

Summary



Welfare Change the New Goods Problem

Consumer's cost function:

$$C(u, p) = \min_q \{p \cdot q \mid f(q) \geq u\}$$

$$e(u) = \sum_{j=1}^n p_j q_j \quad \text{subject to} \quad \sum_{j=1}^n p_j q_j = P \quad \text{and} \quad f(q) \geq u$$

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Valid measures of utility change over the two periods under consideration are the following Hicksian *equivalent and compensating variations*:

$$Q_E(q^0, q^1, p^0) \equiv C(f(q^1), p^0) - C(f(q^0), p^0) ;$$

$$Q_C(q^0, q^1, p^1) \equiv C(f(q^1), p^1) - C(f(q^0), p^1) .$$

Note: Samuelson (1974):

$$Q_S(q^0, q^1, p) \equiv C(f(q^1), p) - C(f(q^0), p) .$$

Hence there is an entire family of cardinal measures of utility change: one measure for each reference price vector p .



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The observable Bennet (1920) variation is the arithmetic average of the Laspeyres and Paasche variations:

$$\begin{aligned}V_B(p^0, p^1, q^0, q^1) &= \frac{1}{2}(p^0 + p^1)(q^1 - q^0) \\ &= p^0(q^1 - q^0) + \frac{1}{2}(p^1 - p^0)(q^1 - q^0) \\ &= V_L + \frac{1}{2} \sum_{n=1}^N (p_n^1 - p_n^0)(q_n^1 - q_n^0)\end{aligned}$$

Bennet variation is equal to the Laspeyres variation V_L plus a sum of N Harberger (1971) consumer surplus triangles of the form $(1/2)(p_n^1 - p_n^0)(q_n^1 - q_n^0)$. Also:

$$V_B(p^0, p^1, q^0, q^1) = V_P + \frac{1}{2}$$

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Recap:

Hicksian equivalent variation can be approximated by V_L

Hicksian compensating variation can be approximated by V_P

Hicks (1942) obtained the Bennet quantity variation V_B as an approximation to the arithmetic average of the equivalent and compensating variations.

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So far, no economic justification for taking the average of V_L and V_P .

Diewert and Mizobuchi (2009)

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Then, for normalized prices, we have the following exact equality:

$$V_B(p^0, p^1, q^0, q^1) = \frac{1}{2} Q_E(q^0, q^1, p^0) + \frac{1}{2} Q_C(q^0, q^1, p^1)$$

i.e., the observable Bennet variation is *exactly equal* to the arithmetic average of the unobservable equivalent and compensating variations.

Hence, a strong justification from an economic perspective for using the Bennet quantity variation. Also, it has strong justification from an axiomatic perspective (Diewert, 2005)

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Now benefits of the introduction of a new good (or service) to a consumer who cannot purchase the good in period 0 but can purchase it in period 1.

Assume (as per Hicks 1940) that there is a shadow price for the new good in period 0 that will cause the consumer to consume 0 units of the new good in period 0.

Let the new good be indexed by the subscript 0 and let the N dimensional vectors of period t prices and quantities for the continuing commodities be denoted by p^t and q^t for $t = 0, 1$.

Period 0 shadow price for commodity 0 is not observed but we make some sort of *estimate* for it, denoted as $p_0^{0*} > 0$ and. The period 0 quantity is observed and is equal to 0; i.e., $q_0^0 = 0$.

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Adapting Proposition 9 of Diewert (2005):

If a superlative index number is chosen for P and Q, V_B approximates V_E to the second order for $q^0=q^1$ and $p^0=p^1$.

The U.S. uses the superlative Fisher Quantity Index for GDP, so:

$$V_E^F \approx \frac{1}{2} p^0 q^0 (1+P^F) (Q^F - 1) - \frac{1}{2} (p^0 + p^1) (q^1 - q^0) = V_B$$

Re-arranging:

$$Q^F - [(p^0 + p^1) (q^1 - q^0)] / [p^0 q^0 (1+P^F)] + 1$$

Note that the numerator is $2 \times V_B$.

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$$\begin{aligned} 2V_B &= (p^0 + p^1)(q^1 - q^0) \\ &= 2p^1(q^1 - q^0) - (p^1 - p^0)(q^1 - q^0) + 2p_0^1 q_0^1 - (p_0^1 - p_0^{0*})q_0^1 \end{aligned}$$

Assuming that the approximation holds exactly, then substitute into:

$$Q^F = [(p^0 + p^1)(q^1 - q^0)]/[p^0(q^0(1+P^F))] + 1$$

If Q^F omits the new good in period 0, and we assume that P^F (the aggregate GDP deflator between adjacent periods) is unaffected by the introduction of the new good, then the (approximate) amount missing from Q^F is:

$$(p_0^{0*} - p_0^1)q_0^1 / [p^0(q^0(1+P^F))]$$

which can simply be added to Q^F if p_0^{0*} is known or can be estimated. P^F will typically fall with the inclusion of the new good, so this is a lower bound on the amount to add.

