

5ggYgga Ybh

5ggYgga YbhHJg_	K Y][\h	8iY'8UHY	7 ci fgY @YUfb]b[Ci hWca Yg 5ggYggYX
%"'HYgh	')	Bichi5dd`]WUV`Y	% <u>*</u> &
&"":]bU`'91 Ua	()	Bch5dd`]WUV`Y	%ž`&ž`'
' "'@UV'F Ydcfh	&\$I	%(#\$(#&\$&&`\$*.\$\$`5A	%ž [*] &

5ggYgga Ybh'% HYgh

5ggYgga Ybh`Yb[h\. 6Ykk YYb - \$`UbX`%&\$`a]bi hYg'ZcfYUW\`HYgh'

Gi Va]gg]cb bchYg. Cb`]bY hYghg

8YUX`]bY'Zcf'UVgc`i hY'ZU]`.''B#5''Zcf'h]a YX'hYghg#i]nnYg

A Uf_g fYh fbYX. "K]h\]b hk c k YY_g Zfca h\Y hYgh"

 $H Y f Y k] ``VY k c W a dcbYbhg. `HYgh%'] b K YY_'(k cfh\ `$i - `fdfUWf) W hYghZcf` HYgh&LUbX` HYgh&'] b K YY_') k cfh\ '') i "'6ch\ `hYghg` UfY Zfca 'h\Y'WbhYbhg` cZK YY_g`% ("$

HYgh%f\$i Łk]``VY`fYU`YUgYX`cb`H\i fgXUmK YY_`(`f%\$`A UfVX\Ł"`Gci``h]cbg`k]```U`gc`VY`[]j Yb`hc`gh XYbhg"`

 $\label{eq:hambers} HYgh\&fl) \ \ L^2]g`cb`H\ fgXUm\% . $$^!. \&\$. \$^K YY_^) `f\%+`A UfVX\L''`A Uf_g`UfY`fYh fbYX`k]h\]b`&`k YY_g`UZhYf h\Y`hYgh'$

H\]g]g bchU Hi fb]hb Ugg][ba Ybh

5ggYgga YbhWf]hYf]U

GYY h\Y Wci fgY W\UbbY`]b A G HYUa g"

5ggYgga Ybh& :]bU`9l Ua

5ggYgga Ybh`Yb[h\. '%\$'a]bi hYg

8YUX`]bY'Zcf'UVgc`i hY'ZU]`.''B#5''Zcf'h]a YX'hYghg#ei]nnYg"

A Uf_g`fYh fbYX. ``H\Y`a Uf_g`k]```VY`fY`YUgYX`Uhih\Y`gUa Y`h]a Y`k\Yb`U```a Uf_g`Zcf`HYfa `&`UfY`fY`YUgYX"

:]bU``YI Ua]bUhjcb ZcW g]b['cb h\Y a UhYf]U``Wcj YfYX`]b K YY_g`) ž+!%\$"

H\]g^]g^bch'U^Hi fb]h]b^Ugg][ba Ybh

5XX]h]cbU`XYhU]`g

H\Y'XUhY'UbX'nja Y'cZh\Y'ZjbU``Yl Ua 'k]```VY'gYhWYbhfU``mVmh\Y'Yl Ua jbUhjcb'i bjh'

5ggYgga Ybh' @UV F Ydcfh

GhUfhiXUhY. '&, #\$' #&\$&& \\$*.\$\$ '5A

5ggYgga Ybh`Yb[h\. &\$'dU[Yg'a Ul]a i a

Gi Va]gg]cb bch/g. Gi Va]gg]cb bch/g k]``VY dcgh/X cb A ccX`Y UbX A G HYUa g"

