

MMAN9452

A UghYfg Dfc YWh6

HYfa "H\fYY" ## "&\$&\$

7ci fgY[·]Cj Yfj]Yk

GhJZZ 7 cbhJWh 8 YhJ] g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWUh]cb	D\cbY
HfUWJY 6UfVYf	<u> </u>	D`YUgY`WcbHJWfiVm Ya U]``cf`HYUa g	>%+ '(\$%5	-',)(\$,%

GW.cc`7cbhJWh′±bZcfa Uh]cb

@cWUh]cb

9a U]`

<u>9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]W</u>yg'-'W ffYbhgh XYbhYbei]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

<u>9b[]bYYf]b['±xi glf]U`'HfU]b]b[</u> '-'±xi glf]U`'lfU]b]b['ei Ygl]cbg

<u>I BGK "Gh Xm5VfcUX"</u>—"gh XmUVfcUX"gh XYbhYbei]f]Yg"fZcf"]bVci bX"gh XYbhgŁ

<u>I_BGK_'9| W.Ub[Y</u>'-'gh XYbhY| W.Ub[Y'Ybei]f]Yg'flcf']bVci bX'gh XYbhgŁ

I BGK : i h fY'Gh XYbhg'-'dchYbhJU'gh XYbhYbei]f]Yg

• Y"["UXa]gg]cbgžZYYgždfc[fUagžWfYX]hlfUbgZYf

<u>GWXcc``CZZJWY</u>'--'GWXcc``[YbYfU``cZZJWY'UXa]b]ghfUh]cb'Ybei]f]Yg

• B6. h\Y fY`Yj UbhhYUa g``]ghYX UVcj Y a i ghVY WcbHUWYX -

gh XYbhYbei]f]Yg

7ci fgY 8YW] g

7 fYX]h'Dc]bhg'('

Gi a a UfmcZh\Y 7ci fgY

H\Y'dfc YWidfcj]XYg'Ub'cddcfhi b]miZcfmci 'hc'Vf]b['hc[Yh\YfYb[]bYYf]b['df]bW]d`Yg`YUfbYX'cj Yf mci f'dfYj]ci g'mYUfg'cZghi XmUbX'Udd`mh\YgY'df]bW]d`Yg'hc']bbcj Uh]j Y`mgc'j Y'dfcV'Ya g'gi W\'Ug'h\Y XYj Y`cda YbhcZU'gdYW]ZJWXYg][bž'dfcWygg'UbX#cf'h\Y']bj Ygh][Uh]cb'cZU'\mdch\Yg]g"'A UghYf'dfc YWg'UfY Wta d`Yl ž'cdYb!YbXYX'dfcV'Ya g'h\UhU``ck 'fcca 'Zcf'mci f'WfYUh]j]mž'UbX'h\Y'UWei]g]h]cbž'UbU`mg]g'UbX]bhYfdfYhUh]cb'cZfYgi 'hg"'H\YfY'UfY'a i 'h]d`Y'dcgg]V`Y'gc'i h]cbg'cf'WtbWi g]cbg'Uhh\Y'ci hgYhUbX'gi ZZMYbh Wta d`Yl]mhc'fYei]fY 'U'XY[fYY'cZdfc YWhd`Ubb]b["'H\Y'dfc YWhfYei]fYg'mci 'hc'Zcfa i `UhY'dfcV'Ya g']b gW]Ybh]ZJWCf'Yb[]bYYf]b['hYfa gž'a UbU[Y'Ub'hYW\b]WU`dfc YWhUbX'Z]bX'gc`i h]cbg'VmUdd`m]b['gW]Ybh]ZJWUbX Yb[]bYYf]b['a Yh\cXg"'Mci 'k]`'U'gc'XYj Y`cd'h\Y'UV]`]mhc'k cf_']b'U'fYgYUfW\'UbX'XYj Y`cda Ybh Ybj]fcba Ybh''Mci 'a i gh']XYbh]ZmU'gi dYfj]gcf'UbX'dfc YWhdf]cf'hc'Ybfc``]b[']b'h\]g'Wti fgY"'H\]g']g'h\Y gYWtbX'Wti fgY'cZh\Y'' 'Wti fgY'dfc'YWhgffi Whf''

