

# **MMAN4200**

5XX]hjj Y A Ubi ZUWh f]b[

HYfa 'Hk c''##'&\$&%

## 7 ci fgY Cj Yfj ]Yk

## GHJZ:7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
L]UcdYb[ '@]		, ,	Fcca " ‰6ž >‰+	

#### GWXcc~7cbhJWh~abZcfa Uhlcb

#### @cWUhjcb

IBGK AYWXUb]WU``UbX`AUbiZUWfif]b['9b[]bYYf]b[

5]bgk cfh\ 'Vi ]`X]b[ '>%+ž@Yj Y`'%

5Vcj Y 7 c ZZYY cb 7 Ua di g

#### < ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7 `cgYX cb 'di V`]W\c`]XUmgž'GW\cc``gW\YXi `YX'Yj Ybhg'UbX'l b]j Yfg]hmG\i hXck b

#### K YV

GWXcc`cZA YWXUb]WU`UbX'A Ubi ZUWi f]b[ '9b[ ]bYYf]b[

9b[]bYYf]b[]Gh XYbhGi ddcfhGYfi]WYg

9b[]bYYf]b[ \*\DXi glf]U\"HfU]b]b[

I BGK Ghi Xm5VfcUX UbX 91 W. Ub[Y flcf]bVci bX ghi XYbhgŁ

I BGK : i h fY Gh XYbhq

#### D\cbY

fŽ\*%&Ł-',),)\$\$-BiWYigGhXYbh<iV

fŽ\*%:&Ł'-',) '+\*\*%'-'9b[]bYYf]b['=bXi glf]U`'HfU]b]b[

fž\*%%&L-',) ''%+- - I BGK Gh Xm5VfcUX UbX I BGK 91 WXUb[Y flcf]bVci bX gh XYbhgL

fŽ\*%"&Ł'-',) '(\$-+'-'GW\cc`'CZZW}}

łł D`YUgY bchY h\Uhih\Y GW\cc`CZZWY k]`bch\_bck k\Yb#Zmci f'Wci fgY Wcbj Ybcf']g'cb WUa di g'cf

Uj U]`UV`Y

#### 9a U]`

9b[]bYYf]b[ 'Gh XYbhGi ddcfhGYfj ]Wg - W ffYbhgh XYbhYbei ]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbz'WUg\"fYei Yghgz'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

9b[]bYYf]b[] \*\DXi glf]U\"HfU]b]b[] - \*\DXi glf]U\"lfU]b]b[ 'ei Ygl]cbg

LBGK Gh Xm5VfcUX - gh XmUVfcUX gh XYbhYbei ]f]Yg flcf ]bVci bX gh XYbhyb

<u>I BGK '91 WXUb[Y</u>'-gh XYbhYl WXUb[Y'Ybei]f]Yg'fZcf']bVci bX'gh XYbhgŁ

LBGK : i hi fY Ghi XYbha - dch Ybh U`ghi XYbh Ybei ]f]Yg

• Y"["UXa ]gg]cbgžZYYgždfc[fUa gžWfYX]hlfUbgZYf

GWXcc`CZZWY'-GWXcc`[YbYfU`cZZWY'UXa]b]qlfUlqcb'Ybei]f]Yq

B 6. 'h\Y 'fY'Yj UbhihYUa g``]ghYX'UVcj Y'a i ghVY'WcbHUWhYX'Zcf'U``'ghi XYbhYbei ]f]Yg"'H\Y'GW\cc``k ]``
cb`mVY'UV'Y'hc'fYZYf'ghi XYbhg'cb'hc'h\Y'fY'Yj UbhihYUa ']ZWcbHUWhYX

7 ci fgY 8 YHJ]`g 7 fYX]hDc]bhg \*\* Gi a a UfmcZh\Y 7 ci fgY

Zjfgh\UbX'YI dYf]YbWY']b'UXX]hjj Y'a Ubi ZJWh f]b["'