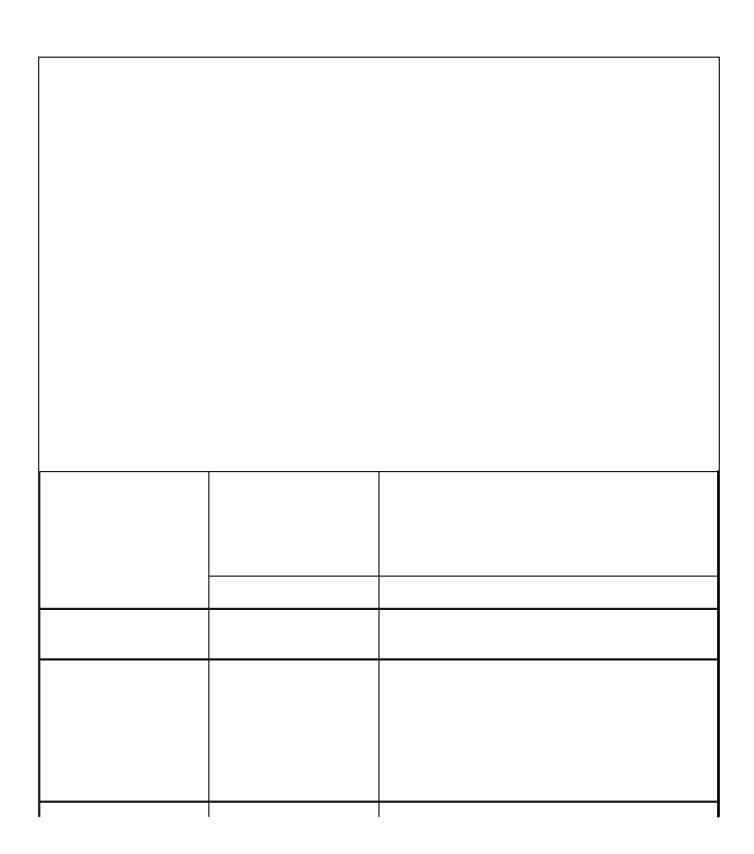
8 i Y`XUhY. '%(#\$(#&\$&&`\$*.\$\$`5 A A Uf_g`fYhi fbYX. '`A Uf_g`k]```VY`fYhi fbYX.'k]h\]b`hk c`k YY_g`Zfca 'h\Y`gi Va]gg]cb`XYUX`]bY"

 $5gg][babhXYgWf]dh]cb'UbX'aUf_]b['Wf]hYf]U'k]``VY'fYU'gYX'cb'AG'HYUag']b'KYY_'+"'H\]g']g'Ub']bX]j]XiU`Ugg][baYbh''$

H\]g']g'bch'U'Hi fb]h]b'Ugg][ba Ybh

5XX]h]cbU`XYhJ]`g

H\Y'XYUX`]bY'Zcf'UVgc`i hY'ZJ]``]g'%#\$(#&\$&&'Uh'*.\$\$'5A ž'Z]j Y'XUmg'UZhYf'h\Y'ghUbXUfX'XYUX`]bY"



K YY_`, .`('5df]`'!`, '5df]`	@YWn fY	HfUbgZYf'Z bWfjcb'fl cf'@H=G-€C'UbX'A ≠ C WJgYgŁ"'7cblfc``UV`Y'WJbcb]WJ``Zcfa "'G]a]`Uf]lm lfUbgZcfa Uhjcb"Gc`i hjcb'cZgHJH'gdUWY'Yei Uhjcb" AUlf]I 'YI dcbYbhjU`"
K YY_''%%'5df]`'!'%) 5df]`	@YWhifY	GtUV]`]miUbU`mg]gžY][Ybj U`iYg"`GtUtY`ZYYXVUW_" Dc`Y`d`UWYaYbti'7cbtfc``UV]`]tm"
K YY_'%\$.'% '5df]`'!'&& 5df]`	@YWhi fY	GHJHY 'Ygh]a Uh]cb"'C VgYfj Yfg"'C VgYfj UV]`]hm' a d`Ya Ybh]b['ghJHY 'ZYYXVUW_'j]U'Ygh]a UHYX ghJHYg" F Yj]g]cb

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

5ggYgga Ybhgi Va]gg]cb UbX a Uf_]b[Wf]hYf]U

G\ci`X'h\Y'WcifgY'\UjY'Ubmbcb!Y`YWfcb]WUggYgga Ybhigi Va]gg]cbž'h\YgY'g\ci`X'\UjY'U'ghUbXUfX GW\cc``WcjYf'g\YYh'

5```gi Va]gg]cbg`UfY`YI dYWYX`lc`VY`bYUh`UbX`WYUf`mgYhci h'`Mci f`fYgi `hg`UfY`h\Y`d]bbUWY`cZU```mci f`\UfX k cf_'UbX`g\ci `X`VY`lfYUh'X`k]h\`Xi Y`fYgdYWf'`DfYgYbh]b[`fYgi `hg`WYUf`m[]j Yg`h\Y`a Uf_Yf`h\Y`VYghW\UbW' cZi bXYfgHUbX]b[`mci f`a Yh\cX/Yj Yb`]Zh\Y`bi a Yf]WU``fYgi `hg`UfY`]bWcffYWf