Welfare Change the Free Goods Problem

Consider a consumer whose preferences over N market goods and services and M commodities that are available to the household with no visible charge.

Utility function $f(x,z)$: where $x \in \mathbb{R}_+^N$ and $z \in \mathbb{R}_+^M$ are vectors which represent the consumption of market commodities and of free commodities respectively.

We assume that $f(x,z)$ is defined over the nonnegative orthant in \mathbb{R}^{N+M} and has the following properties:

- (i) continuity,
- (ii) quasiconcave in x and y , and
- (iii) $f(x,z)$ is increasing if all components of x increase and increasing if all components of z increase.

Welfare Change the Free Goods Problem

Define two cost functions that are dual to f . The first cost function is the consumer's *regular cost function* that is the solution to the following cost minimization problem which assumes (hypothetically) that the household faces positive prices for market and free goods and services so that $p \gg 0_N$ and $w \gg 0_M$:

$$C(u,p,w) \sum \min_{x,z} \{p \cdot x + w \cdot z : f(x,z) \geq u, x \geq 0_N, z \geq 0_M\}.$$

The *conditional cost function* minimizes the cost of market goods and services needed to achieve utility level u , conditional on having the vector $z \geq 0_M$ of free goods and services at its disposal:

$$c(u,p,z) \sum \min_x \{p \cdot x : f(x,z) \geq u, x \geq 0_N\}.$$

Welfare Change the Free Goods Problem

Decompose the first cost function into a two-stage minimization problem using the second cost function:

$$\begin{aligned} C(u,p,w) &= \min_{x,z} \{p'x + w'z : f(x,z) \geq u; x \geq 0_N, z \geq 0_M\} \\ &= \min_z \{c(u,p,z) + w'z : z \geq 0_M\}. \end{aligned}$$

Suppose $z^* \geq 0_M$ solves this cost minimization problem and suppose further that $c(u,p,z^*)$ is differentiable with respect to the components of z at $z = z^*$.

Then the first order necessary conditions for z^* to solve the cost minimization problem imply that:

$$\nabla_z c(u,p,z^*) = -w.$$

Welfare Change the Free Goods Problem

With $z = z^*$, we can find an x solution which we denote by x^* ; i.e., x^* is a solution to:

$$c(u, p, z^*) \sum \min_x \{p|x: f(x, z^*) \geq u, x \geq 0_N\}.$$

It can be seen that (x^*, z^*) is a solution to the regular cost minimization problem defined by so that:

$$\begin{aligned} C(u, p, w) \sum \min_{x, z} \{p|x + w|z: f(x, z) \geq u, x \geq 0_N, z \geq 0_M\} \\ = p|x^* + w|z^*. \end{aligned}$$

Thus the imputed marginal valuation prices $w \sum 4 \subseteq_z c(u, p, z^*) \geq 0_M$ are appropriate prices to use when valuing the services of free goods in order to construct cost of living indexes or measures of money metric utility change.



Welfare Change the Free Goods Problem

Marginal willingness to sell function for free good m :

$$W_m(u, p, z) = \frac{\partial c(u, p, z)}{\partial e_m} ; \quad m = 1, \dots, M.$$

where e_m is a unit vector of dimension M with a 1 in component m and zeros elsewhere for $m = 1, \dots, M$.

Survey methods could be used in order to obtain approximate measures for these marginal willingness to sell functions.

Let $W(u, p, z)$ denote the vector $[W_1(u, p, z), \dots, W_M(u, p, z)]$.

It can be seen that $W(u, p, z)$ is a discrete approximation to the marginal valuation price vector $w = \frac{\partial c(u, p, z)}{\partial z}$

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Welfare Change the Free Goods Problem

Attempts to find prices for “free” goods include:

Brynjolfsson and Oh (2012)

“We develop a new framework to measure the value of free services using the insight that even whenpe (

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Summary

Defined a framework for measuring welfare change.

Drew on the work of Hicks (1941-42), Bennet (1920) and Diewert and Mizobuchi (2009).

Derived an explicit term that is the value of a new good on welfare change.

That is, we get a measure of the contribution to welfare of a new good, and hence the extent of welfare change mismeasurement if it is omitted from statistical agency collections.

Showed how to work out a lower bound on the addition to real GDP growth from the introduction of a new good.

We then re-work the theory allowing for there to be “free” goods (with an implicit or imputable price).