

MECH4100

A YWXUb]WU`8Yg][b'&

HYfa "Hk c"#"&\$&%

7ci fgY[·]Cj Yfj]Yk

GhJZZ 7 cbhJWh 8 YhJ] `g

7cbj Ybcfg

| B Ua Y | 9a U]` | 5j U]`UV]`] m | @cWJhjcb | D\cbY |
|----------------------------|-----------------------------------|----------------------|--|-------|
| ≢YbY`F YbUi X! 5ggYa Uh |]"fYbUi XUggYa Uh4 i bgk "YXi "Ui | | 5]bgk cft\\ 6i]`X]b[`>%+ž F cca `&\$, 7 | |

@YWhi fYfg

| B Ua Y | 9a U]` | 5j U]`UV]`]lm | @cWJhjcb | D\cbY |
|-----------|----------|---------------|----------|-------|
| 8Ub]Y`9[(|]bgk cfh | | | |
| | | | | |
| | | | | |

fŽ*%%&L'-',) " %+-'-'I BGK "Gh Xm5VfcUX"UbX"I BGK "91 WXUb[Y"fZcf"]bVci bX"gh XYbhgŁ

 $f\check{Z}^*\%\&\&`-',)`(\$-+\check{-}`GWXcc``CZZ]WYH$

 $\label{thm:condition} $$H^*D^*YUgY^*bcH'^*h.Uh^*h.Y^*GW.cc^*CZZWY^*k]^*^*bch_bck^*k.Yb\#Zmcif^*WcifgY^*WcbjYbcf^*]g^*cb^*WUadig^*cfUjU]^*UV^*Y$$$

9a U]`

9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]W/g'-W ffYbhgh XYbhYbei]f]Yg

•

7ci fgY 8YW] g

7 fYX]hDc]bhg**

Giaa UfmcZh\Y 7ci fgY

±0 'th\]g'Wci fgYz'ghi XYbhg'k]``i bXYfhU_Y'U'a U'c'f'XYg][b'dfc'YWh'Ug'U'hYUa z'YbUV`]b['th\Ya 'hc'XYj Y`cd WcbWYdhi U``UbX'XYhJ]`YX'gc`i h]cbg'Zcf'Ub'Yb[]bYYf]b['dfc'YWbzi h]`n]b['UbX'Yl hYbX]b['th\Y]f'_bck `YX[Y'UbX g_]``g'UWei]fYX'h\fci [\ci hth\Y'XY[fYY'dfc[fUa a Y"'H\]g']bWi XYg'dfc'YWhVugYX'XYg][b'dfcWyggz'dfc'YWhau UbU[Ya Ybhz'XYg][b'Zcf'a Ubi ZUWhi f]b[z'fY`]UV]`]mz'`UZZcfXUV]`]mz'`UbX'gi ghU]bUV]`]mn'

CLIVAKYUJHUJ KŽĮģiti f\\ULDJinYWA\&Zir\ck\JWHUPf\b[Utd)Kc\BrithWBfUFAi]b\b]\k\fta\Y`cj YfU```UWfj Jh]Yg`cZh\Y`hYUa ž`Ug`**%&**```**%**@ WccfX]bUhY`h\Y`cj YfU```X]fYWf]cb`cZh\Y`hYUa ž`UbX`fYdcfhih\Y]f`dYfZcfa UbWY`h\fci [\'dfchchmdYa Ubi ZJWh f]b[`UbX`hYgh]b[žk f]hhYb`XY`\\ %• Y`

| | • | |
|--|---|--|

Fygci fWyg

FYWca a YbXYX FYgci fWyg

Gi [[YghYXfYUX]b[fughUfh]b[dc]bhcb`mŁ

91 W*``Ybh'UXj]\W`z`XYg\\f]dh\cbg`UbX`[i]\XY`]b\Ygž\VUg\X`cb`dfU\\f]\\U\`YI d\Yf]\Yb\\``Uf\Y`[]j \Yb']b'? S] [` \Y`E f[d\[` \Y 7` \Y[` \We \Vm5\b\XmF cggž\k \]\\\ '\\W\b'\Y\'c\Vh\]\b\Y\`\Ug'\U'\Z\Y\`\Xck b`c\UX'\Zc``ck]\b[\d\[[\Y .

