

MTRN3020

A cXY``]b['UbX'7 cblfc`'cZA YVX Ulfcb]WGmghYa g

HYfa 'C bY''##'&\$&%

7ci fgY[.]Cj Yfj]Yk

fŽ*%%&E-',) " %+- - I BGK "Gh Xm5VfcUX"UbX"I BGK "91 WXUb[Y ffIcf]bVci bX gh XYbhgk

fŽ*%"&Ł"-',) "(\$-+"-"GW\cc`"CZZJWY}}

 $\label{thm:linear} $$H^*D^*UgY^*bchY^*h\Uhih\Y^*GW\&cc^*CZ_J^*W'^k]^*bch_bck^*k^Yb\#Zmcif^*WcifgY^*WcbjYbcf^*]g^*cb^*WUadig^*cf^*Uj^*UJ^*UV^*Y}$

9a U]`

9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]Wg - W ffYbhgh XYbhYbei]f]Yq

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

<u>9b[]bYYf]b[⊕Xi glf]U`HfU]b]b[</u> - ⊕Xi glf]U`ffU]b]b[ei Ygl]cbg

LBGK Gh Xm5VfcUX - gh XmUVfcUX gh XYbhYbei]f]Yg fZcf]bVci bX gh XYbhgŁ

IBGK '91 WkUb[Y'-ghi XYbhYl WkUb[Y'Ybei]f]Yg'flcf']bVci bX'ghi XYbhgŁ

I BGK : i h fy Gh XYbhg - dch bh U gh XYbh Ybei]f] Yg

• Y"["UXa]gg]cbgžZYYgždfc[fUagžWfYX]hlfUbgZYf

<u>GWkcc``CZZWY</u>'-GWkcc``[YbYfU``cZZWY'UXa]b]qlfUllqcb'Ybei]f]Yq

• B6. ThY fY Yj UbhthYUa g``]ghYX UVcj Y a i ghVY WcbtUWYX Zcf U```gh XYbhYbei]f]Yg

7ci fgY 8YW] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

H\]grigi VYWihYUW\Ygrh\Yrighi XYbhi\ck 'hc 'XYg][b 'UbX'XYj Y`cd 'Wcblfc``gmghYa g']b 'X]gWfYhY!fja Y`Xca U]b 'h\Uh WUb 'VY'i gYX']b 'a chjcb 'Wcblfc``gmghYa g"'A UhYf]U``Wcj YfYX']bWi XYg/F Yj]g]cb 'cZWcbfjbi ci g!fja Y`Wcblfc` gmghYa g'UbX'XYg][b 'hcc`g'gi W\ 'Ug'fcch`cW gž 6cXY'a Yh\cXg'UbX'@Ud`UWY 'hfUbgZcfa "'GhUfYX'@Ud`UWY 'hfUbgZcfa gz'n!hfUbgZcfa g"'8]gWfYfjg]b['Wcbfjbi ci g!fja Y`gmghYa g"'GhUV]`]mz'gdYYX`cZfYgdcbgY`UbX UVWY fUWh''7 cblfc``Yf'XYg][b i g]b[/fcchl`cW g'a Yh\cXz'X]fYWh'UbX']bX]fYWh'UbU`mfjWU``a Yh\cXg'UbX`VcXY a Yh\cXg"'C VgYfj UV]`]mz'Wcblfc``UV]`]md''GhUhY`YghJa Uhcfg'UbX`XYg][b cZcVgYfj Yfg"

7 ci fgY 5]a g

H\]g`Wci fgY`k]```[]j Y`mci `U`h\cfci [\`i bXYfgHUbX]b[`cZWca di hYf!Wcblfc``YX`gmghYa g"`#g`WcfY`WcblYbh\WUb VY`VfcUX`m`WUhY[cf]nYX`]bhc`a Uh\Ya Uh]WU``a YUbg`cZa cXY``]b[`A YW\Ulfcb]WGmghYa gža cXY``j U`]XUh]cbž XYg][b`cZX][]HU``Wcblfc``Yfg`i g]b[`U'j Uf]YhmcZX]ZYfYbha Yh\cXg`UbX`h\Y`]a d`Ya YbhUh]cb`cZ`Wcblfc``Yfg`cb fYU'!`]ZY`gmghYa g''H\Y`gmghYa g'VY]b[`a cXY``YX`UbX`Wcblfc``YX`UfY``Uf[Y`mia ch]cb`Wcblfc``gmghYa g''

