

MMAN9453

A UghYfg Dfc YWi7

HYfa 'C bY''##'&\$&%

7ci fgY'Cj Yfj]Yk

GHJZ:7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
HfUW]Y 6UfVYf	\mathbf{e}	J	>%+ '(\$%5	
		Ya U]``cf'j]U'HYUa g		

GWkcc~7cbhJWh~abZcfa Uh]cb

@cWUhlcb

I BGK 'A YVX.Ub]WU``UbX'A Ubi ZUWi f]b['9b[]bYYf]b[

5]bgk cfh\ 'Vi]`X]b['>%+ž'@Yj Y`'%

5Vcj Y 7 c ZZYY cb 7 Ua di g

< ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7 `cgYX cb 'di V`]W\c`]XUmgž GWkcc``gWkYXi `Y%Wj Ybhg \$bXT b]j Yfg]hmG \ZMXKcdbb7 Ua di g

K YV

GWXcc`cZA YWXUb]WU`UbX'A Ubi ZUWi f]b['9b[]bYYf]b[

9b[]bYYf]b[]Gh XYbhGi ddcfhGYfi]WYa

9b[]bYYf]b['=bXi ghf]U`'HfU]b]b[

LBGK Ghi Xm5VfcUX UbX 91 VX Ub[Y flcf]bVci bX ghi XYbhgŁ

I BGK : i hi fY Ghi XYbhg

D\cbY

fŽ*%&Ł-',),)\$\$-BiWYigGhXYbh<iV

fŽ*%%&L'-',) '+**%'-'9b[]bYYf]b['=bXi glf]U`'HfU]b]b[

fŽ*%%&L-',) ''%+- - I BGK Gh Xm5VfcUX UbX I BGK '91 WX Ub[Y fZcf]bVci bX gh XYbhgL

 $f\tilde{Z}^*\%.\&L^{-}',).(\$-+-\$$ ZYY.cb.7 Ua di g

9a U]`

<u>9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]W</u>yg'-'W ffYbhgh XYbhYbei]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

<u>9b[]bYYf]b['±xi glf]U`'HfU]b]b[</u> '-'±xi glf]U`'lfU]b]b['ei Ygl]cbg

<u>I BGK "Gh Xm5VfcUX"</u>—"gh XmUVfcUX"gh XYbhYbei]f]Yg"fZcf"]bVci bX"gh XYbhgŁ

<u>I_BGK_'9| W.Ub[Y</u>'-'gh XYbhY| W.Ub[Y'Ybei]f]Yg'flcf']bVci bX'gh XYbhgŁ

I BGK : i h fY'Gh XYbhg'-'dchYbhJU'gh XYbhYbei]f]Yg

• Y"["UXa]gg]cbgžZYYgždfc[fUagžWfYX]hlfUbgZYf

<u>GWXcc``CZZJWY</u>'--'GWXcc``[YbYfU``cZZJWY'UXa]b]ghfUh]cb'Ybei]f]Yg

• B6. h\Y fY`Yj UbhhYUa g``]ghYX UVcj Y a i ghVY WcbHUWYX -

gh XYbhYbei]f]Yg

7ci fgY 8YW] g

7 fYX]hDc]bhg'('

Gi a a UfmcZh\Y 7ci fgY

H\Y'h\Yg]g'dfcj]XYg'Ub'cddcfh b]miZcfimci 'hc'Vf]b['hc[Yh\Yf'Yb[]bYYf]b['df]bW]d'Yg'YUfbYX'cj Yf mci f'dfYj]ci g'mYUfg'cZghi XmUbX'Udd`mh\YgY'df]bW]d'Yg'hc']bbcj Uhjj Y`mgc'j Y'dfcV'Ya g'gi W\'Ug'h\YXY XYj Y`cda YbhcZU'gdYW]ZJWXYg][bž'dfcWYgg'UbX#cf'h\Y']bj Ygh][Uhjcb'cZU'\mdch\Yg]g"H\Yg]g'dfcYWg'UfY Wca d'Yl ž'cdYb!YbXYX'dfcV'Ya g'h\UhU``ck 'fcca 'Zcfimci f'WfYUhjj]mž'UbX'h\Y'UWei]g]hjcbž'UbU`mg]g'UbX]bhYfdfYhUhjcb'cZfYgi 'hg"'H\YfY'UfY'a i 'hjd'Y'dcgg]V'Y'gc'i hjcbg'cf'WcbWi g]cbg'UbX'gi ZZJW]YbhWca d'Yl]mhcfYei]fY'U'XY[fYY'cZdfcYWid'Ubb]b["'H\Y'h\Yg]g'fYei]fYg'mci 'hc'Zcfa i 'UhY'dfcV'Ya g']b'gW]Ybh]ZJWcfYb[]bYYf]b['hYfa gž'a UbU[Y'U'hYW\b]WU'dfcYWiUbX'Z]bX'gc`i hjcbg'VmUdd`m]b['gW]Ybh]ZJWUbX'Yb[]bYYf]b[a Yh\cXg"'Wci 'k]``U'gc'XYj Y`cd'h\Y'UV]`]mhc'k cf_']b'U'fYgYUfW\'UbX'XYj Y`cda YbhYbj]fcba Ybh''Wci 'a i gh]XYbh]ZmU'gi dYfj]gcf'UbX'dfcYWidf]cf'hc'Ybfc``]b[']b'h\]g'Wci fgY"'H\]g']g'h\Y'h\]fX'Wci fgY'cZh\Y'h\fYY Wci fgY'h\Yq]q'qhfi Whi fY"

