

MMAN2300

9b[]bYYf]b['A YW\Ub]Wg'&

HYfa 'Hk c''##'&\$&%

fŽ*%~&Ł~-',) ~(\$-+~-~~GW\cc~~CZZW}}

 $\label{thm:condition} $$H^*D^*YUgY^*bch'Y^*h.Uhhh.Y^*GWkcc^*CZZWY^*k]^*^bch_bck^*k.Yb\#Zmcif^*WcifgY^*WcbjYbcf^*]g^*cb^*WUadig^*cfUjU]^*UV^*Y$$$

9a U]`

9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]W/g - W ffYbhgh XYbhYbei]f]Yg

•

7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`Wcj Yfg`h\Y`dfUWf)WU``Udd`]WUh]cb`cZYb[]bYYf]b[`a YW\Ub]Wg`UbX`a YW\Ub]WU``j]VfUh]cbg"`Hcd]Wg]bWi XY`h\Y`Zc``ck]b[..'D`UbY`_]bYa Uh]Wg`UbX`_]bYh]Wg`cZf][]X`VcX]Yg/Yei Uh]cbg`cZa ch]cbžk cf_`UbX YbYf[m/*±hfcXi Wh]cb`hc`a YW\Ub]WU``j]VfUh]cb/: fYY`UbX`ZcfWYX`fYgdcbgYg`cZg]b[`Y`XY[fYY!cZ\ZYYXca gdf]b[!a Ugg!XUa dYf`gmghYa gžj]VfUh]cb']gc`Uh]cb/< Ufa cb]WUbU`mg]g/@bYUf'j]VfUh]cbg`cZa i `h]!XY[fYY!cZ\ZYYXca 'gmghYa g"

7 ci fgY 5]a g

 $\label{eq:continuity} $$H^{g'W'} fgY'U]a\ g'hc'XYj'Y`cd'mci f'i\ bXYfgHUbX]b[\ cZhY'a\ YWVUb]Wg'cZd`UbUf'f][\]X'VcX]Ygž'a\ YWVUb]ga\ gž'UbX'j]VfUhcfmigmghYa\ g''$

.

7 ci fgY @YUfb]b[Ci hWca Yg

5 ZhYfigi WWYggZ ``mVcad`Yh]b['h\]g'WcifgYžmci 'g\ci `X'VY'UV`Y'hc.

@YUfb]b['Ci HWca Y	95 GW[Y %7ca dYHYbW]Yg
%"'9 d`U]bžXYgWf]VY'UbX'Udd`mdf]bW]d`Yg'UbX'Wca dcbYbhg'cZ 9b[]bYYf]b['A YW\Ub]Wg'i g]b['U'fUb[Y'cZhYW\b]ei Yg	D9%"%ž"D9%"&ž"D9&"%ž"D9' "&
&"`9Id`U]b`UbX`XYgWf]VY`df]bW]d`Yg`UbX`WcadcbYbhg`cZaYW\Ub]WU`j]VfUh]cbg	D9%"%ž"D9%"&ž"D9&"%ž"D9' "&
' "'8]gWfb'h\Y'fY`Yj Ubhdf]bWjd`Yg'h\Uha i ghVY'Udd`]YX'hc'XYgWf]VY h\Y'Yei]`]Vf]i a 'cf'a ch]cb'cZYb[]bYYf]b['gmghYa g'UbX'X]gWf]a]bUhY VYfk YYb'fY`Yj Ubh'UbX']ffY`Yj Ubh']bZcfa Uh]cb']b'h\Y'WcbhYl h	D9%"%žD9%"&žD9&"%
("'8 Ya cbglfUlY'Ub'UV]`]lmlrc'Wca a i b]\WlY'WYUf'mUbX'dfYW]gY`m UVci hlYWkb]\WJ`a UlHYfg'fY`UlYX'rc'9b[]bYYf]b['A YWkUb]\Wg	D9%"*ž'D9' "&

 $\label{eq:hammer_hammer} $$H\Y''H'UWX]b[''gH'UHY[']Yg''H'UHK]$$``VY''I gYX'']bWIXY.$

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'HJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
K YY_`m <ca cf_<br="" yk="">DfcV`Ya g</ca>	%* I	8 UhYgʻgdYWZ]YXʻ]b XYgWf]dh]cb	%ž &ž ' ž')
A ccX`Y`E i]nnYg`	%(1	8 UhYgʻgdYWJZYXʻ]b XYgWfJdhJcb	%ž &ž' ž')
@UVcfUhcfm7`Ugg 5gg][ba Ybhg	' \$ı	8 UhYgʻgdYW]Z]YXʻ]b XYgWf]dh]cb	%ž'&ž'' ž'(ž')
:]bU``91 Ua `			

@Yb[h\.~hc~VY~Wcaaib]WUhYX

8YHJ]`g.