#### 5XX]hjcbU`7ci fgY ±bZcfa Uhjcb

H\Y "YWi fYg Zca 'k YY\_"% hc'( 'k ]" ZcWi gcb'j Uf]ci g'UXX]hj Ya Ubi ZJWi f]b[ 'HYW.b]ei Yg'i d'hc'XUhY'k \YfY mci 'k ]" [U]b 'VUg]W\_bck 'YX[Y'UVci hih\Y'\]ghcfmžXYj Y`cda YbhUbX'Z bXUa YbhU`Yb[]bYYf]b[ 'UgdYWgcZ h\]g'h\W.b]ei Y" H\Y "YWi fY']b k YY\_-, 'k ]" 'VY'U'dfYgYbhUhcb'cb 'h\Y'`UV'dfc YWi'H\Y "YWi fY']b k YY\_- - 'k ]" Wcj Yf'UXX]hj Ya Ubi ZJWi f]b[ 'dfcW/ggcdh]a ]nUhcbž]bWi X]b[ 'a UhYf]U`g'Zcf'UXX]hj Ya Ubi ZJWi f]b[ 'dfcW/ggcdh]a ]nUhcbž]bWi X]b[ 'a UhYf]U`g'Zcf'UXX]hj Ya Ubi ZJWi f]b[ 'ZuVf]WUhYX'Wca dcbYbhgžUbX'Udd']\WhcbgcZUXX]hj Ya Ubi ZJWi f]b[ 'ZuVf]\WhYX'Wca dcbYbhgžUbX'Udd']\WhcbgcZUXX]hj Ya Ubi ZJWi f]b[ 'ZuVf]\Whz'UbX'Ufhg`UbX'XYg][ b"'K YY\_- - 'k ]``'U`gc'Wcj Yf'UXX]hj Ya Ubi ZJWi f]b[ 'XYg][ b''k \YfY'mci 'k ]``' gY'Wca a Yf\yU``mUj U]`UV`Y'gcZhk UfY 'hc'XYg][ b''UXj UbWiX'ghfi Wi fYg Zcf'UXX]hj Ya Ubi ZJWi f]b[ "H\Y'YWi fYg']b k YY\_-'% k ]``']bhfcXi Wi Wi ffYbha U'cf'Udd`]\WJhcbgcZUXX]hj Y

## 5ggYgga Ybh

## 5ggYgga YbhHUg\_g

5ggYgga Ybh'hUg_	K Y][ \h	8iY'8UHY	Gh XYbh@YUfb]b[ CihWca Ygʻ5ggYggYX
: ]bU``YI Ua '	(\$1	Bch5dd`]WUV`Y	%ž'&ž'' ž'(
C b`]bY'ei ]nnYg'	%\$ı	9UWX H\i fgXUm'%\$ Ua GmXbYmHa Y	%ž'&ž' ž'(
5gg][ ba Ybh%	&) ı	K YY_',	%ž'&ž'' ž'(
@UV dfc YWh	&) ı	K YY_',	%ž'&ž'' ž'(

## 5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% : ]bU`Yl Ua

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\., ei Ygh]cbgž&\ci fg

8YHJ]`g. &'\ci fg

5XX]h]cbU`XYhU]`g.

8 YUX`]bY 'Zcf'UVgc`i HY 'ZU]`']g 'b#U

5ggYgga Ybh& Cb`]bY ei ]nnYg

GHJfhiXUhY. '9UVX K YXbYgXUm'% 'Ua 'GmXbYmh]a Y

@Yb[h\.~&rei]nreiYgh]cbgrZcfrYUWkrkYY\_žr%r\cif

8YHJ]`g.

Cb`]bY'ei ]nnYg'VUgYX'cb'k YY\_`m'`YWfi fYg

#### 5XX]h]cbU`XYhJ]`g.

8 YUX`]bY`Zcf`UVgc`i hY`ZJ]``]g`YUW\`H\i fgXUm'%\$`Ua `GmXbYmh]a Y"`A Uf\_g`k ]``VY`fYh fbYX`k ]h\`cbY`k YY\_"

Gi Va ]gg]cb bch/g. Gi Va ]h/X j ]U A ccX Y

5ggYgga Ybh' . 5gg][ ba Ybh%

GhUfh'XUhY. K YY\_'(

@Yb[h\.") \$\$ k cfXg

#### 8 YW] g.

H\]g`Ugg][ ba YbhfYei ]fYg`YUW.`ghi XYbhcf`U`[ fci d`cZghi XYbhg`fXYdYbX]b[ `cb`h\Y`bi a VYf`cZh\Y`Ybfc``YX ghi XYbhgŁhc`k f]hY`Ub`YggUmVUgYX`cb`[ ]j Yb`hcd]Wg`UVci h`UXX]h]j Y`a Ubi ZUWhi f]b[ "`H\Y`hcd]Wg`k ]```VY dfcj ]XYX`hc`h\Y`ghi XYbhg`]b`k YY\_\_`( "

#### 5XX]h]cbU`XYhU]`q.

8 Y U X ] b Y ' Z c f ' U V g c `i ' h Y ' Z U ] ` ' ] g ' ) ' X U mg ' U Z h Y f ' X i Y ' X U h Y " A U f \_ g ' k ] ` ` ' V Y ' f Y h ' f b Y X ' k ] h ' h k c ' k Y Y \_ g "