H\Y`Wci fgY`\Ug``UVcfUhcfmiYl dYf]a Ybhg`f]Łhc`a cXY``Ub`]bj YfhYX`dYbXi`i a `gmghYa `UbX`hc`XYg][b`U WUgg]\W``Wcbhfc``Yf`f]]Łhc`XYg][b`X][]hU``Wcbhfc``gmghYa g`Zcf`gdYYX`UbX`dcg]h]cb`Wcbhfc``f][g"

H\Y`Wci fgYg`]b`h\Y`A YW\Ulfcb]Wg`X]gW]d`]bY`UfY`Vi]`hi d`cb`Zci f`X]ZYfYbhUfYUg. a YW\Ub]WU``XYg][bž Wca di h]b[žY`YWfcb]Wg`UbX`a]WfcdfcWfggcfgžUbX`Wcblfc``gmghYa g"`H\Y``UlfYf`h\fYY`UfYUg`UfY`]bhYffY`UhYXž UbX`h\]g`Wci fgY`Zcfa g`U`WcfbYfglrcbY`cZh\Y`Z bXUa YbhU``Wci fgYg`cb`k\]W\`h\Y`A YW\Ulfcb]W9b[]bYYf]b[Wci fgY`UhI B`GK``]g`Vi]`hi dcb"`5 'h\cfci [\`i bXYfgHUbX]b[`cZh\Y`Wcblfc``cZXmbUa]WU``a YW\Ub]WU``gmghYa g hc`UW\]Yj Y`XYg]fYX`a ch]cbg`]g`YggYbh]U``Zcf'h\Y`XYg][b`UbX`XYj Y`cda YbhcZUbmgcd\]gh]WUhYX`A YW\Ulfcb]W GmghYa "`I g]b[`h\Y`Z bXUa YbhU``WUgg]WU``Wcblfc``gmghYa '_bck `YX[Y`[U]bYX`]b`h\Y`h\]fX`mYUfž'h\]g`Wci fgY Vi]`Xg`mci f`_bck `YX[Y`cb`XYg][b]b[`UbX`]a d`Ya Ybh]b[`Wca di hYf!Wcblfc``YX`gmghYa g"`7 cblfc``gmghYa g dfcj]XY`U`a Yh\cX]WU``k UmcZWUffm]b[`ci hih\Y`a ch]cb`Wcblfc``h\Uh'U`gc`bYYXg`dfc[fUa a]b[`UbX Wca di h]b["5g`gi W\`h\Y`Wcblff]Vi h]cbg`Zfca `h\]g`Wci fgY`hc`h\Y`A YW\Ulfcb]W9b[]bYYf]b[`XY[fYY`dfc[fUa UfY`YqqYbh]U``UbX`] hU`"

7diffgevendulfpjb[weihweayg	g]Yj Y	f'h\Y'XY	/Υ.
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5 ZYfgi WWggZ "mWca d'Yhlb[h\]g'Wci fgYžmci g\ci `X'VY'UV'Y'hc.

@YUfb]b['Ci HWca Y	95 GHJ[Y %7ca dYHYbW]Yg

@YUfb]b['Ci hWca Y	95 GW[Y %7ca dYH/bW]Yg
Udd`]WUh]cb`Ybj]fcba Ybh'UbX`VY`UV`Y`hc`]a d`Ya Ybh'h\Y`XYg][bYX Wcblfc``gmghYa g"	