A Uf_]b['[i]XY`]bYg`Zcf`Ugg][ba Ybhgi Va]gg]cbg`k]```VY`dfcj]XYX`Uhh\Y`gUa Y`h]a Y`Ug`Ugg][ba YbhXYhU]`g hc`Ugg]ghk]h\`a YYh]b['UggYggUV`Y`fYei]fYa Ybhg"`Gi Va]gg]cbg`k]```VY`a Uf_YX`UVWcfX]b['hc`h\Y`a Uf_]b[[i]XY`]bYg`dfcj]XYX"

K cf_'gi Va]hhYX``UhY`k]h\ci hUb`Uddfcj YX`YI hYbg]cb`Vmh\Y`Wci fgY`WccfX]bUhcf`cf`XY`Y[UhYX`Ui h\cf]hm]g gi VYWhc`U``UhY`dYbU`hmcZ&\$`dYf\Wbhf&\$i \ \E'cZh\Y`a Ul]a i a 'a Uf_'dcgg]V`Y`Zcf`h\UhUggYgga Ybh']hYa ž dYf`WU`YbXUf`XUmž`Zcf`U`a]b]a i a 'cZnYfc'a Uf_g"

H\Y``UhY`dYbU`hm`]g`Udd`]YX`dYf`WU`YbXUf`XUmfcf`dUfhih\YfYcZzž]bWi X]b[`k\YY_YbXg`UbX`di V`]W\c`]XUmgž'h\Uhih\Y`UggYgga Ybh`]g`cj YfXi Y"`

K cf_'gi Va]hhYX'UZhYf'h\Y''XYUX`]bY'Zcf'UVgc`i hY'ZU]''']g'bchUWWdhYX'UbX'U'a Uf_'cZnYfc'k]``'VY'Uk UfXYX Zcf'h\Uh'UqqYgqa Ybh']hYa "': cf'Yl Ua d`Y.

- Mci f'Wci fgY'\Ug'Ub'UggYgga Ybhildg_'k cfh\'U'hclU'cZ' \$'a Uf_g'fA Ul 'Dcgg]V'Y'A Uf_Ł
- Mci 'gi Va]hh\Y'UggYgga Ybh&'XUmg'UZhYf'h\Y'Xi Y'XUhY
- H\Y`UggYgga Ybh]g`a Uf_YX`Ug`i gi U``UbX`UW\]Yj Yg`U`gWcfY`cZ&\$`a Uf_g`f5k UfXYX`A Uf_Ł
- H\Y``UhY`dc`]Wni]g`Udd`]YX`i g]b['@UhY`A Uf_'1 `5k UfXYX`A Uf_'! 'fB Umgł DYbU`hmidYf`8 Umłł A UlDcgg]V`Y`A Uf_"'Mci f`UX^ ghYX`Z]bU``gWcfY`]g`, 'a Uf_g`f&\$'! 'ff&ł \$"&Ł!'

: cf'gca Y'UggYgga Ybh']hYa gž'U'`UhY'dYbU`hma Umbch'VY'Uddfcdf]UhY"'H\YgY'UfY'WYUf'm']bX]\WUhYX']b'h\Y Wci fgY'ci h']bYz'UbX'gi W\'UggYgga Ybhg'fYWY]j Y'U'a Uf_'cZnYfc']Zbch'Wca d`YhYX'Vmh\Y'gdY\\JZ]YX'XUhY" 91 Ua d`Yq']bWi XY.

%" K YY_`mcb`]bY'hYghg'cf``UVcfUhcfmk cf_'k cfh\'U'ga U``'dfcdcfh]cb'cZh\Y'gi VYWha Uf_žcf

- &" C b`]bY'ei]nnYg'k \YfY'Ubgk Yfg'UfY'fY`YUgYX'hc'gh XYbhg'cb'Wca d`Yh]cbžcf
- "DfcZYgg]cbU``UggYgga YbhiHug_gž'k\YfY`h\Y`]bhYbh]cb`]g`hc`WfYUhY`Ub`Ui h\Ybh]WUggYgga Ybhih\Uh\Ug`Ub`UVgc`i hY`gi Va]gg]cb`XUhYž'cf
- ("DUgg# U] UggYgga YbhHJg_g"