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

5ggYgga Ybhgi Va]gg]cb UbX a Uf_]b[Wf]hYf]U

G\ci`X'h\Y'WcifgY'\UjY'Ubmbcb!Y`YWfcb]WUggYgga Ybhigi Va]gg]cbž'h\YgY'g\ci`X'\UjY'U'ghUbXUfX GW\cc`WcjYf'g\YYh'

5```gi Va]gg]cbg`UfY`YI dYWYX`hc`VY`bYUh`UbX`WYUf`mgYhci h'`Mci f`fYgi `hg`UfY`h\Y`d]bbUWY`cZU```mci f`\UfX k cf_'UbX`g\ci `X`VY`hfYUhYX`k]h\`Xi Y`fYgdYWf`DfYgYbh]b[`fYgi `hg`WYUf`m[]j Yg`h\Y`a Uf_Yf`h\Y`VYghW\UbW\ cZi bXYfghUbX]b[`mci f`a Yh\cX/Yj Yb`]Zh\Y`bi a Yf]WU``fYgi `hg`UfY`]bWcffYWf

A Uf_]b['[i]XY`]bYg`Zcf`Ugg][ba Ybhgi Va]gg]cbg`k]```VY`dfcj]XYX`Uhh\Y`gUa Y`h]a Y`Ug`Ugg][ba YbhXYhU]`g hc`Ugg]ghk]h\`a YYh]b['UggYggUV`Y`fYei]fYa Ybhg"`Gi Va]gg]cbg`k]```VY`a Uf_YX`UVWcfX]b['hc`h\Y`a Uf_]b[[i]XY`]bYg`dfcj]XYX"

@UhY 'dc`]Wh

K cf_'gi Va]hhYX``UhY`k]h\ci hUb`Uddfcj YX`YI hYbg]cb`Vmh\Y`Wci fgY`WcfX]bUhcf`cf`XY`Y[UhYX`Ui h\cf]hm]g gi VYWhhc`U``UhY`dYbU`hmcZ&\$`dYfWYbhf&\$i \ \!cZh\Y`a Ul]a i a 'a Uf_'dcgg]V`Y`Zcf`h\UhUggYgga Ybh]hYa ž dYf`WU`YbXUf`XUm'

 $\label{eq:conversion} \text{K cf_'gi Va]} \text{httx:'UZhtf:h.Y:'XYUX`]bY:'Zcf:'UVgc`i htt:'ZU]'':]g'bchUWWtdhtx:'UbX:'U'a Uf_'cZ'nYfc'k]```VY'Uk UfXYX Zcf:h.Uh:'UgqYgga Ybh']hta " } \\$

: cf`gca Y`UggYgga Ybh`]hYa gž`U``UhY`dYbU`hma UmbchVY`Uddfcdf]UhY"`H\YgY`UfY`WYUf`m`]bX]\WUhYX`]b`h\Y Wci fgY`ci h`]bYž`UbX`gi W\`UggYgga Ybhg`fYWY]j Y`U`a Uf_`cZnYfc`]ZbchWca d`YhYX`Vmh\Y`gdY\\JZ]YX`XUhY" 9I Ua d`Yq`]bWi XY.

%" K YY_`mcb`]bY`hYghg`cf``UVcfUhcfmk cf_`k cfh\`U`ga U```dfcdcfh]cb`cZh\Y`gi VYWha Uf_žcf &" C b`]bY`ei]nnYg`k \YfY`Ubgk Yfg`UfY`fY`YUgYX`hc`gh XYbhg`cb`Wca d`Yh]cbžcf

- "DfcZYgg]cbU``UggYgga YbhihUg_gž'k\YfY`h\Y`]bhYbh]cb`]g`hc`WfYUhY`Ub`Ui h\Ybh]WUggYgga Ybhih\Uh\Ug`Ub`UVgc`i hY`gi Va]gg]cb`XUhYž'cf
- ("DUgg# U]`UggYgga YbhHJg_g"