H\Y'YI dYf]a YbhU`'WcbhYbh'UbX'h\Y'UggcWJUhYX'j]XYc'WcbhYbh'k]``\Y'd'h\Y'ghi XYbhg'UddfYWJUhY'\ck 'Wcbhfc` gmghYa g'k cf_"'\#\'U'[fci d'gYh]b[ž'ghi XYbhg'k]``\VY'UV`Y'hc'cVgYfj Yž\ck 'Wcbhfc``Yfg'cZh\Y]f'dYYfg'cdYfUhY k]h\'X]ZZYfYbh'Wcbhfc``Yf'dUfUa YhYfg''XYa cbghfUh]b['X]ZZYfYbh'VY\Uj]ci fg''

HYUWX]b[GhfUhY[]Yg

HYUW]b[cZh]g Wci fgY]g h fci [\ A]WcgcZhHYUa 7 `Uggfcca g" H\Y a Ucf]lmcZh Y `YWi fY WchYbh]g Uj U] `U] `UY `UgdfY! fYWcfXYX j]XYcg" H\Y ghi XYbhg `UfY `YI dYWYX hc k UhW h YgY dfY! fYWcfXYX j]XYcg `UbX Wca d `YhY a]bcfei]nnYg `VYZcfY h Y `YWi fY ha Y" H\Y a]bcfei]nnYg k]```Uk UfX a Uf_g" 5 ZhYf k UhW]b[h Y j]XYc `WcbhYbhzh Y ghi XYbhg `UfY `U`gc YI dYWYX hc `UhhYa dhih Y `gUa d`Y `dfcV`Ya g`[]j Yb hc h Ya `VYZcfY h Y `YWi fYžk \]W k]```VY X]gW ggYX `UbX k cf_YX ci h Xi f]b[h Y `YWi fYg" 8 i f]b[h Y `YWi fY ha Y `U`Vf]YZ YI d`UbUh]cb `cZh Y k YY_`mWcbhYbh]g `U`gc `[]j Yb `UbX h Yb `ghi XYbhg `[YhUb `cddcfhi b] mk cf_ci h gUa d`Y dfcV`Ya g" `Hi hcf]U``WUggYg k]```U`gc `hU_Y d`UWY]b `A]WcgcZhHYUa g`WUggfcca g" @UVcfUhcfmYI YfW]gYg k]`` VY YI d`U]bYX `UbX `XUhU qYhq Zcf]bX]j [Xi U``ghi XYbhq k]``VY Ydfcj]XYX"

H\Y'hi hcf]U``gYgg]cbg`UfY`XYg][bYX'hc`\Y`d`mci `i gY'hcc`g`gi W.`Ug`A Uh\UV'hc`gc`j Y`Wca d`YI `Wcblfc``gmghYa dfcV`Ya g"'=h`]g`YggYbh]U``h\Uhmci `\Uj Y`UWWgg`hc`A Uh\UV`Xi f]b[`U```cb`]bY`gYgg]cbg`gc`h\Uhmci `\Uj Y`UWWgg`hc`A Uh\UV`Xi f]b[`U```cb`]bY`gYgg]cbg`gc`h\Uhmci `\Wb a U]bhU]b`U`gYUa `Ygg`Wcbh]bi Uh]cb`cZmci f``YUfb]b["H\Y'dfcj]g]cb`cZh\Y``YUfb]b[`Ybj]fcba Ybh]b`h\Y cb`]bY`hi hcf]U``gYgg]cbg`]g`hc`ZJW]`]hUhY`XYj Y`cd]b[`WcbZ]XYbWY`]b`a UbU[]b[`XYg][b`HUg_g`Ug`dfc`YWg"`H\Y WcbhYbhXY`]j YfYX`]b`h\Y``YWhi fYg`k]```VY`i gYX`hc`XYg][b`Wcblfc``Yfg`UbX`h\Yb`hc`Udd`mh\Ya `hc`Wcblfc``a cXY`g`cZfYU`!`]ZY`gmghYa g"

5XX]h]cbU`7ci fgY +bZcfa Uh]cb

 $H\Y'ghi\ XYbhg'k\ \c'\Wca\ Y'hc'\Zc'`ck'h\]g'\Wci\ fgY'\UfY'fYei\]fYX'hc'\Uj\ Y'df]cf''_bck'YX[\ Y'\Zfca''A\ A\ 5B' \ \&\$$ @]bYUf'GmghYa\ g'\UbX'7\ cbhfc``cf'\U'g]a\]'Uf'\Wci\ fgY'''$