7 ci fgY 5]a g

H\Y'h\Yg]g'dfcj]XYg'Ub'cddcfhi b]hmZcf'h\Y'ghi XYbhhc'Vf]b['hc[Yh\Yf'Yb[]bYYf]b['df]bW]d'Yg'YUfbYX'cj Yf h\Y]f'dfYj]ci g'mYUfg'cZghi XmUbX'Udd'mh\YgY'df]bW]d'Yg'hc']bbcj Uhji Y'mgc'j Y'dfcV'Ya g'gi W\'Ug'h\Y XYj Y'cda YbhcZU'gdYW]ZWXYg][bž'dfcW\'gg'UbX\\chi\Y']bj Ygh][Uhjcb'cZU'\mdch\Yg]g''H\Yg]g'dfc'YWg'a i gh VY'Wca d'Yl ž'cdYb!YbXYX'dfcV'Ya g'h\UhU'`ck 'fcca 'Zcf'ghi XYbhWYUhjj]hmž'UbX'h\Y'UWei]g]h]cbž'UbU'mg]g UbX']bhYfdfYhUhjcb'cZfYgi 'hg''H\YfY'a i ghVY'a i 'h]d'Y'dcgg]V'Y'gc'i h]cbg'cf'WcbWi g]cbg'Uhih\Y'ci hgYhUbX gi ZZ[W]YbhWca d'Yl]hmhc'fYei]fY'U'XY[fYY'cZdfc'YWhd'Ubb]b['Zfca 'h\Y'ghi XYbh''H\Y'h\Yg]g'fYei]fYg'h\Y ghi XYbhhc 'Zcfa i 'UhY'dfcV'Ya g']b'Yb[]bYYf]b['hYfa gž'a UbU[Y'Ub'Yb[]bYYf]b['dfc'YWhUbX'Z]bX'gc'i h]cbg'Vm Udd'm]b['Yb[]bYYf]b['a Yh\cXg''Ghi XYbhg'U'gc'XYj Y'cd'h\Y]f'UV]']mhc'k cf_']b'U'fYgYUfW\'UbX'XYj Y'cda Ybh Ybj]fcba Ybh'

7ci fgY @YUfb]b[Ci hWca Yg

- %" 8 Yj Y`cd`U`XYg][b`cf`U`dfcW'gg`cf`]bj Ygh][UhY`U`\mdch\Yg]g`Zc``ck]b[`]bXi ghfm'UbX`dfcZYgg]cbU` Yb[]bYYf]b[`ghUbXUfXg"`
- &" 7 f]h]wU``mfYZYWicb'U'gdYW]U`]ghiVcXmcZ_bck `YX[Y'fY`UhYX'hc'h\Y]f'h\Yg]g'hcd]w''
- ' " 5dd`mgWYbfjZWUbX'Yb[]bYYf]b['a Yh\cXg'hc'gc`j Y'Ub'Yb[]bYYf]b['dfcV'Ya "'
- ("5bU`mgY`XUHU`cV'YWhj Y`mi g]b[ei UbhjHuhj Y`UbXa Uh\Ya UhjWU`a Yh\cXg"
-) "8 Ya cbglfUHY cfU`UbX k flHYb Wca a i blWUHcb lb dfcZygglcbU`UbX `UmXca U]bg":

HYUW]b[GhfUhY[]Yg

H\Y`Wci fgY`]g`HJi [\h'Ug`Ub`]bX]j]Xi U``fYgYUfW\`dfc`YWžhc`XYj Y`cd`U``Yj Y``cZfYgYUfW\`g_]``g`UbX Ui hcbca m'

5ggYgga Ybh

5ggYgga Ybh'HUg_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y [.] 8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
A UghYf dfc YWhUbX fYdcfh	%\$ı	% #%#&\$&\$`\$).\$\$`DA	%ž˙(ž˙)

5ggYgga Ybh8YhJ]`g

5ggYgga Ybh% 'A UghYf'dfc YWh'UbX fYdcfh

8YHJ]`g.

F Ygci fWYg

FYWca a YbXYX FYgci fW/g

BchUj U]`UV`Y

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

@UVcfUhcfmK cf_g\cd \dag{b}Zcfa Uh]cb

 $7\,C\,J = 8\,!\% B\,C\,H9. H\Y'GWkcc```UVcfUlrcfm'ZJWJ']hJYg'UfY'cdYb'hc'fYgYUfWk'hkYg]g'gh'XYbhg']b'H&/\kYjYfZ'UWW'gg']g``]a]h'X"''Hc'Wcad`mk]hk'[cjYfbaYbhfg'gcWJU'`X]ghUbWJb['fYei]fYaYbhgZY''['"Z'('geiUfY'aYhYfg'dYfdYfgcbZYUWk'fcca'\Ug'U'aUl]aia'WddUWJmm']a]h'Zk\]Wk'k]```VY'acb]hcfYX'Vmh\Y'gidYfj]g]b[dfcZYgg]cbU``ghUZZ''D`YUgY'WkYW_k]h\hkY'`UVcfUlrcfmcZJWf']b'mcif'UfYU'VYZcfY'UWW'gg]b['UbX'cVgYfjY'hkY']a]h'UbX'U```ch\Yf'gUZYmdfYWJihJcbg''$