п

7 ci fgY 5]a g

H\Y'h\Yg]g'dfcj]XYg'Ub'cddcfh b]mmZcf'h\Y'ghi XYbhirc'Vf]b['hc[Yh\Yf'Yb[]bYYf]b['df]bW]d'Yg''YUfbYX'cj Yf h\Y]f'dfYj]ci g'mYUfg'cZghi XmUbX'Udd'mh\YgY'df]bW]d'Yg'hc']bbcj Uhjj Y`mgc'j Y'dfcV'Ya g'gi W\'Ug'h\Y XYj Y`cda YbhicZU'gdYW]Z]WXYg][bž'dfcWYgg'UbX#cf'h\Y']bj Ygh][Uh]cb'cZU'\mdch\Yg]g''H\Yg]g'dfc'YWg'a i gh VY'Wca d'Yl ž'cdYb!YbXYX'dfcV'Ya g'h\Uh'U`ck 'fcca 'Zcf'ghi XYbhiWfYUhjj]hmž'UbX'h\Y'UWei]g]h]cbž'UbU'mg]g UbX']bhYfdfYhUh]cb'cZfYgi 'hg'''H\YfY'a i ghVY'a i 'h]d'Y'dcgg]V'Y'gc'i h]cbg'cf'WcbWi g]cbg'UbX'gi ZZJW]Ybh Wca d'Yl]hmhc'fYei]fY'CZdfc'YWid'Ubb]b['Zfca 'h\Y'ghi XYbh''H\Y'h\Yg]g'fYei]fYg'h\Y'ghi XYbhinc Zcfa i 'UhY'dfcV'Ya g']b'Yf]b['hYfa gž'a UbU[Y'Ub'Yb[]bYYf]b['dfc'YWiUbX'Z]bX'gc'i h]cbg'VmUdd'm]b[Yb[]bYYf]b['a Yh\cXg''Ghi XYbhg'U`gc'XYj Y`cd'h\Y]f'UV]']mhc'k cf_']b'U'fYgYUfW\'UbX'XYj Y`cda Ybh Ybj]fcba Ybh'

7ci fgY @YUfb]b[Ci hWca Yg

5 ZHYf'gi WWYggZ ``m'Wca d`Yh]b['h\]g'Wci fgYž'mci 'g\ci `X'VY'UV`Y'hc.

@YUfb]b['Ci HWca Y	95 GW[Y %7ca dYHYbW]Yg
%":8Yj Y`cd`U`XYg][b`cf`U`dfcWYgg`cf`]bj Ygt][UtY`U`\mdct\Yg]g Zc``ck]b[`]bXi gtfmUbX'dfcZYgg]cbU``Yb[]bYYf]b[`gtUbXUfXg"	D9%"%
&"`7 f]h]WU``mifYZYWicb`U`gdYWJU`]ghVcXmcZ_bck`YX[Y`fY`UhYX`hch\Y]f`h\Yg]g`hcd]W'`	D9%"
' "'5dd`mgWjYbtjZjWUbX`Yb[]bYYf]b['a Yt\cXg`tc`gc`j Y`Ub Yb[]bYYf]b['dfcV`Ya "'	D9&"&
("'5bU'mgY'XUHU'cVYWNj Y'mig]b['eiUbh]HUhj Y'UbX'aUh\YaUh]WU' aYh\cXg"'	D9%"&
) "'8 Ya cbglfUlY'cfU`'UbX'k f]ltYb'Wca a i b]WUl]cb']b'dfcZYgg]cbU`UbX``UmXca U]bg"'	D9' "&

HYUWX]b['GhfUhY[]Yg'