H\]g`]g`Ub`]bX]j]Xi U`UggYgga YbH'
Hk c'ei]nnYg`fk Y][\h'+i YUW\Ł'hc'VY`gi Va]hYX`cb`A ccX`Y"
A ccX`Y`ei]n`%fDUfhi5`8 mbUa]WgŁ`]g`gW\YXi `YX`Uh`*da !+da `fGmXbYmh]a YŁžK YXbYgXUm`]b`k YY_'' `fW*
>i bY`&\$&\L''H\]g`ei]n`k]```]bWi XY`h\Y`WcbhYbhWcj YfYX`]b`k YY_g`%\&"
A ccX`Y`ei]n`&`fDUfhi6`J`]VfUh]cbŁ`]g`g\W\YXi `YX`Uh`*da !+da `fGmXbYmh]a YŁžK YXbYgXUm`]b`k YY_', `f&%'>i `m
&\$&\L'''H\]g`ei]n`k]```]bWi XY`h\Y`WcbhYbhWcj YfYX`]b`k YY_g`)!+ž'd`i g`UbmWcbhYbhZfca `h\Y`XmbUa]\Wg`dUfh
fk YY_g`%\) Ł`fY`Yj Ubhhc`j]\YfUh]cb"
Ghi XYbhg`k]```VY`fYei]fYX`hc`gi Va]hVch\.

•

H\Y`YI Ua``k`]```VY`WcbXi WYX`cb`]bY`fA ccX`YŁ"`Bc``UHY`gi Va`]gg]cb`]g`U``ckYX`Zcf`h\]g`UggYgga YbH``@UHY gi Va`]gg]cbg`k`]```bchVY`UVWYdHYX`UbX`k`]```fYWY]j Y`U`nYfc`f\$Ł'a Uf_"
: i fh\Yf`XYHJ]`g`k`]```VY`dfcj]XYX`VYZcfY`h\Y`YI Ua"

5 hm/b XUb VW F Yei]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$

7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YtUV`Y

8 UhY	HmdY	
0 01 11	11101	

	K cf_g\cd	A cbXUm'%; i `m! 'Hi YgXUm% '>i `m '%ghDfcV'Ya Gc'j]b['GYgg]cb']b'h\Y'k YY_ DUfh6. 'J]VfUh]cb'5bU`mg]g : fYY'G8C: 'j]VfUh]cb'gc`i h]cb'UbX'UbU`mg]g H\i fgXUm'% '>i `m!': f]XUm'* '>i `m '&bX'DfcV'Ya
	K cf_g\cd	Gc`j]b['GYgg]cb']b'h\Y'k YY_ DUfh'6.'J]VfUh]cb'5bU`mg]g : cfWYX'G8C: 'j]VfUh]cb
K YY_', .'%-'>i `m!'&' '>i `n	n@YWhifY	A cbXUm% '>i `m'%gh@YWi fY']b'thY'k YY_ DUfh6.'J]VfUt]cb'5bU`mg]g : cfWfX'G8C: 'j]VfUt]cb'!'Gd

]cX.n.

@YWhi fr	Hi YgXUm' '5i [i gh'&bX'@YWnfY']b'h\Y'kYY_ DUfhg'5/6.'DfUW]WYfYj]Yk
K cf_g\	A cbXUm&'5i [i gh!'Hi YgXUm' '5i [i gh'%ghDfcV'Ya Gc'j]b['GYgg]cb']b'h\Y'k YY_ DUfh6.'J]VfUh]cb'5bU'mg]g J]VfUh]cb'fYWJd'YI YfW]gYg
K cf_g\	H\i fgXUm) '5i [i gh!': f]XUm*'5i [i gh'&bX'DfcV`Ya Gc`j]b['GYgg]cb']b'h\Y'k YY_ DUffg'5/ 6. 'DfUWf]WY'fYj]Yk

UggYgga YbhdYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc`Udd`mZcf`GdYWU``7cbg]XYfUhjcb'df]cf'hc`gi Va]hhjb['Ub UggYgga Ybhcfg]hjb[Ub Yl Ua "

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi 'Y</u>žk\|W\'a YUbg'h\Uh|Zmci 'UhhYa dh'Ub'YI Ua 'cf'gi Va |h'U'd|YWY'cZ UggYgga Ybhžmci 'UfY'XYWUf]b['mci fgY`ZZhYbci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`mZcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYVJU``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhJfm UggYgga Ybhž'd'YUgY'gYY'h\Y']bZcfa Uhjcb'cb'l BGK 'g'GdYWJU`'7cbg]XYfUhjcb'dU[Y"

