

MMAN9001

A $UghYf^*cZ9b[]bYYf]b[~GW]YbWW^*Dfc^YWh5$

HYfa 'Hk c''##'&\$&%

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUh]cb	D\cbY
Fcb [.] 7\Ub	f"W. Ub4 i bgk "YXi "Ui	#i]g fyWca a ybxyx mci 'Ya U] 'h\y Wci fgy Wcbj ybcf h a U_Y 'U'gdyWJZJW Uddc]bha ybh]Zmci byyx hc X]gW gg Ub]a dcffUbh]ggi yg"	5]bgk cfh\ 6i]`X]b[-',)%)')

GWXcc`7cbHJWhi±bZcfa Uh]cb

@cWUh]cb

I BGK 'A YVX.Ub]WU``UbX'A Ubi ZUWf f]b['9b[]bYYf]b[

5]bgk cft\\ 'Vi]`X]b['>%+ž'@Yj Y`'%

5Vcj Y 7 c ZZYY cb 7 Ua di g

<ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

K YV

GWXcc``cZA YWXUb]WU``UbX`A Ubi ZUWi f]b['9b[]bYYf]b[

fž*%*&Ł'-',) '(\$-+'-'GW\cc`'CZ]\\\I

 $\label{thm:linear_continuity} $$ \frac{1}{D}YUgY'bchY'h\Uhih\Y'GW\&cc`'C\Z_JWY'k\]``bch_bck'k\Yb\#Zmci\ f'Wci\ fgY'Wcbj\Ybcf']g'cb'WUa\ di\ g'cf'Uj\UJ'UV'Y}$

9a U]`

9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]WYg - W ffYbhgh XYbhYbei]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

9b[]bYYf]b[*\DXi glf]U``HfU]b]b[-- *\DXi glf]U``lfU]b]b['ei Ygl]cbg

<u>I BGK "Gh Xm5VfcUX"</u>—gh XmUVfcUX gh XYbhYbei]f]Yg flcf]bVci bX gh XYbhgŁ

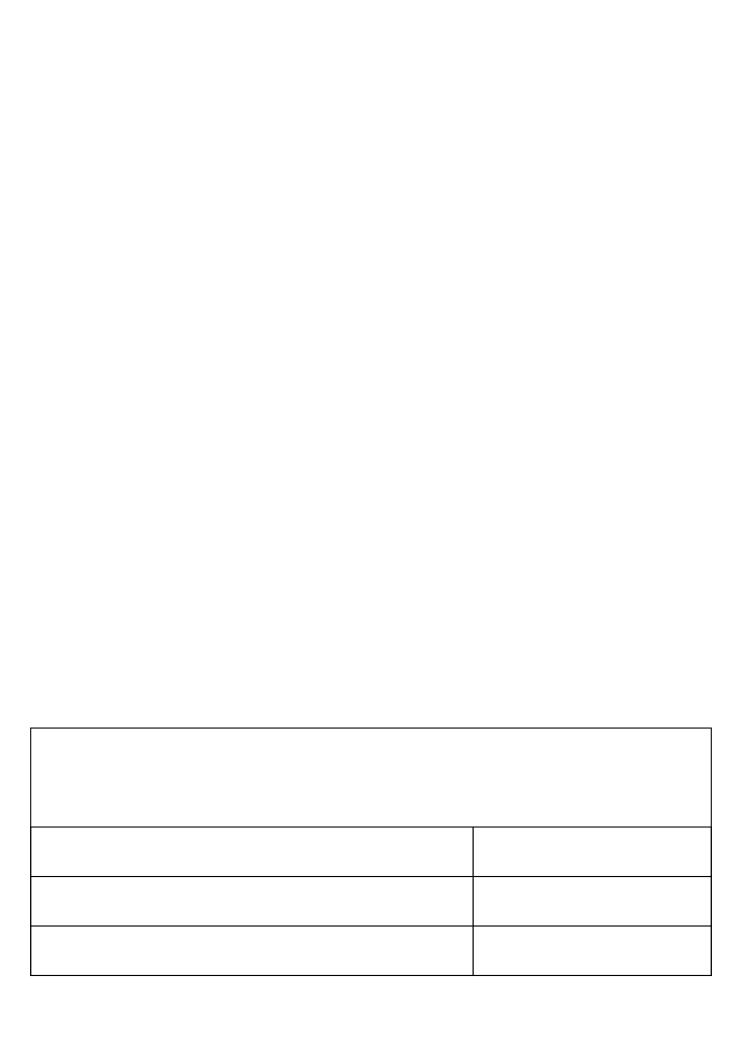
<u>I BGK '91 WXUb[Y</u>'-ghi XYbhYl WXUb[Y'Ybei]f]Yg'flcf']bVci bX'ghi XYbhgŁ