 $H\Y`Wci fgY`]g`HUi [\hiUg`Ub`]bX]j`]Xi U``fYgYUfWX`dfc YWfz`hc`XYj Y`cd`U``Yj Y``cZfYgYUfWX`g_]``g`UbXUi hcbca m'$

F Ygci fWyg

FYWca a YbXYX FYgci fWfg

BchUj U]`UV`Y

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

@UVcfUhcfmK cf_g\cd \pdzcfa Uhlcb

7 C J =8!% 'B C H9. 'H\Y'GW\cc```UVcfUhcfmiZJW]']h]Yg'UfY'cdYb'hc'fYgYUfW\'h\Yg]g'gh XYbhg/\ck Yj Yfž'UWW'gg]g``]a]hYX"'Hc'Wca d`mk]h\'[cj Yfba Ybhfg'gcW]U`X]ghUbW]b['fYei]fYa YbhgžY"["ž'('gei UfY'a YhYfg'dYf dYfgcbžYUW\'fcca '\Ug'U'a Ul]a i a 'WUdUW]hm`]a]hž'k \]W\'k]``'VY'a cb]hcfYX'Vmh\Y'gi dYfj]g]b[dfcZYgg]cbU`'ghUZ''D`YUgY'W\YW_'k]h\'h\Y'`UVcfUhcfmicZ]WYf]b'mci f'UfYU'VYZcfY'UWW'gg]b['UbX'cVgYfj Y'h\Y `]a]h'UbX'U``'ch\Yf'qUZYhmdfYWUi hlcbq"

•

 $\label{eq:maighgyy} $$Mci 'a i ghgyy_'[i]XUbW' UbX' Uddfcj U``Zfca 'mci f'H\Yg]g'gi dYfj]gcf'df]cf'hc' Ubm'`UVcfUhcfm'UWfj]h]Yg'' $$$

.

 $\label{thm:condition} $$HY^*UVcfUhcf]Yg^*UfY^hY^*fYgdcbg]V]^]hmcZhY^*ghUZ]b!W.Uf[Yž^*UbX^mci a i ghcdYfUhY^k]hY]bhYY^*UWWdhYX dfUWf]WYg^*cZhY^*UVcfUhcfmWcbWfbYX''@UVcfUhcfmghUZUfY^\][\`mg_]`YX^*UbX^\Y^dZ\'AZ' \``UXj UbhU[Y cZhY]fYl dYf]YbW/\ck Yj Yfz^mci g\ci `X'bchYl dYWh`UVcfUhcfmghUZhc'hU_Y`fYgdcbg]V]`]hmZcf'mci f'hYg]g cf'WUffmci hk cf_'Zcf'mci ''$

.

.

: cf'a cfY']bZcfa Uhjcb'UVci h'UVWYgg]b['h\Y'GW\cc''g''UVcfUhcf]Ygž'd'YUgY'j]g]hh\Y'GW\cc'\
k YVg]hY.`\hdg.#k k k "Yb[]bYYf]b["i bgk "YXi "Ui #a YV\Ub]\W'!Yb[]bYYf]b[#Ygci f\Wg#UV!U\W\gg!\ck!hc!\
Zcfa q

K cf g\cd

.

Mci 'a i ghgYY_'[i]XUbW''UbX'Uddfcj U`'Zfca 'mci f'H\Yg]g'gi dYfj]gcf'df]cf'hc'fYei Ygh]b['Ubmik cf_g\cd UWn]j]h]Yg"

.

5```ghi XYbh`UWhj]hjYg`fYei]f]b['a Ubi ZJWhi fY`]b`h\Y`A YW\Ub]WJ``UbX`A Ubi ZJWhi f]b['9b[]bYYf]b['fA A 9Ł k cf_g\cd'g\ci `X`VY`X]gW ggYX`k]h\ 'h\Y`k cf_g\cd'dYfgcbbY``Uhh\Y`]bWYdhjcb'cZh\Y`k cf_"'H\Y`k cf_g\cd dYfgcbbY``a i ghi\Uj Y`h\Y`cddcfhi b]mhc`UXj]gY`UbX`]bZi YbWY`h\Y`XYg][b`hc`\Y`d`a]b]a]gY`UggYa V`mž a Ubi ZJWhi fY`cf`Z bWhjcbU``dfcV`Ya g"