Mci a i ghgYY_[i]XUbWYUbX'Uddfcj U``Zfca 'mci f'H\Yg]g'gi dYfj]gcf'df]cf'hc'Ubm`UVcfUhcfmUWfj]h]Yg"

 $\label{thm:convergence} H\Y``UVcfUhcf]Yg`UfY`h\Y`fYgdcbg]V]`]hmicZh\Y`ghUZ]b! W\Uf[Yž`UbX`mci`a i ghicdYfUhY`k]h\]b`h\Y`UWW\dhYX dfUWf]W\g`cZh\Y``UVcfUhcfmiwb\WfbYX''`@UVcfUhcfmighUZUfY`\][\`mig_]``YX`UbX`\Y`dZ`\'hU_Y`Z````UXj UbhU[Y cZh\Y]f`YI dYf]Yb\W'\ck Yj Yfžmci 'g\ci`X`bchYI dY\Wi`UVcfUhcfmighUZhc`hU_Y`fYgdcbg]V]`]hmiZcf`mci f`h\Yg]g cf`\Wffmici hk cf_`Zcf`mci ''$

= mci f'dfc YWi]bj c'j Yg``UVcfUhcfmik cf_žWcbHUWih\Y'cZ]Wf!]b!W\Uf[Y'fC = Ł'cZh\Y'`UVcfUhcfmi]b'k \]W\'mci k]```VY'k cf_]b['Ug'gccb'Ug'dcgg]V'Y'hc'X]gWigg'mci f'fYei]fYa Ybhg"'H\Ymk]```]ggi Y'mci 'k]h\'U'@UVcfUhcfm 5\WWgg'5ddfcj U'`f@55Ł'Zcfa 'k \]W\'mci 'a i gh'Wca d'YhY'UbX'fYh fb'hc'h\Y'C =7"

K cf q\cd

5```ghi XYbh`UWhj]hjYg`fYei]f]b['a Ubi ZJWhi fY`]b`h\Y`A YW\Ub]WJ``UbX`A Ubi ZJWhi f]b['9b[]bYYf]b['fA A 9Ł k cf_g\cd'g\ci `X`VY`X]gW ggYX`k]h\ 'h\Y`k cf_g\cd'dYfgcbbY``Uhh\Y`]bWYdhjcb'cZh\Y`k cf_"'H\Y`k cf_g\cd dYfgcbbY``a i ghi\Uj Y`h\Y`cddcfhi b]mhc`UXj]gY`UbX`]bZi YbWY`h\Y`XYg][b`hc`\Y`d`a]b]a]gY`UggYa V`mž a Ubi ZJWhi fY`cf`Z bWhjcbU``dfcV`Ya g"

H\Y'k cf_g\cd']g'i gi U``m]b'j Yfm\][\'XYa UbX"'=\mathrix mci 'fYei]fY'h\Y'k cf_g\cd'hc'a Ubi \mathrix Yei]da Ybh YggYbh]U`hc'mci f'h\Yg]g\tin\Y'b'a U_Y'gi fY'h\Uhmci 'X]gW gg'mci f'fYei]fYa Ybhg'Ug'YUf'mUg'dcgg]V`Y'k]h\'h\Y K cf_g\cd#@UVcfUhcfmA UbU[Yf"\Mci 'g\ci `X'dfcj]XY'Yb[]bYYf]b['XfUk]b[g'k\]W\"UfY'\mathrix ZfghUddfcj YX'Vm h\Y'`UVcfUhcfmC=\mathrix "\Mci 'g\ci `X'a U_Y'Yj YfmY\mathrix Zfhhc'a]b]a]gY'h\Y'K cf_g\cd'cUX'Vma cX]\mathrix Mb['Yl]gh]b[Yei]da YbhfUh\Yf'h\Ub'Vi]\X]b['\mathrix Zfca 'bYk \tin\Uhx'Vm_YYd]b['mci f'XYq][bg'q]a d'Y"

UggYgga YbhdYfZcfa UbWfZmci "UfY Y`][]V`Y`nc`Udd`mZcf`GdYWJU``7 cbg]XYfUhjcb`df]cf`nc`gi Va]htjb[`Ub