I BGK : i h fY'Gh XYbhg'-'dchYbhJU'gh XYbhYbei]f]Yg

• Y"["UXa]gg]cbgžZYYgždfc[fUa gžWfYX]hlfUbgZYf

<u>GWkcc``CZJWY</u>'-GWkcc``[YbYfU``cZJWY'UXa]b]ghfUh]cb'Ybei]f]Yg

• B6. h\Y fY Yj UbhhYUa g``]ghYX UVcj Y a i ghVY WcbhUWYX Zcf'U``gh XYbhYbei]f]Yg"'H\Y GW\cc``k]`` cb`mVY UV`Y hc fYZYf'gh XYbhg'cb'hc h\Y fY`Yj UbhHYUa]ZWcbhUWYX



HYUWX]b[GhfUhY[]Yg

H\YfY`]gʻbcʻZcfa U``HYUW\]b[`Vi hh\Y`ghi XYbhg``YUfbʻZca `Vch\`]bHYfbU``UbX`YI HYfbU``gci fWYg"``H\Y gi dYfj]gcfž'ch\Yf`UWJXYa]WgʻUbX``UVcfUhcfm#k cf_g\cd`gHJZUfY`h\Y`]bHYfbU``gci fWYgž'k \YfYUgʻh\Y`@]VfUfmž]bHYfbYhUbX`]bXi ghfma YbhcfgʻUfY`h\Y`YI HYfbU``gci fWYg"`H\YfY`UfY`bc`gYh`YWhi fYgʻZcf`h\]gʻWci fgYž'Vi hU bi a VYf`cZk cf_g\cdg`k]```VY`dfcj]XYX`hc`Ugg]ghighi XYbhgʻhc`Wca d`YhY`h\Y]f`h\Yg]gʻhc`U`\][\`gHUbXUfX"`5`` k cf_g\cdg`k]```VY`fYWcfXYX`UbX`a UXY`Uj U]`UV`Y`hc`ghi XYbhgʻcb`]bY"

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
Dfc[fYgg'9jU'i Uh]cb'k]h\ A Ybhcfg'	%) ।	K YY_`m	%ž'&ž' ž'(
Dfc YWnDfYgYbrUrjcb	%) ।	K YY_`m	%ž'&ž'' ž'(
Dfc[fYgg'F Ydcfh	+\$1	K YY_`%\$´: f]XUm	%ž'&ž'' ž'(

5ggYgga Ybh8YHJ]`g

 $5ggYgga\ Ybh\%\ Dfc[fYgg\ 9j\ U`i\ Uh]cb\ k\]h\ A\ Ybhcfg$

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

U"'8YgWf]dh]cb'cZh\Y'hUg_"

 $9 \verb|UWX 'hYUa 'k] \verb|``'dfYgYbh'h\Y]f'_Ym'Z|bX]b[g'k YY_`m'hc'h\Y]f'a Ybhcfg'UbX#cf'Wci fgY'Wcbj YbYf''$