 $\label{eq:hamiltonian} $$H\setminus g^*]g^*U^**ib]hc^{Z}W^*fX]hflc^{Z}W^*fX]hflc^{Z}W^*fX]hflc^{Z}U^*g^*UbX^!bjc^{Y}Yg^*Ucifg^*dYf^kYY_f^k& L^cZgW_YXi^YX^*cb^{D}Y^*W^bhUW^*HY^*bcfaU^*kcf_^cUX^YldYW^Uhlcbg^cZU^*ghXYbhUfY^Uddfcl]aUhY^m&)^*\cifg^*dYf^*hYfa^{Z}cf^YUWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}g^*UWX^!C^{Z}$

Mci 'g\ci `X'U]a 'hc'gdYbX'UVci h%) '\#k 'cb'h\]g'Wci fgY"'H\Y'UXX]h]cbU``h]a Y'g\ci `X'VY'gdYbh]b'a U_]b[gi fY'h\Uhmci 'i bXYfghUbX'h\Y'`YWh fY'a UhYf]U'ž'Wca d`Yh]b['h\Y'gYhUgg][ba Ybhgž'\land\final fh\Yf'fYUX]b[ž'UbX fYj]g]b['\alphacf'UbmYl Ua]bUh]cbg"

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'HJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
Ei]n'.'@YWhifY'J]XYc Ei]nnYg'	%\$I	Bch5dd`]WUV`Y	
Ei]n'.'A U'cf'Ei]n'DUfh%	%\$I	H65	%ž'&
@UV`%∵ ≠j YfhYX`DYbXi`i a 9I dYf]a Ybh	%\$I	&-#\$'#&\$&%'%%)- 'DA	%ž &
@UV`&.`GdYYX`7 cblfc` 9I dYf]a Ybh	%) ।	%*#\$(#&\$&%`%%)- DA	%ž'&ž''
Ei]n'.'A U'cf'Ei]n'DUfh&'	%) ၊	H65	%ž'&ž''
@UV" . 'Dcg]h]cb'7 cbhfc` 91 dYf]a Ybh	&\$I		

8 YHU]`g. H\Y`ghi XYbhg`k]```VY`fYei]fYX`hc`XYg][b`U`Wcbhfc``Yf`UbX`]a d`Ya Ybhi]hcb`Ub`UWh U``YI dYf]a YbhU`f]["H\Y`YI dYf]a YbhU``f][Wcbg]ghg`cZUb`]bj YfhYX`dYbXi`i a `UbX`h\Y`Wcbhfc``gmghYa g`XYg][bYX`VmYUWX]bX]j]Xi U``ghi XYbhi]g`YI dYWhYX`hc`\c`X`h\Y`dYbXi`i a `ghYUXmk\]Y`h\Y`WUfhcb`k\]W\`]h]g`a ci bhYX`a cj Yg"H\Y`XUhU`Wc``YWhYX`ZcfYUW\`ghi XYbhk]``VY`a UXY`Uj U]`UV`Y`Zcf'h\Ya `UbX`h\Yma i ghWca d`YhY`U`fYdcfh'

Gi Va]gg]cb bch/g. Gh XYbhg a i ghgi Va]hU fYdcfhUg dYf h Y Yl dYf]a YbhgdYW[Z]WUh]cb"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$

5ggYgga Ybh(. @UV & GdYYX 7cbffc 9l dYf]a Ybh

GhJfhiXuhY. "H\Y "`UVcfUhcfm'WUggYg'k]" "HJ_Y'd'UWY'Xi f]b['k YY_'+

8 YHU]`g. H\Y`ghi XYbhg`k]```VY`fYei]fYX`hc`XYg][b`U`Wcblfc``Yf`UbX`]ad`YaYbh]hcb`Ub`UWhU`YldYf]aYbhU`f]["'H\Y`YldYf]aYbhU`f][o`U`Wbg]ghg`cZU`gdYYX`Wcblfc``gmghYa`UbX`h\Y`Wcblfc``gmghYag`XYg][bYX`VmYUW\]bX]j]XiU``ghi XYbh]g`YldYWhYX`hc`aU]bHJ]b`h\Y`gdYYX`cZh\Y`gmghYa`XYgd]hY``cUX`jUf]Uh]cbg"'H\Y`XUhU\Wc`YWhYX`Zcf`YUW\ghi XYbhk]``VY`aUXY`UjU]`UV`Y`Zcf`h\Ya`UbX`h\YmaighWad`YhY`U`fYdcfh'