D`YUgY`bchY`h\Uhgh XYbhg`k]``bchVY`fYei]fYX`hc`dfc † bch

dfW

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W`hifY`cZ`YUfb]b[`]bZcfaYX`Vm`UWJXYa]W]bhY[f]hm'`5``IBGK `ghiXYbhg`UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWJXYa]W]bhY[f]hm'D`U[]Uf]gaibXYfa]bYg`UWJXYa]W]bhY[f]hm`UbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{\WSeaXafZWbeS`VbSee[YfZW_aX\Sekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``h]a Y'a UbU[Ya Ybh]g'Ub']a dcfhUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fW/g']b'dfYdUf]b['U```UggYgga Ybh'hUg_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg_]``g'''H\Yma UmUg_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf_icfidUm]b[igca YcbYihciXcimci fik cf_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha]gWcbXi WhdfcWYXi fYg"dXZ

5WUXYa]W±bZcfaUhjcb

7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$

- A ccX`Y
- <u>@UV'5WWYgg</u> 7 ca di hb[': UW]hYg

5ddYbX]I . '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±bHYbXYX'@YUfb]b['Ci HWca Yg	
?bck `YX[Y`UbX'g_]```VUgY	
D9%%7ca dfY\Ybg]j Yžfh\YcfmVUgYX'i bXYfgfUbX]b['cZfh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]WU`'gWJYbWYg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbfU`g'Udd`]WUV`Y'hc'h\Y'Yb[]bYYf]b['X]gWJd`]bY	
D9%"&"7cbWYdh U``i bXYfgHJbX]b['cZh\Y'a Uh\Ya Uh]Wgžbi a Yf]WJ``UbU`mg]gžgHJh]gh]Wgž'UbX Wca di hYf`UbX`]bZcfa Uh]cb`gW]YbWYg`k\]W\`i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" =: XYdh\ i bXYfghUbX]b[icZgdYW]U`]ghVcX]YgicZ_bck iYX[Yik]h\]bih\ YiYb[]bYYf]b[X]gW]d`]bY	
D9%"('8]gWfba YbhcZ_bck `YX[Y`XYj Y`cda YbhUbX`fYgYUfVX 'X]fYWf]cbg`k]h\]b`h\Y Yb[]bYYf]b['X]gW]d`]bY	
D9%") '? bck `YX[Y`cZYb[]bYYf]b['XYg][b`dfUWf]WY`UbX`WcbHYl hi U``ZUWhcfg`]a dUWf]b['h\Y Yb[]bYYf]b['X]gW]d`]bY	
D9%"* I bXYfgHJbX]b['cZh\Y'gWcdYž'df]bW]d`Ygž'bcfa gž'UWWci bHJV]`]h]Yg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[]bYYf]b['dfUWf]WY']b'h\Y'gdYW]Z]WX]gW]d`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUhjcb`cZYgHUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl 'Yb[]bYYf]b[`dfcV`Ya gc`j]b[
D9&"&": `i YbhiUdd`]WUh]cb'cZYb[]bYYf]b['hYW\b]ei Ygž'hcc`g'UbX'fYgci fWYg	
D9&" '5dd`]WUh]cb`cZgmghYa Uh]WYb[]bYYf]b['gmbh\Yg]g'UbX'XYg][b'dfcWYggYg	
D9&"('5dd`]WUh]cb'cZgmghYa Uh]WUddfcUVXYg'hc'h\Y'WcbXi WhUbX'a UbU[Ya YbhcZYb[]bYYf]b[dfcYWng	
DfcZYgg]cbU`'UbX'dYfgcbU`'UHff]Vi HYg	
D9' "%'9h\]WU`'WcbXi WiUbX'dfcZYgg]cbU`'UWWci bhUV]`]hm	
D9' "&'9ZYWNjjY'cfU``UbX'k f]hhYb'Wcaaib]WUhjcb']b'dfcZYgg]cbU``UbX'`UmXcaU]bg	
D9' "' '7 fYUhjj Yž']bbcj Uhjj Y'UbX'dfc!UWhjj Y'XYa YUbci f	
D9' "('DfcZ/gg]cbU`i gY'UbX'a UbU[Ya YbhcZ]bZcfa Uh]cb	
D9' ") 'C fXYf`ma UbU[Ya YbhcZgY`Zë'UbX`dfcZYgg]cbU``WcbXi Wh	
D9' "* '9ZZYW¶j Y'HYUa 'a Ya VYfg\]d'UbX'HYUa '`YUXYfg\]d	