91 Ua]bUh]cbq

Mci 'a i gh'VY'Uj U]`UV`Y'Zcf'U```ei]nnYgž'hYghg`UbX`YI Ua]bUh]cbg": cf'Wci fgYg'h\Uh\Uj Y'Z]bU``YI Ua]bUh]cbgž h\YgY'UfY'\Y`X'Xi f]b['h\Y'I b]j Yfg]hmYI Ua]bUh]cb'dYf]cXg.': YVfi Ufm'Zcf'Gi a a Yf'HYfa ž'A Um'Zcf'H\ž 5i [i gh'Zcf'H\&ž'UbX`Bcj Ya VYf#8 YWYa VYf'Zcf'H' "

D'YUgY'j]g]ha ml BGK 'Zcf'Dfcj]g]cbU`'91 Ua]bUh]cb'h]a YhUV'Y'di V']g\'XUhYg": cf'Z fh\Yf']bZcfa Uh]cb'cb

YI Ua gžďYUgY gYY h\Y <u>91 Ua g</u> k YVdU[Y"

GdYWJU`7cbg]XYfUhJcb

⊋mci `\Uj Y`YI dYf]YbWX`Ub`]``bYgg`cf`a]gUXj Ybhi fY`VYmcbX`mci f`Wcblfc``h\Uhk]```]bhYfZ/fY`k]h\`mci f UggYgga YbhidYfZcfa UbWYž'mci `UfY`Y`][]V`Y`hc`Udd`mZcf`GdYWJU``7cbg]XYfUh]cb`df]cf`hc`gi Va]hh]b[`Ub UggYgga Ybhicf`g]hh]b[`Ub`YI Ua "

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi 'Y</u>ž'k \]W\ 'a YUbg'h\Uh']Zmci 'UhhYa dh'Ub'YI Ua 'cf'gi Va]h'U'd]YW\'cZ UggYgga Ybhž'mci 'UfY'XYWUf]b['mci fgY`ZZ]hYbci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`mZcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY'h\Y`]bZcfa Uh]cb`cb`I_BGK 'g`<u>GdYWJU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhgh XYbhg'k]``bch'VY'fYei]fYX'rc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbW'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'VYWUi gY'cZ'7CJ=8!% 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WU`'Wfh]Z]WUh'g'Zfca 'UbmcbY'XYYa YX'rc'VY'U'dcg]h]j Y'WUgYž cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'Wfh]Z]WUh'g'UfY'X]ZZ]W 'hhc'cVhU]b'UbX'di hUb'i bbYWggUfmghfU]b'cb qh XYbhg'UbX'a YX]WU`'qhUZZ''

 $5 dd^{3}WUh^{3}cbg^{3}Zcf^{3}gdYW^{3}U^{3}Wcbg^{3}XYfUh^{3}cb^{3}K^{3}^{3}YfUh^{3}cb^{3}K^{3}YfUh^{3}cb^{3}K^{3}YfUh^{3}cb^{3}K^{3}YfUh^{3}cb^{3}K^{3}YfUh^{3}cb^{3}K^{3}YfUh^{3}Cf^{3}G^{3}VH^{3}Cf^{3}G^{3}VH^{3}Ch^{3}K^{3}YfUh^{3}Ch^{3}YfUh^{3}Ch^{3}X^{3}YfUh^{3}Ch^{3}YfUh^{3}Yf$

GdYWJU 7cbg]XYfUh]cb Ci hWca Yg

5ggYgga Ybhg`\Uj Y`XYZJi`hGdYWJU``7cbg]XYfUhJcb'ci hWca Yg"'H\Y`XYZJi`hci hWca Y`Zcf'h\Y`UggYgga Ybhk]`` VY`UXj]gYX`k \Yb'mci 'Udd`mZcf'GdYWJU``7cbg]XYfUhJcb"'6Y`ck ']g'h\Y``]ghcZdcgg]V`Y'ci hWca Yg.