Gi Va]gg]cb bch/g. H\Y gh XYbhg a i ghk f]h/ U fYdcfhUg dYf h\Y dcg]h]cb Wcblfc Yl dYf]a YbhgdYWJZWUh]cb

Hi fb]h]b gYh]b[. H\]g Ugg][ba Ybh]g gi Va]hhYX h\fci [\ Hi fb]h]b UbX gh XYbhg Xc bchgYY Hi fb]h]b g]a] Uf]hmfYdcfhg"

5ggYgga Ybh). Ei]n. A U'cf Ei]n DUfh&

GhUfh:XUhY. '\$-#\$(#&\$&%'\$*.\$\$'DA

8 YHU]`g. H\]gʻei]nʻk]```VYʻWcbXi WHXX`]bʻK YY_`, ʻ: f]XUmVYHk YYbʻ* ʻUbXʻ+ʻda "ʻH\]gʻ]gʻUʻWca di `gcfmei]n UbX`k]```Wcj YfʻU```WcbHYbhWcj YfYX`]bʻk YY_gʻ(ʻ!·+ʻf]bWi X]b[ʻa UHYf]U``dfYgYbHYX`]bʻdfY!`YWfi fYʻj]XYcgž `YWfi fYʻj]XYcgž'hi hcf]U``WUggYgʻUbX``UVʻWUggYgʻ]ZUbmŁ"'H\Y`WcbX]h]cbgʻi bXYf`k \]WX`h\]gʻei]n`k]```VY WcbXi WHXX`k]```VY`Ubbci bWYX`WcgYf`hc`h\Y`ei]n"``

5ggYgga Ybh'*. @UV'' . Dcg]h]cb 7cbhfc`'9l dYf]a Ybh

GhJfhiXUhy. 'H\Y 'YI dYf]a Ybhik]``'hU_Y 'd`UWY ']b 'k YY_'-

8 YHJ]`g. H\Y`ghi XYbhg`k]```VY`fYei]fYX`hc`XYg][b`U`Wcblfc``Yf`UbX`]a d`Ya Ybhi]hicb`Ub`UWi U``YI dYf]a YbHU`f]["'H\Y`YI dYf]a YbHU`f]['Wcbg]ghg`cZU'dcg]h]cb`Wcblfc``gmghYa 'UbX`h\Y`Wcblfc``gmghYa g`XYg][bYX`VmYUW\]bX]j]Xi U``ghi XYbhi]g`YI dYWYX`hc`dcg]h]cb`U`fchUh]b['Ufa 'UVW' fUhY`m''H\Y`XUHU`Wc``YWYX`Zcf`YUW\`ghi XYbh k]```VY`a UXY`Uj U]`UV`Y`Zcf`h\Ya 'UbX`h\Yma i ghiWca d`YhY`U`fYdcfh'