•

H\Y'k cf_g\cd']g'i gi U``m']b'j Yfm'\][\'XYa UbX"'=\mathref{Trei}]fY'h\Y'k cf_g\cd'hc'a Ubi \mathref{Z}\whiter fy' Yei]da Ybh YggYbh]U``hc'mci f'h\Yg]g\tink\Yb'a U_Y'gi fY'h\Uhmci 'X]g\W gg'mci f'fYei]fYa Ybhg'Ug'YUf'mUg'dcgg]V`Y'k]h\'h\Y K cf_g\cd#@UVcfUhcfmA UbU[Yf''Mci 'g\ci `X'dfcj]XY'Yb[]bYYf]b['XfUk]b[g'k\]\W\'UfY'\namedfy \mathref{Z}\fgh'Uddfcj YX'Vm h\Y'`UVcfUhcfmC=\mathref{Tree}''Mci 'g\ci 'X'a U_Y'Yj YfmY\mathref{Z}\fgh'hc'a]b]a]gY'h\Y'K cf_g\cd''cUX'Vma cX]\mathref{Z}\fgh'b['Yi]gh'b['Yei]da YbhfUh\Yf'h\Ub'Vi]\X]b['\mathref{Z}\fca 'bYk \tink\Ym_YYd]b['mci f'XYg][bg'g]a d'Y

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYWU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWU gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WD`'Wfh]ZJWUh'g'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYž cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'Wfh]ZJWUh'g'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWfggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

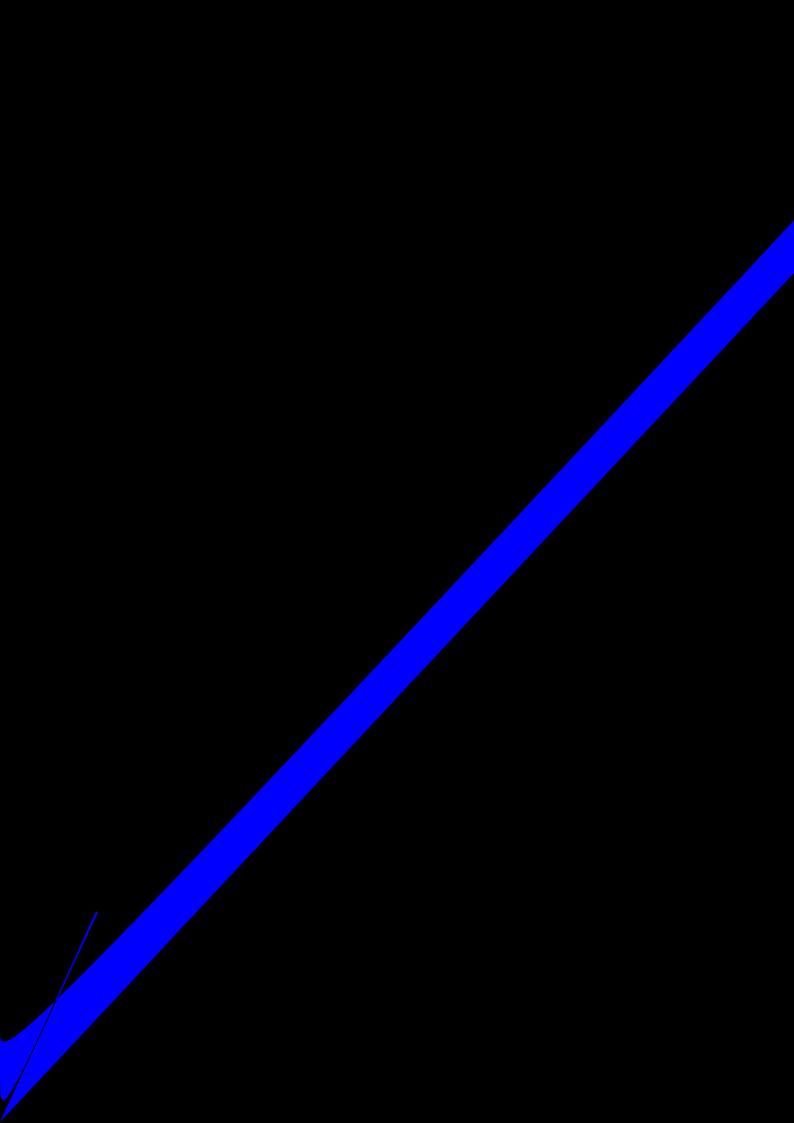
 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUh$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg``UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{`WSege[YfZW_aXSekagdai`ž$