 $V"`Bi\ a\ VYf`cZUggYgga\ Ybhg`]bWi\ XYX`]b`h\]g`UggYgga\ Ybh'WUhY[\ cfm']$

: fca ' †

```
Z'K \Yb'A Uf_g'UfY'fYh fbYX.
Cb!h\Y!gdchZYYXVUW_
["'<ck 'hc'gi Va ]gg]cb'h\Y'UggYgga Ybh
H\Y'Yj U'i Uhjcb'k ]``'VY'XcbY'`]j Y'cb'A G'HYUa g
5ggYgga Ybh& Dfc YWhDfYgYbhUh]cb
GhUfhXUhY. Bch5dd`]WUV`Y
@Yb[h\. %) a ]bi hYg
8 YW] g.
U"'8 YgWf]dh]cb'cZh\Y'hUg_"
9UWX 'hYUa 'k ]```dfYgYbhih\Y]f`_YmZ]bX]b[g'k YY_`mhc'h\Y]f'a Ybhcfg'UbX#cf'Wci fgY'Wcbj YbYfz'Vi hih\Y'hYUa
k]``VY`UggYggYX`cb`h\Y]f`dfYgYbhUh]cb`g_]``g`UbX``Yj Y``cZcf[Ub]gUh]cb"
V"'Bi a VYf'cZUggYgga Ybhg']bWi XYX']b'h\]g'UggYgga YbhWUhY[cfm"
: fca 'K YY_'( 'hc'K YY_'- 'f]bWi g]j YŁ
WK \Yh\Yfh\Y UggYgga Ybh]g U; fci d Dfc YWh
#i]g'U'[fci d'UggYgga Ybh!'a Ul '+'dYf'hYUa "
X" 5ggYgga Ybh\f]l\f]U
H\Y`a Uf_]b[ 'Wf]hYf]U`Zcf'h\]g`UggYgga Ybhk ]```VY`a UXY`Uj U]`UV`Y`cb`A ccX`Y#A G`HYUa g
Y"'8YUX`]bY'Zcf'UVgc`i hY'ZJ]`.'+'WU`YbXUf'XUmg'UZhYf'h\Y'Xi Y'XUhY
B#5
```

Z'K \Yb'A Uf_g'UfY'fYh fbYX.

```
Cb!h\Y!gdchZYYXVUW_
["'<ck hc gi Va ]gg]cb h\Y UggYgga Ybh
H\Y'dfYgYbhUhjcb'k]``VY'XcbY'`]j Y'cb'A G'HYUa g
5ggYgga Ybh' Dfc[fYgg F Ydcfh
GhUfhXUhY. Bch5dd`]WUV`Y
@Yb[h\.')!%$'dU[Yg'Ž'Zfcbh'/ YbX'a UffYf
8 YW] g.
U"8 YgWf]dh]cbcZh\YhUg_"
Gh XYbhgi Va ]hg Ub ]bX]j ]Xi U`fYdcfh! ci h]b]b[ '_YmZ]bX]b[ g cZDfc YWh5 UbX dfcXi WY U XYhU]YX d`Ub Zcf
Dfc YWn6 ]b h\Y bYI hhYfa "
V"'Bi a VYfcZUggYgga Ybhg']bWl XYX']b'h\]g'UggYgga YbhWUhY[cfm'
Cb`m%! Uhih\Y YbX cZh\Y hYfa
W'K\Yh\Yf'h\Y'UggYgga Ybh]g'U'; fci d'Dfc'YWf'
Вс"
X"5ggYgga YbhWf]hYf]U
\label{eq:hamiltonian} $H\Y'a\ Uf\_b[\ 'Wf]hYf]U'Zcf'h\]g'UggYgga\ Ybhk\]```VY'a\ UXY'Uj\ U]`UV'Y'cb'A\ ccX`Y\#A\ G'HYUa\ g
Y"'8YUX`]bY'Zcf'UVgc`i hY'ZJ]`.'+'WU'YbXUf'XUmg'UZhYf'h\Y'Xi Y'XUhY
+ "WU`YbXYf`XUmg`UZhYf`h\Y`XiY`XUhY"
Z'K \Yb'A Uf_g'UfY'fYh fbYX.
```

I dcb fY YUgY cZh Y ZbU fygi h

.