Gi Va ]gg]cb bchYg. Gi Va ]gg]cb j ]U A ccXY

5ggYgga Ybh ( @UV dfc YWh

GHJfhXUhY. K YY (

@Yb[h\.'%'[fciddfygYbhUh]cb'UbX'dfcYWhfYdcfh

#### 8 YW] g.

: cfih\Y'cb`]bY'@UV'dfcYWfighi XYbhgik]``VY'X]j ]XYX']bhc'gYj YfU`'[ fci dg'UbX'U'ga U``žZYl ]V`Y'dfcYWfik]``VY [ ]j Yb'hc'YUW\ [ fci d"'9UW\ 'dfcYWfik]``VY'ZcW gYX'cb'dc`ma Yf'cfia YhU`'UXX]hjj Y'a Ubi ZJWfi f]b[ 'k\YfY'mci k]``bYYX'hc'XYg][ b'U'fYU``Wca dcbYbhZcf'' 8'df]bhjb["'Mci 'k]``U'gc'bYYX'hc'HU`\_'UVci h\ck 'mci f'[ fci d k cf\_'hc[Yh\Yf'hc'XYg][ b'UbX'ZJVf]WUhY'h\Y'Wca dcbYbhi g]b[ '' 8'df]bhYfg']b'mci f'cb`]bY'[ fci d'dfYgYbhUh]cb" H\Y'UggYgga YbhZcf'h\Y'dfcYWfik]``VY'VUgYX'cb'h\Y'hYUa'g'k cf\_žmci f'i bXYfghUbX]b['cZh\Y'' 8'df]bhJb[ dfcWfggžUbX'mci f'[ fci d'cb`]bY'dfYgYbhUh]cb"

#### 5XX]h]cbU`XYhU]`g.

8 YUX`]bY 'Zcf'UVqc`i HY 'ZJ]``]q') 'XUmq'UZHYf'Xi Y 'XUHY"'A Uf\_q'k ]``VY 'fYhi fbYX'k ]h\ 'k c'k YY\_q"

Gi Va ]gg]cb bchYg. Gi Va ]gg]cb j ]U HYUa g

## FYgci fW/g

## FYWca a YbXYX FYgci fWfg

U"'I BGK "@]VfUfmik YVg]hY. `\hdg.#k k k "`]VfUfmii bgk "YXi "Ui # V"'A ccX`Y. `\hdg.#\a ccX`Y"hY`hii bgk "YXi "Ui #c[]b#\bXYI "d\d

#### 7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

: YYXVUW\_cb^h\Y`Wci fgY`]g`[ Uh\YfYX`dYf]cX]WU``mi g]b[ 'j Uf]ci g`a YUbgž]bWi X]b[ 'h\Y`I BGK a m9l dYf]YbW'`dfcW'ggž]bZcfa U``X]gW gg]cb^]b^h\Y`Z]bU``WUgg`Zcf`h\Y`Wci fgYžUbX`h\Y`GW\cc`'g Gh XYbh#GhUZZa YYh]b[ g"'Mci f`ZYYXVUW\_`]g`hU\_Yb`gYf]ci g`mžUbX`Wcbh]bi U``]a dfcj Ya Ybhg`UfY`a UXY`hc`h\Y Wci fgY`VUgYXž]b`dUfhžcb`gi W\`ZYYXVUW\_"

#### @UVcfUhcfmK cf\_g\cd \DZcfa Uh]cb

Cb`]bY`dfc^YWg`k]```VY`Ugg]ghYX`Vmh\fYY`XYacbghfUhcfg`UbX`gcZkUfY`dUW\_U[Y`k]```U`gc`VY`dfcj]XYX`Zcf

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]hhJb['UbUggYgga Ybhicf'g]hhJb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b[ 'Zcf`GdYW]U``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYWJU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b[ 'cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}XYfUhlcb^{3}xYfUhlcb^{3}xYfUhlcb^{3}XYfUh$ 

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa] ha Ybhhc`ZcghYf]b[``U`W`hi fY`cZ`YUfb]b[`]bZcfa YX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghi XYbhg`\Uj Y`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'UbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[e VWX]`WWSege[`YfZWiadVead[VWSeaXafZWbeS`VbSee[`YfZW_aXSekagdai`ž$ 

D`U[]Uf]ga `]g`U'hmdY'cZ]bhY``YWi U``h\YZh''±i\Wb'hU\_Y'a UbmZcfa gžZfca `XY`]VYfUhY'W\YUh]b['hc'UWW]XYbhU``m Wcdm]b['Zfca 'U'gci fW'k]h\ci h'UW\_bck `YX[Ya Ybh''l BGK '\Ug'dfcXi WYX'U'k YVg]hY'k]h\'U'k YU`h\'cZ fYgci fW'g'hc'gi ddcfhighi XYbhg'hc'i bXYfghUbX'UbX'Uj c]X'd`U[]Uf]ga žj]g]h'

## 5WUXYa]W±bZcfaUhjcb

## 7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$ 

- A ccX`Y
- <u>@UV'5WWYgg</u> 7 ca di hb[': UW]hYg

# $5\,ddYbX]I.~9b[~]bYYfg~5i~ghfU`]U'f95Ł'DfcZYgg]cbU`~9b[~]bYYf~7\,ca~dYhYbWm~~\chl_bXL_bfXd$

Dfc[fUa '±ohYbXYX'@YUfb]b['CihWca Yg				