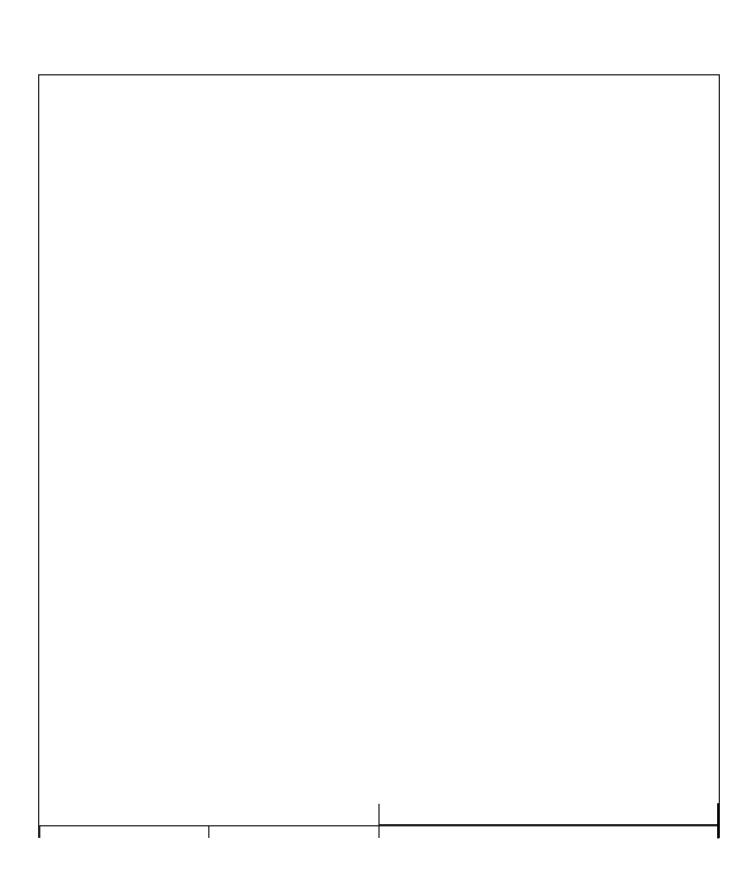
Gi Va]qq]cb bchYq. H\Y gh XYbhq a i ghk f|hY U fYdcfhUq dYf h\Y dcq|h|cb Wcblfc`Yl dYf]a YbhqdYW[Z]WUh|cb

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhf]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\"Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$

5ggYgga Ybh+. Ei]n . A U'cf Ei]n DUfh'

GhUfhiXUhY. '&' #\$(#&\$&%'\$*.\$\$'DA

8YHU]`g. H\]gʻei]nʻk]```VY`WcbXi WhYX`cbʻK YY_`%\$`: f]XUmVYkk YYbʻ* `UbX`+`da "`H\]gʻ]gʻU`Wca di `gcfmei]n UbX`k]```Wcj Yf`U```WcbHYbhWcj YfYX`]b`k YY_gʻ, `!`%\$`f]bWi X]b['a UHYf]U``dfYgYbHXX`]b`dfY!`YWi fY`j]XYcgž `YWi fY`j]XYcgžhi hcf]U``WUggYgʻUbX``UV`WUggYgʻ]ZUbmb"`H\Y`WcbX]h]cbgʻi bXYf`k \]W\`h\]gʻei]n`k]```VY WcbXi WHXXk]```VY`Ubbci bWXX`WcgYf`hc`h\Y`ei]n"``



K YY_''%&'5df]`'!'%*	5ggYgga Ybh	K YY_'-'A]bcf'ei]n
5df]`	Hi H@UV	Dcg]hjcb 7 cblfc 9 dYfja Ybh
	@YWhi fY	±bX]fYWfi8Yg][b.'8]gWfYhY'7cbhfc``Yf'8Yg][b'lg]b[

5d* AY fell !%

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

DfY fYWcfXYX "YWfi fY j]XYcg UbX "YWfi fY bchYg k] "VY a UXY Uj U] UV Y cb A ccX Y "

FYWca a YbXYX FYgci fW/g

H\YZc``ck]b[hYI hVcc_g a UmVY i gYXZcf fYZYfYbW\"

%"8 cfgYmž >"ž"7 cbhjbi ci gʻUbX`8]gWfYhY`7 cbhfc``GmghYa g"ž'A W fUk`<]`` &"; c`hYbž'>"`UbX`5"`J Yfk Yfž''7 cbhfc``GmghYa '8 Yg][b`UbX`G]a i `UhJcb"`A W fUk`<]``

