

8. A parsimonious and yet flexible model, *continued*

It turns out that the model of equation (62) is equivalent to (11), since there is a one-to-one [illegible]

[illegible]

(64) $\beta_T = \beta_K \mu_K + (1 - \beta_K)\mu_L$

(65) $\phi_{KT} = \phi_{KK}(\mu_K - \mu_L)$

(66) $\phi_{TT} = \phi_{KK}(\mu_K - \mu_L)^2 + \lambda$

[illegible]

[illegible]

(68) $\mu_L = \beta_T - \beta_K \dfrac{\phi_{KT}}{\phi_{KK}}$

(69) $\lambda = \phi_{TT} - \dfrac{\phi_{KT}^2}{\phi_{KK}}$

8. A parsimonious and yet flexible model, *continued*

For TFP we now get:

$$\ln T_{t,t-1} = \beta_K \mu_K + (1-\beta_K)\mu_L + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)(\ln x_{K,t} - \ln$$

(70)

TFP is numerically identical to (15). ... This esti

$$\ln T^L_{t,t-1} = (1-\beta_K)\mu_L + \frac{1}{2}(1-\beta_K)\lambda(2t-1)$$

$$\mu_L)\lambda(3t^2 - 3t + 1)$$

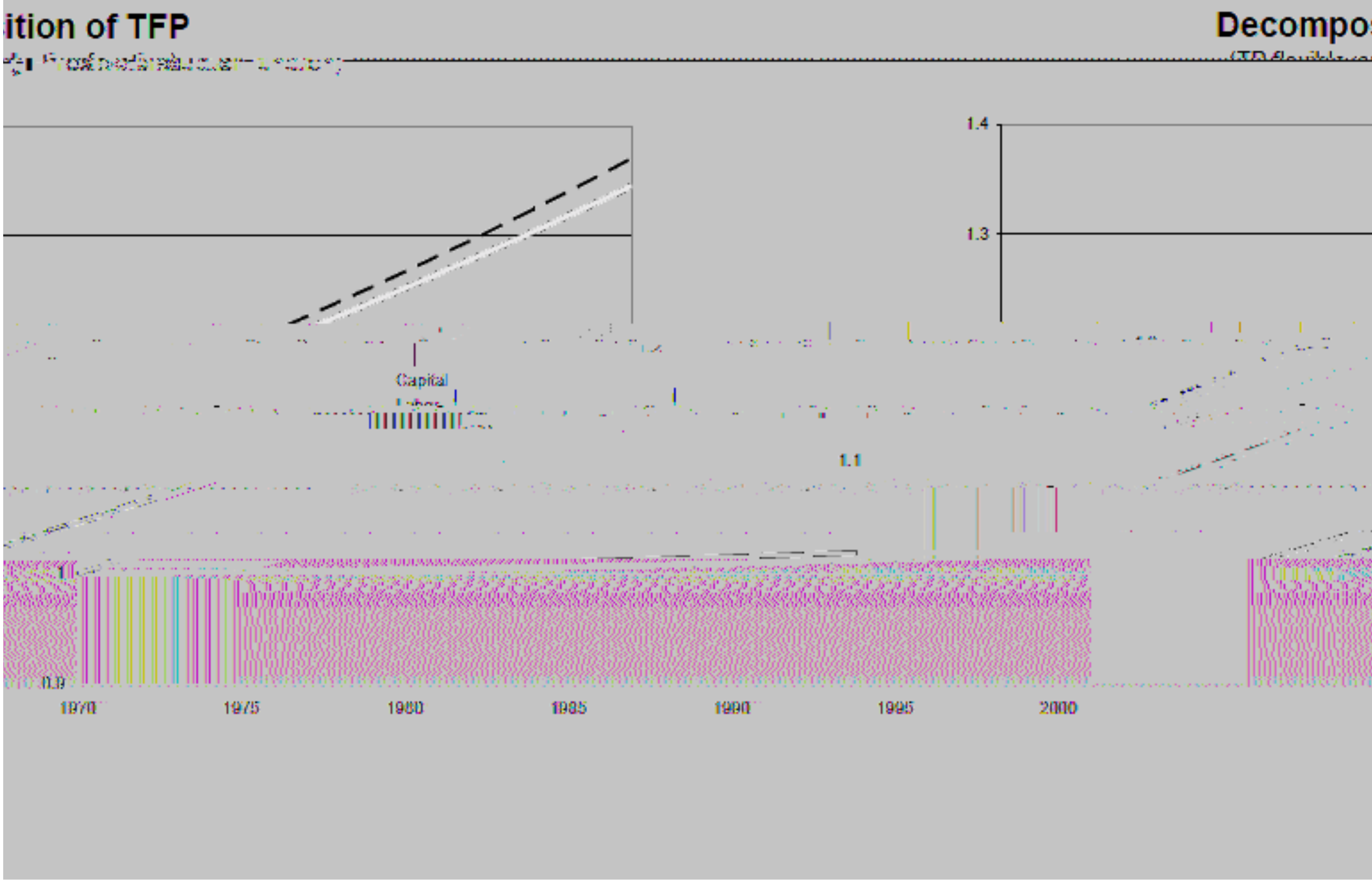
$$+\frac{1}{2}\phi_{KK}(\mu_K - \mu_L)\mu_L(2t-1) + \frac{1}{2}\phi$$