["'<ck'hc'gi Va]gg]cb'h\Y'UggYgga Ybh

H\Y`gi Va]gg]cb`k]```VY`a UXY`cb`A ccX`Y"

.

FYgci fWyg

FYWca a YbXYX FYgci fW/g

7 cbhYbhicb h\Y 'Wci fgY 'A ccX`Y 'dU[Y 'k]``'VY 'i dXUhYX 'cZhYb 'k]h\ 'h]dgž X]gW gg]cbg 'UbX 'fYgci fWYgž gc 'mci UfY 'ghfcb[`mUXj]gYX hc 'a U_Y 'gi fY 'mci 'W\YW_Zcf'U```i dXUhYg"

Mci WUb U`gc ZlbX i gYZ `fYgci fWrg cb h\Y I B GK `]VfUfmk YVg]hY"

U" I BGK @|VfUfmk YVg]|Y. \hdg.#k k k "`]VfUfm'i bgk "YXi "Ui #

V" A ccXY. \hdg.#a ccXY"hYhIi bgk "YXi "Ui #c[]b#bXYI "d\d

7ci fgY 9j U'i Uh]cb UbX 8Yj Y'cda Ybh

: YYXVUW_cbih\YiWci fgYi]gi[Uh\YfYXidYf]cX]WU``mii g]b[ij Uf]ci gia YUbgž]bWi X]b[ih\YiI BGK a m9I dYf]YbWiidfcWiggž]bZcfa U``X]gWi gg]cbi]bih\YiZ]bU``WUggiZcfih\YiWci fgYžUbXih\YiGWcci'g Ghi XYbh#GhUZZa YYh]b[g"iMci fiZYYXVUW_i]gihU_YbigYf]ci g`mžUbXiWcbh]bi U``]a dfcj Ya YbhgiUfYia UXYihcih\Y Wci fgYiVUgYXž]bidUfhžicbigi W\iZYYXVUW_"i=bih\]giWci fgYžfYWibh]a dfcj Ya YbhgifYgi h]b[iZfca ighi XYbhZYXVUW_i]bWi XY.`\Uj]b[iga U``YfihYua ig]nYgihci]a dfcj Yighi XYbhg!a YbhcfiYb[U[Ya Ybhi

 $\label{thm:condition} $$ \ensuremath{\mathsf{U}} = \ensuremath{\mathsf{U}} =$

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi 'Y</u>ž'k \]W\ 'a YUbg't\\Uh']Zmci 'UhhYa dh'Ub'YI Ua 'cf'gi Va]h'U'd]YW\'cZ UggYgga Ybhž'mci 'UfY'XYWUf]b['mci fgY`ZZ]hYbci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`m'Zcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYVJU``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg'Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`l BGK 'g`<u>GdYVJU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhigh XYbhg'k]``bchVY'fYei]fYX'hc'dfcj]XY'UbmXcW a YbhUfmYj]XYbW'hc gi ddcfhUVgYbWg'Zfca 'UbmWUggYg'a]ggYX'

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg``UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVWY] WWSege[YfZW_aXSekagdai`ž$

‰

5WUXYa]W±bZcfaUhjcb

7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$

- A ccX`Y
- <u>@UV:5WWgg</u>
- 7 ca di hb[: UW]hYq
- Gh XYbhF Ygci fWg
- 7 ci fqY 'Ci h]bYq
- : UW `hmiHfUbg]hcbU``5ffUb[Ya Ybhq'Zcf'7CJ=8!%
- <u>A U YfadUW</u>
- I BGK 'Hla YHUV'Y
- <u>I BGK '< UbXVcc</u>
- <u>9ei]hUV`Y`@YUfb]b['GYfj]WYg</u>

a U[Y⁻7fYX]h

GmbYf[]Yg^]b^GcibX^&\$%*

7F = 7CG

7 F = 7 C G 'Dfcj]XYf'7 cXY. '\$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci bhfm

 $K Y : UW\underline{b}ck : YX[Y : h, Y : 6YXY[U : dYcd : Y : k \setminus c : UfY : h, Y : hfUX]h\underline{b}cb : W : ghcX]Ubg : cZh, Y : UbXg : cb : k \setminus JWX : I B G K : Ybg]b[hcb : WUa di g :]g : cWUhYX"$