7 ci fgY 9j U`i Uhcb UbX 8Yj Y`cda Ybh

: YYXVUW_cb^h\Y`Wci fgY^]g^[Uh\YfYX'dYf]cX]WU``mi g]b['j Uf]ci g^a YUbgž]bWi X]b['h\Y'I BGK a m9I dYf]YbW' dfcW'ggž]bZcfa U``X]gW gg]cb^]b^h\Y`Z]bU``WUgg^Zcf^h\Y`Wci fgYžUbX'h\Y`GW\cc^*g Gh XYbh#GhUZZa YYh]b[g"'Mci f^ZYYXVUW_]g^hU_Yb^gYf]ci g`mžUbX'Wcbh]bi U``]a dfcj Ya Ybhg^UfY a UXY^hc^h\Y Wci fgY^VUgYXž]b^dUfhžcb^gi W\ZYYXVUW_"

±0 'h\]g'Wci fgYž'fYWYbh]a dfcj Ya Ybhg'fYgi `h]b['Zfca 'ghi XYbh'ZYYXVUW_']bWi XY'Wca d`YhY'X][]hU`'i d`]Zh]b['cZh\Y'Wci fgY"

@UVcfUhcfmK cf_g\cd \(\dag{b}\)Zcfa Uhlcb

5````UVcfUhcfm'WUggYg'k]```HU_Y'd`UWY']b'Fcca '&\$('cZK]``]g'5bbYl''

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

5ggYgga Ybhgi Va]gg]cb UbX a Uf_]b[Wf]hYf]U

G\ci`X'h\Y'WcifgY'\UjY'Ubmbcb!Y`YWfcb]WUggYgga Ybh'gi Va]gg]cbž'h\YgY'g\ci`X'\UjY'U'ghUbXUfX GW\cc`'WcjYf'g\YYh'

5```gi Va]gg]cbg`UfY`YI dYWYX`lc`VY`bYUh`UbX`WYUf`mgYhci h'`Mci f`fYgi `hg`UfY`h\Y`d]bbUWY`cZU```mci f`\UfX k cf_'UbX`g\ci `X`VY`lfYUh'X`k]h\`Xi Y`fYgdYWf'`DfYgYbh]b[`fYgi `hg`WYUf`m[]j Yg`h\Y`a Uf_Yf`h\Y`VYghW\UbW' cZi bXYfgHUbX]b[`mci f`a Yh\cX/Yj Yb`]Zh\Y`bi a Yf]WU``fYgi `hg`UfY`]bWcffYWf

A Uf_]b['[i]XY`]bYg`Zcf`Ugg][ba Ybhgi Va]gg]cbg`k]```VY`dfcj]XYX`Uhh\Y`gUa Y`h]a Y`Ug`Ugg][ba YbhXYhU]`g hc`Ugg]ghk]h\`a YYh]b['UggYggUV`Y`fYei]fYa Ybhg"`Gi Va]gg]cbg`k]```VY`a Uf_YX`UVWcfX]b['hc`h\Y`a Uf_]b[[i]XY`]bYg`dfcj]XYX"

@UhY 'dc`]Wh

K cf_'gi Va]hhYX``UhY`k]h\ci hUb`Uddfcj YX`YI hYbg]cb`Vmh\Y`Wci fgY`WccfX]bUhcf`cf`XY`Y[UhYX`Ui h\cf]hm]g gi VYWihc`U``UhY`dYbU`hmcZ&\$`dYfWbhf&\$i \ \L'cZh\Y`a Ul]a i a 'a Uf_'dcgg]V`Y`Zcf`h\UhUggYgga Ybh]hYa ž dYf`WU`YbXUf`XUm'

 $\label{thm:condition} $$HY^UHY'dYbU'mm]g'Udd']YX'dYf'WUYbXUf'XUmf]bWi X]b['k YY_YbXg'UbX'di V']W\c`]XUmgŁ'h\Uh'h\Y UggYgga Ybh']g'cj YfXi Y'''H\YfY']g'bc'dfc!fUhU'cZh\Y''UhY'dYbU'mmZcf'gi Va]gg]cbg'a UXY'dUfhk Umh\fci [\'UXUm'$