Table 1

Parameter estimates

	(11)	(31)	(53)	(62)	
	8.38851	8.39259	8.38944	8.38851	α_0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Figure 3


ition of TFP
Decompos
1.4
1.3
Capital
1.1
1
0.9
1970
1975
1980
1985
1990
1995
2000

9. The impact of technological change on factor rental prices reexamined, *continued*"

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9. The impact of technological change on factor rental prices reexamined, *continued*"

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Let ε_{ij} be the inverse elasticity of factor demand

$$(73) \qquad \varepsilon_{ij} \equiv \frac{\partial \ln \tilde{w}_i(\tilde{x}_K, \tilde{x}_L, p)}{\partial \ln \tilde{x}_j}, \quad i, j \in \{K, L\}$$

$$(74) \qquad \varepsilon_{iK} + \varepsilon_{iL} = 0, \quad i \in \{K, L\}$$

It is well known that:

$$(75) \qquad \varepsilon_{KL} = \psi_{KL} s_L$$

$$(76) \qquad \varepsilon_{LK} = \psi_{KL} s_K$$

9. The impact of technological change on factor rental prices reexamined, *continued*

We thus get for the total change in the rental price of an efficiency unit of capital:

(80) [illegible]

and similarly for labor:

[illegible]

9. The impact of technological change on factor rental prices reexamined, *continued*"

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9. The impact of technological change on factor rental prices reexamined, *continued*"

¦

10. Generalization to an arbitrary number of inputs, *continued*

10. Generalization to an arbitrary number of inputs, *continued*

We thus find [illegible]

(85) [illegible]

[illegible] (86)

(87) $\lambda + \sum^{J-1} \sum^{J-1} \phi$ [illegible]

[illegible]

from (85), and λ by (87)

11. Conclusions

!

! **five different**

!

!

!

t *t*

! *t* *t*

!

!

11. Conclusions, continued

- We have shown that in the case of a TP-flexible Translog production function TFP can always be interpreted as the outcome of disembodied, factor augmenting technological change
- Indeed, we have proposed a convenient way to derive the factor-augmenting rates of technological change from the estimates of such a Translog production function
- We have found that technological change is almost Harrod-neutral in the case of the United States, so that TFP is overwhelmingly explained by labor

11. Conclusions, *continued*

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Thank you for your attention!

Growth factors 1970-2001

Quantity of capital services:!	"#	2.25706
! "#$%&%'()*(+#,)-(./-0&1/.2!	"$	1.70513
3-&1/()*()"%4"%2!	%	3.76623
! "#$%&%'()*()"%4"%2!	&	2.52563
3-&1/()*(+#,)-(./-0&1/.2(	'$	5.50201
5)%#+(*#1%)-(4-)6"1%&0&%'2!	(	1.37071
Capital component of TFP:	T_K	1.01850
Labor component of TFP:	T_L	1.34581
Capital efficiency:	!#	1.06789
Labor efficiency:	!$	1.50832
7#,)-(.8#-/2!	)$	0.98628
Output per unit of labor:	! ! ! ! !$_L$	1.48119