5ddYbX]I . '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±bHYbXYX'@YUfb]b['Ci HWca Yg	
?bck `YX[Y`UbX'g_]```VUgY	
D9%%7ca dfY\Ybg]j Yžfh\YcfmVUgYX'i bXYfgfUbX]b['cZfh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]WU`'gWJYbWYg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbfU`g'Udd`]WUV`Y'hc'h\Y'Yb[]bYYf]b['X]gWJd`]bY	
D9%"&"7cbWYdh U``i bXYfgHJbX]b['cZh\Y'a Uh\Ya Uh]Wgžbi a Yf]WJ``UbU`mg]gžgHJh]gh]Wgž'UbX Wca di hYf`UbX`]bZcfa Uh]cb`gW]YbWYg`k\]W\`i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" =: XYdh\ i bXYfghUbX]b[icZgdYW]U`]ghVcX]YgicZ_bck iYX[Yik]h\]bih\ YiYb[]bYYf]b[X]gW]d`]bY	
D9%"('8]gWfba YbhcZ_bck `YX[Y`XYj Y`cda YbhUbX`fYgYUfVX 'X]fYWf]cbg`k]h\]b`h\Y Yb[]bYYf]b['X]gW]d`]bY	
D9%") '? bck `YX[Y`cZYb[]bYYf]b['XYg][b`dfUWf]WY`UbX`WcbHYl hi U``ZUWhcfg`]a dUWf]b['h\Y Yb[]bYYf]b['X]gW]d`]bY	
D9%"* I bXYfgHJbX]b['cZh\Y'gWcdYž'df]bW]d`Ygž'bcfa gž'UWWci bHJV]`]h]Yg'UbX'Vci bXg'cZ gi gHJ]bUV'Y'Yb[]bYYf]b['dfUW]WY']b'h\Y'gdYW]Z]WX]gW]d`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUhjcb`cZYgHUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl 'Yb[]bYYf]b[`dfcV`Ya gc`j]b[
D9&"&": `i YbhiUdd`]WUh]cb'cZYb[]bYYf]b['hYW\b]ei Ygž'hcc`g'UbX'fYgci fWYg	
D9&" '5dd`]WUh]cb`cZgmghYa Uh]WYb[]bYYf]b['gmbh\Yg]g'UbX`XYg][b'dfcWYggYg	
D9&"('5dd`]WUh]cb'cZgmghYa Uh]WUddfcUVXYg'hc'h\Y'WcbXi WhUbX'a UbU[Ya YbhcZYb[]bYYf]b[dfcYWng	
DfcZYgg]cbU`'UbX'dYfgcbU`'UHff]Vi HYg	
D9' "%'9h\]WU`'WcbXi WiUbX'dfcZYgg]cbU`'UWWci bhUV]`]hm	
D9' "&'9ZYWNjjY'cfU``UbX'k f]hhYb'Wcaai b]WUhjcb']b'dfcZYgg]cbU``UbX'`UmXcaU]bg	
D9' "' '7 fYUhjj Yž']bbcj Uhjj Y'UbX'dfc!UWhjj Y'XYa YUbci f	
D9' "('DfcZ/gg]cbU`i gY'UbX'a UbU[Ya YbhcZ]bZcfa Uh]cb	
D9' ") 'C fXYf`ma UbU[Ya YbhcZgY`Zë'UbX`dfcZYgg]cbU``WcbXi Wh	
D9' "* '9ZZYW¶j Y'HYUa 'a Ya VYfg\]d'UbX'HYUa '`YUXYfg\]d	