 $\label{eq:conversion} \text{K cf_'gi Va]} \text{httx:'UZhtf:h.Y:'XYUX`]bY:'Zcf:'UVgc`i htt:'ZU]'':]g'bchUWWtdhtx:'UbX:'U'a Uf_'cZ'nYfc'k]```VY'Uk UfXYX Zcf:h.Uh:'UgqYgga Ybh']hta " } \\$

: cf`gca Y`UggYgga Ybh`]hYa gž`U``UhY`dYbU`hma UmbchVY`Uddfcdf]UhY"`H\YgY`UfY`WYUf`m`]bX]\WUhYX`]b`h\Y Wci fgY`ci h`]bYž`UbX`gi W\`UggYgga Ybhg`fYWY]j Y`U`a Uf_`cZnYfc`]ZbchWca d`YhYX`Vmh\Y`gdY\\JZ]YX`XUhY" 9I Ua d`Yq`]bWi XY.

%" K YY_`micb``]bY`hYghg`cf``UVcfUhcfmik cf_`k cfh\`U`ga U```dfcdcfh]cb`cZh\Y`gi VYWha Uf_žcf &" C b`]bY`ei]nnYg`k \YfY`Ubgk Yfg`UfY`fY`YUgYX`hc`ghi XYbhg`cb`Wca d`Yh]cbžcf

- ' "DfcZYgg]cbU``UggYgga Ybh'hUg_gž'k\YfY`h\Y`]bhYbh]cb`]g`hc`WfYUhY`Ub`Ui h\Ybh]WUggYgga Ybh'h\Uh\Ug`Ub`UVgc`i hY`gi Va]gg]cb`XUhYžcf
- ("DUgg# U]`UggYgga YbhHJg_g"

91 Ua]bUh]cbg

Mci 'a i gh'VY'Uj U]`UV`Y'Zcf'U```ei]nnYgž'hYghg'UbX`YI Ua]bUh]cbg": cf'Wci fgYg'h\Uh\Uj Y'Z]bU``YI Ua]bUh]cbgž h\YgY'UfY'\Y`X'Xi f]b['h\Y'I b]j Yfg]hmYI Ua]bUh]cb'dYf]cXg.': YVfi UfmZcf'Gi a a Yf'HYfa ž'A UmZcf'H%ž 5i [i gh'Zcf'H&ž'UbX`Bcj Ya VYf#8 YWYa VYf'Zcf'H' "

D`YUgY'j]g]ha m BGK 'Zcf'Dfcj]g]cbU``91 Ua]bUh]cb'h]a YhUV`Y'di V`]g\'XUhYg": cf'Z fh\Yf']bZcfa Uh]cb'cb Yl Ua gž'd`YUgY'gYY'h\Y'<u>91 Ua g</u>'k YVdU[Y"

GdYWJU`7cbg]XYfUhJcb

⊋mci `\Uj Y`YI dYf]YbW/X`Ub`]``bYgg`cf`a]gUXj Ybh fY`VYmcbX`mci f`Wcblfc``h\Uhk]```]bhYfZYfY`k]h\`mci f

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYWU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWU gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WD`'Wfh]ZJWUh'g'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYž cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'Wfh]ZJWUh'g'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWfggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}xYfUh$

5 WUXYa]W<cbYglmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg``UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVWY] WWSege[YfZW_aXSekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``h]a Y'a UbU[Ya Ybh']g'Ub']a dcffUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh''Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\ž'XfUZh]b['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fWYg']b'dfYdUf]b['U``'UggYgga Ybh'lUg_g''

⊋d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci i'UfYi]biZ[fghmYUfžmci fi`YVWi fYfik]`icZYfimci i'Ugg]gHUbWYihc]a dfcj Yimci fi'UWUXYa]Wg_]``g"'H\Yma Um'Ug_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWYgžiUhhYbXih\Yi@YUfb]b[7 YbhfYžcfiqca Yha YqifYqi Va]himci fik cf_ik]h\ih\YidfcV`Ya iZ||YX"'<ck Yj Yfia cfYiqYf]ci qi]bqhUbWYqi]biZ[fqh

a dcfhUbh'@]b_g