91 Ua]bUh]cbg

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYWU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WD`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WD`'ghJZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUhlcb^{$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[`U`W`hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg`\UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{\WSeaXafZWbeS`VbSee[YfZW_aX\Sekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``h]a Y'a UbU[Ya Ybh]g'Ub']a dcftUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fWYg']b'dfYdUf]b['U``'UggYgga Ybh'lUg_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg_]``g'''H\Yma UmUg_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf_icfidUm]b[igca YcbYihciXcimci fik cf_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]\\makewaa Ybhg#gh XYbha]g\\cbXi \\makebXi \\makebXi fYg"dXZ

5WUXYa]W±bZcfaUhjcb

7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$

- A ccX`Y
- <u>@UV'5WWYgg</u> 7 ca di hb[': UW]hYg

5ddYbX]I . '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

| Dfc[fUa =bhybXYX=@YUfb]b['Ci hWca Yg | |
|--|--|
| ?bck `YX[Y`UbX'g_]```VUgY | |
| D9%"%7ca dfY\Ybg]j Yž'h\YcfmVUgYX'i bXYfghUbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]\WJ`'g\WJYb\Wg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbhJ`g'Udd`]\WJV`Y'hc'h\Y'Yb[]bYYf]b['X]g\WJd`]bY | |
| D9%"&"7cbWYdh U``i bXYfgHJbX]b['cZh\Y'a Uh\Ya Uh]Wgžbi a Yf]WJ``UbU`mg]gžgHJh]gh]Wgž'UbX Wca di hYf`UbX`]bZcfa Uh]cb'gW]YbWYg'k\]W\'i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY | |
| D9%" '=b!XYdh\'i bXYfghUbX]b['cZgdYW]U`]ghVcX]Yg'cZ_bck `YX[Y'k]h\]b'h\Y'Yb[]bYYf]b[X]gW]d`]bY | |
| D9%'('8]gWfba YbhcZ_bck `YX[Y 'XYj Y`cda YbhUbX'fYgYUfWX 'X]fYWf]cbg'k]h\]b'h\Y Yb[]bYYf]b['X]gW]d`]bY | |
| D9%) '? bck 'YX[Y'cZYb[]bYYf]b['XYg][b'dfUWjWY'UbX'WcbhYl hi U`'ZJWcfg']a dUWjb['h\Y Yb[]bYYf]b['X]gWjd`]bY | |
| D9%"* I bXYfgHJbX]b['cZh\Y'gWcdYž'df]bW]d`Ygž'bcfa gž'UWVci bHJV]`]h]Yg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[]bYYf]b['dfUWfJWY']b'h\Y'gdYW]ZJWX]gW]d`]bY | |
| 9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm | |
| D9&"%5dd`]WUh]cb`cZYgHUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j]b[| |
| D9&"&": `i YbhiUdd`]WUh]cb'cZYb[]bYYf]b['hYVXb]ei Ygžhcc`g'UbX'fYgci fWYg | |
| D9&" '5dd`]WUh]cb`cZgmghYa Uh]WYb[]bYYf]b[`gmbh\Yg]g`UbX`XYg][b`dfcWYggYg | |
| D9&"('5dd`]WUh]cb'cZgmghYa Uh]WUddfcUWXYg'hc'h\Y'WcbXi WhUbX'a UbU[Ya YbhcZYb[]bYYf]b[dfcYWng | |
| DfcZYgg]cbU`'UbX'dYfgcbU`'Ultf]Vi hYg | |
| D9' "%'9h\]WU`'WcbXi Wh'UbX'dfcZYgg]cbU`'UWWci bHJV]`]hm | |
| D9' "&'9ZYWAj Y'cfU`'UbX'k f]MYb'Wcaaib]WUAjcb']b'dfcZYgg]cbU`'UbX'`Um'XcaU]bg | |
| D9' "' '7 fYUhjj Yž']bbcj Uhjj Y'UbX'dfc!UWhj Y'XYa YUbci f | |
| D9' "('DfcZ/gg]cbU`i gY'UbX'a UbU[Ya YbhcZ]bZcfa Uh]cb | |
| D9' ") 'C fXYf'mia UbU[Ya YbhcZgY`Zi'UbX`dfcZygg]cbU``WcbXi Wh | |
| D9' "* '9ZZYWNjj Y'NYUa 'a Ya VYfg\]d'UbX'NYUa 'YUXYfg\]d | |