D`U[]Uf]ga `]g`U`lmdY`cZ]bhY``YWi U``lh\YZh'`\#i\WUb`HU_Y`a UbmZcfa gžZfca `XY`]VYfUhY`\W\YUlf]b[`lc`U\W]XYbhU``m \\cdrigot f\W`k]lh\ci hU_bck `YX[Ya Ybh'`I BGK `\Ug`dfcXi \\X'X`U`k YVg]lhY`k]lh\ci hU_bck `YX[Ya Ybh'`I BGK `\Ug`dfcXi \\X'X`U`k YVg]lhY`k]lh\`U`k YU`h\`cZ fYgci f\W`g`lc`gi ddcfhighi XYbhg`lc`i bXYfghUbX`UbX`Uj c]X`d`U[]Uf]ga žij]g]h 'ghi XYbh'i bgk "YXi "Ui \\d'\U[]Uf]ga "\\Y`@YUfb]b[`7 YbhY`Ugg]ghg`ghi XYbhg`k]lh\`I bXYfghUbX]b[`U\WUXYa]\W]bhY[f]hmUbX`\ck 'bchlc'd`U[]Uf]gY" \\YmU`gc`\c`X`k cf_g\cdg`UbX`\WUb`\Y`d`ghi XYbhg`cbY!cb!cbY"

Mci 'UfY'U'gc fYa]bXYX'h\Uh\WUfYZ ``h]a Y`a UbU[Ya Ybh]g Ub']a dcffUbhdUfhcZgh XmUbX'cbY'cZh\Y



a dcfhUbh'@]b_g

- A ccX`Y
- <u>@UV:5\W\/aa</u>
- < YU`h\ 'UbX'GUZY m
- <u>7 ca di h|b[: UW|^]h|Yq</u>
- Gh XYbhF Ygci fWg
- 7 ci faY 'C i hlbYa
- 9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]\Wg:7YblfY
- A U YfadUW
- I BGK 'H]a YHUV'Y
- I BGK '< UbXVcc
- I BGK 'A YVX Ub]WU`'UbX'A Ubi ZUWi f]b['9b[]bYYf]b[
- <u>9ei]hUV`Y`@YUfb]b['GYfj]WYg</u>

a U[Y'7fYX]h

GmbYf[]Yg^]b^GcibX^&\$%*

7F = 7CG

7F = 7CG Dfcj]XYf 7cXY. \$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci blfm

 $K Y : UW\underline{b}ck `YX[Y : h, Y : 6YXY[U : dYcd `Y : k \c : UfY : h, Y : hfUX]h\underline{b}cb U ``W ghcX]Ubg : cZh, Y ``UbXg : cb : k \] WX : I B G K ? Ybg]b[hcb : WUa di g :]g ``c WUhYX''$

5ddYbX]I. '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWmGHJbXUfX

Dfc[fUa '±bhYbXYX'@YUfb]b['CihWca Yg	
?bck `YX[Y'UbX'g_]```VUgY	
D9%%7ca dfY\Ybg]j Yž'h\YcfmVUgYX'i bXYfgHJbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]WJ`'gWJYbWYg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbHJ`g'Udd`]WJV`Y'hc'h\Y'Yb[]bYYf]b['X]gWJd`]bY	
D9%%.7cbWrdhi U`i bXYfghUbX]b[cZh\Ya Uh\Ya Uh\Ya Uh]Wgžbi a Yf]WU`UbU`mg]gžghUh]gh]WgžUbX Wca di hYf'UbX`]bZcfa Uh]cb'gW]YbWrg'k\]W\i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" '±0!XYdh\'i bXYfgh\bX]b['cZgdYW]U`]ghVcX]Yg'cZ_bck `YX[Y`k]h\]b'h\Y'Yb[]bYYf]b[X]gW]d`]bY	
D9%"('8]gWYfba YbhcZ_bck 'YX[Y'XYj Y`cda YbhUbX'fYgYUfW\'X]fYWIjcbg'k]h\]b'h\Y Yb[]bYYf]b['X]gWJd`]bY	
D9%") "? bck "YX[Y"cZYb[]bYYf]b["XYg][b"dfUWf]VY" "UbX" WcbhYl hi U" "ZJWcfg"]a dUWf]b["h\YYb[]bYYf]b["X]gW]d"]bY	
D9%"* I bXYfgHJbX]b['cZh\Y'gWcdYž'df]bWjd`Ygž'bcfa gž'UWVci bHJV]`]hjYg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[]bYYf]b['dfUWjWY']b'h\Y'gdYWjZJWX]gWjd`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUhjcb`cZYghUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j]b[
D9&"&": `i Ybh'Udd`]WUh]cb'cZYb[]bYYf]b['hYVXb]ei Ygž'hcc`g'UbX'fYgci fWfg	
D9&"' '5dd`]\WInjcb'cZgmghYa Unj\WYb[]bYYf]b['gmbh\Yg]g'UbX'XYg][b'dfc\WggYg	