CihwcaY	91 d`UbUhjcb	9l Ua d`Y
H]a Y'YI hYbg]cb	Gh XYbhdfcj]XYX a cfY ha Y hc gi Va]hh Y UggYgga Ybh	Y"["'%a cfY'k YY_'cZh]a Y'[fUbhYX'hc'gi Va]h'U'fYdcfh
Gi dd`Ya YblUfm UggYgga Ybh	Gh XYbhdfcj]XYX'Ub U'HYfbUHY'UggYgga YbhUhU 'UHYf'XUHY#]a Y	Y"["'U'gi dd`Ya YblUfmYl Ua ']g'gVXYXi `YX'Xi f]b['lh\Y gi dd`Ya YblUfmYl Ua 'dYf]cX'cZh\Y'hYfa
Gi Vghjhi hY jhYa	H\Y`a Uf_`Zcf`h\Y`a]ggYX UggYgga Ybh`]gʻgi Vgh]h hYX k]h\`h\Y`a Uf_`cZUbch\Yf UggYgga Ybh	Y"["'a Uf_'Zcf'Ei]n'%'Udd`]YX'U`gc'Udd`]YX'Ug'a Uf_'Zcf Ei]n'&za YUb]b[']ZU'ghi XYbhUW\]Yj YX'U'a Uf_'cZ &\$# \$'Zcf'Ei]n'%'UbX'k Ug'[fUbhYX'GdYW]U` 7 cbg]XYfUh]cb'Zcf'Ei]n'&z'U'a Uf_'cZ&\$# \$'k ci`X'VY Udd`]YX'Zcf'Ei]n'&z'YhW
91 Ya dhicb	5```Wci fgY'a Uf_g'UfY fYWU'W`UHYX'YI Wi X]b['H\]g UggYgga YbhUbX']hg k Y][\H]b[Y"["'H\Y'Wci fgY'\Ug'Ub'UggYgga Ybhghfi Wh fY'cZ '!'5gg][ba Ybhg'' \$i ž '!'@UV'fYdcfh' \$i ž '!':]bU''9I Ua'(\$i " -Zh\Y'@UV'fYdcfh]g'a]ggYX'UbX'gh XYbh]g'[fUbhYX GdYW]U''7 cbg]XYfUh]cbžh\Yb'h\Y'UggYgga Ybh ghfi Wh fY'a Um'VY'fYk Y][\hYX'Ug'Zc''ck g. '!'5gg][ba Ybhg')\$i '!':]bU''9I Ua')\$i Ug'h\ci [\'h\Y'@UV'fYdcfhX]X'bchYI]gh
B c b ! g r U b X U f X	7 ci fgY 7 ccfX]bUhcfi]g WcbhJWhYX Zcfh\Yici hWca Y k \YbigdYWJU`Wcbg]XYfUh]cb]g [fUbhYX Ugih\Yici hWca Y X]ZZYfgicbiU`WJgY!Vm\WJgY VUg]g	Y"["'md]WD`Zcf'[fcid'UggYggaYbhg'k\YfY'h]aY YI hYbg]cb'gidd`YaYbhUfmUggYggaYbhWci`X'VY [fUbhYX'hc'h\Y'[fcid'aYaVYfž'h]aY'YI hYbg]cb'Wci`X VY'[fUbhYX'hc'h\Y'k\c`Y'[fcidž'YhW'7`Uf]Zmk]h\'mcif 7 cifgY'7 cbjYbcf'Zcf

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg``UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{`WSege[YfZW_aXSekagdai`ž$

D`U[]Uf]ga i]g`U`hmdY`cZ]bhY``YWi U``h\YZh'`±iWUb`hU_Y`a UbmZcfa gžZfca `XY`]VYfUhY`W\YUh]b[`hc`UWV]XYbhU``m Wcdm]b[`Zfca `U`gci fWYk]h\ci hUW_bck `YX[Ya Ybh'`I BGK `\Ug`dfcXi WYX`U`k YVg]hY`k]h\`U`k YU`h\`cZ fYgci fWYg`hc`gi ddcfhighi XYbhg`hc`i bXYfghUbX`UbX`Uj c]X`d`U[]Uf]ga žij]g]h`ghi XYbh'i bgk "YXi "Ui #d`U[]Uf]ga "H\Y`@YUfb]b[`7 YbhY`Ugg]ghg`ghi XYbhg`k]h\`i bXYfghUbX]b[`UWUXYa]W]bhY[f]hmUbX`\ck 'bchhc'd`U[]Uf]gY" H\YmU`gc`\c`X`k cf_g\cdg`UbX`WUb`\Y`d`ghi XYbhg`cbY!cb!cbY"

Mci 'UfY'U'gc'fYa]bXYX'h\Uh'WUfYZ\ ``h]a Y'a UbU[Ya Ybh']g'Ub']a dcfhUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd'U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi ZZ]W]Ybh'h]a Y'Zcf

• 5 WUXYa]W< cbYghmUbX^D^U[]Uf]ga