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK `ghiXYbhg`UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{`WSege[YfZW_aXSekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``h]a Y'a UbU[Ya Ybh]g'Ub']a dcfhUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fW/g']b'dfYdUf]b['U```UggYgga Ybh'hUg_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg_]``g'''H\Yma UmUg_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf_icfidUm]b[igca YcbYihciXcimci fik cf_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha]gWcbXi WhdfcWYXi fYg"dXZ

5 WUXYa]W±bZcfa Uh]cb

7 fYX]hdc]bhg

7 ci fgY'WfYX]hi]g'WU'W'`UhYX']b'I b]hg!C Zi 7 fYX]hifl C 7 L"'H\Y'bcfa U``k cf_`cUX'YI dYWfUh]cb'Zcf'cbY'I C 7]g'UddfcI]a UhY`m'&) '\ci fg'dYf'hYfa "'H\]g']bWi XYg'WUgg'WcbhUWh'\ci fgž'df]j UhY'ghi Xmž'ch\Yf'`YUfb]b[UWfj]]h]Ygž'dfYdUfUh]cb'UbX'h]a Y'gdYbh'cb'U``'UggYggUV`Y'k cf_"

A cgh'Wci fgYk cf_'Wci fgYg'Uhl BGK 'UfY'*'I C 7 'UbX']bj c`j Y'Ub'Ygh]a UhYX'%) \$`\ci fg'hc'Wca d`YhY'''9UW\ Wci fgY']bWi XYg'U'dfYgWf]VYX'bi a VYf'cZ\ci fg'dYf'k YY_'fl\#\ Ł'cZgW\YXi `YX'ZJ\WY!hc!ZJ\W''UbX#cf'cb`]bY WcbhJ\W''5bmUXX]h]cbU`'hja Y'VYmcbX'h\Y'dfYg\fy]VYX'WcbhJ\Wi\ci fg'g\ci `X'VY'gdYbh]b'a U_]b['gi fY'h\Uhmci ii bXYfghJbX'h\Y''YWfi fY'a UhYf]U'ž'Wca d'Yh]b['h\Y'gYhUgg][ba Ybhgž'Z fh\Yf'fYUX]b['ž'UbX'fYj]g]b['Zcf UbmYl Ua]bUh]cbg"

Cb!WUa di g WUgg UHYbXUbW

Di V`]WX]gHJbW]b[`WcbX]h]cbg'a i gh'VY`Zc``ck YX`Zcf'U```H' `ZJ\WY!hc!ZJ\WY`WUggYg":Hc`Ybgi fY`h\]gž'cb`m ghi XYbhg`Ybfc``YX`]b`h\cgY`WUggYg'k]```VY`U``ck YX`]b`h\Y`fcca "`7`Ugg`fcghYfg`k]```VY`UhtUW\YX`hc WcffYgdcbX]b[`fcca g`UbX`W]fW``UhYX`Ua cb[``UV`XYa cbghfUhcfg"'Bc'cj Yf!Ybfc`a Ybh]g`U``ck YX`]b`ZJ\WY!hc! ZJ\WY`WUgg"'Ghi XYbhg`Ybfc``YX`]b`cb`]bY`WUggYg`WJb`gk Ud`h\Y]f`Ybfc`a YbhZca 'cb`]bY`hc'ch\Yf`UXX]h]cbU`ž' Vi h``]a]hYXž'bi a VYf`cZcb!WJa di g`WUggYg`VmGi bXUmž'K YY_`%"D`YUgY`fYZYf`hc`mci f`Wci fgYfg`A]\WcgcZh HYUa g`UbX`A ccX`Y`g]hYg`Zcf'a cfY`]bZcfa Uh]cb`UVci h`WUgg`UhhYbXUbWY`Zcf']b!dYfgcb`UbX`cb`]bY`WUgg gY\W]cbg#J\W]j]h]Yg"

 $\label{thmuh} $$ Mci f'Yu'h' `UbX'h'Y'YU'h' `cZh'cgY']b'mci f'WUgg']g'Wf]hJWU'`m]a dcfhUbh''Mci 'a i ghghUmUh'\ca Y']Zmci 'UfY g]W_'cf'\Uj Y'VYYb'UXj]gYX'hc'gY'Z]gc`UhY'VmBGK_'\YU'h' `cf'[cj Yfba Ybh'Ui h'\cf]hJYg'''7 i ffYbh'U'Yfhg'UbX'U `]gh'cZ\chgdchg'WUb'VY'Zci bX'$

a dcfhUbh'@]b_g