

# SOLA4951, SOLA9451

FYgYUfWX 'H\Yg]g'5



## 7ci fgY'8YW]`g

I b]hg~cZ7fYX]h~(~

Gi a a UfmcZh\Y 7ci fgY

H\Y'h\Yg]g'dfcj ]XYg'Ub'cddcfhi b]miZcf'mci 'hc'Vf]b[ 'hc[Yh\Yf'Yb[]bYYf]b[ 'df]bW]d`Yg''YUfbYX'cj Yf mci f'dfYj ]ci g'mYUfg'cZghi XmUbX'Udd`mh\YgY'df]bW]d`Yg'hc']bbcj Uhjj Y`mgc'j Y'dfcV`Ya g'gi W\'Ug'h\Y XYj Y`cda YbhcZ'U'gdYW]ZJWXYg][ bž'dfcWygg'UbX#cf'h\Y']bj Ygh][ Uhjcb'cZU'\mdch\Yg]g"'H\Yg]g'dfc YWg'UfY Wca d`Yl ž'cdYb!YbXYX'dfcV`Ya g'h\UhU``ck 'fcca 'Zcf'mci f'WfYUhjj ]mž'UbX'h\Y'UWei ]g]hjcbž'UbU'mg]g'UbX ]bhYfdfYhUhjcb'cZfYgi 'hg"'H\YfY'UfY'a i 'hjd'Y'dcgg]V'Y'gc'i hjcbg'cf'WcbWi g]cbg'Uhh\Y'ci hgYhUbX'gi ZZJW]Ybh Wca d`Yl ]mhc'fYei ]fY'U'XY[ fYY'cZdfc'YWhd'Ubb]b[ "'H\Y'h\Yg]g'fYei ]fYg'mci 'hc'Zcfa i 'UhY'dfcV'Ya g']b gw]Ybh]ZJWcf'Yb[]bYYf]b[ 'hYfa gž'a UbU[ Y'Ub'hYW\b]WU'dfc'YWhUbX'Z]bX'gc'i hjcbg'VmUdd`m]b[ 'gw]Ybh]ZJWUbX Yb[]bYYf]b[ 'a Yh\cXg"'Mci 'k]'`'U'gc'XYj Y`cd'h\Y]f'UV]`]mhc'k cf\_']b'U'fYgYUfW\'UbX'XYj Y`cda Ybh Ybj ]fcba Ybh''Mci 'a i gh]XYbh]ZmU'gi dYfj ]gcf'UbX'dfc'YWhdf]cf'hc'Ybfc``]b[ ']b'h\]g'Wci fgY''H\]g']g'h\Y'Z]fgh Wci fgY'cZh\Y'' 'Wci fgY'h\Yg]g'gffi Wh fY''

# 7ci fgY 8YW] g

 $\label{thm:condition} $$HY'bcfa\ U''k\ cf_cux'YI\ dYWUhjcbg'cZU'ghi\ XYbh'ufY'UddfcI\ ]a\ UhY'm'&)\ '\ci\ fg'dYf'hYfa\ 'Zcf'YUW\'I\ C7 ž\ ]bWi\ X]b[\ 'WUgg'Wcbh'uWi\ci\ fgž'ch\Yf''YUfb]b[\ 'UWhjj\ ]hJYgž'dfYdUfUhjcb''UbX'hJa\ Y'gdYbh'cb''U'''UggYggUV'Y\ k\ cf_'''$ 

 $\label{eq:linear_continuous_con$ 

H\Yg]g; i ]XY`]bYg

# 5ggYgga Ybh 5ggYgga YbhCj Yfj ]Yk

- %" Dfc YWiD`Ub`—'h\]g`k]``Wca df]gY`U`%!&`dU[Y`XcW`a YbhYl d`U]b]b[`h\Y`^ gh]Z]WUh]cb`Zcf`h\Y]f`dfcYWž U`fci [\``Umci hcZU`d`Ub`cZk cf\_`h\fci [\ci hih\Y`dfcYWž`]bWi X]b[`UbmgcZk UfYža Yh\cXg`YhWž h\YmbYYX`hc`VY`hfU]bYX`cb"`fH\Yq]q`5Ł
- &" @|hYfUhifY`UbX`Dfc[fYgg`fYj]Yk—'h\]g`g\ci`X`Wcadf]gY`h\Y`fY`YjUbh`]hYfUhifY`UbX`VUW\_[fcibX`cZh\Y`hcd]Wzh\Y`dfcV`Ya`ghUhYaYbhUbX`ach]jUh]cb`Zcf`h\Y`kcf\_'UbX`U`XYhU]`YX`fYgYUfW\`d`Ub":cfXYhU]`q`cZk\Uhhc`]bWiXY`qYY`VY`ck`]b`5qqYqqaYbh'

 $\label{thm:continuous} $H\Yg]g'6."H\Y'df]a$ Ufm]bhYbh]cb'VY\]bX'H\Yg]g'6']g'hc'Ybgi fY'ghi XYbhg'ghUmcb''hfUW_'k]h\'h\Y]f'dfc'YWhg UbX'dfc'YWhk cf_'Ug'h\Ymdfc[fYgg'h\fci[\'h\Y'mYUf'']$ 

- %" Dfc[fYgg'i dXUhY'—'U'Zcfa 'cb'a ccX'Y'k \YfY'mci 'i dXUhY'mci f'dfc[fYggž'UbX'hU'\_'UVci h'Ubm cVghUWYg'cf'W\Ub[Yg'hc'mci f'cf][]bU`'d'Ub"
- &" GYa ]bUf`DfYgYbhUh]cb`—`H\Y`gYa ]bUf`g\ci``X`]bWi XY`cj YfU```U]a `cZdfc^YWb`]bhYbXYX`ci hWca Ygž`U dfc[fYgg`fYdcfh`]bWi X]b[`U`XYhU]`YX`a Yh\cXc`c[mb`UbX`dfY`]a ]bUfmfYgi `hg"
- '"Dfc[fYgg'FYdcfh-'h\]g'g\ci`X'Wca df]gY'U'h\Yg]g'HUV'Y'cZWcbhYbhg'ci h]b]b['h\Y'ghfi WhifY'cZh\Y h\Yg]g"'5'&!)'dU[Y'gi a a Ufmcb'[cU'g'UbX'HJg\_g'UWWca d']g\YX'UbX'Z hifY'k cf\_

 $H (Yg) g 7 . H (Yg) g 7 . H (Yg) g h (Y dfc YWik cf_"H (Y _YmXY^) j YfUV (Y ) g h (Y K f) h (Y dfc YWik cf_"H (Y _YmXY^) j YfUV (Y ) g h (Y K f) h (Y ) f (Y ) f$ 

%" DUff]W]dUf]cb'-'UggYggYg'h\Y'gh'XYbhg'Wca a ]ha Ybh'UbX'Yb[U[Y\$

# CjYfj]Yk 'cZU``XY`]jYfUV`Yg']g'VY`ck ž'gdYW]Z]WXiY'XUhYg'Zcf'YUW\'hYfa ž'Ug'kY``'Ug'h\Y'Wf]hYf]U'k]``'VYZci bX']b'YUW\'hYfa g'Wci fgY'ci h]bY"

	5ggYgga Ybhg	8i Y K YY_	7 cblf]Vi l]cb"lc"Z]bU"a Uf_	5 ggYggYX 'Vm
H\Yg]g <sup>-</sup> 5	%" Gi Va ]h bca ]bUh]cb Zcfa %" Dfc YVViD`Ub	%	@cgg cZ&i ci hcZ%\$i dUfr]WdUh]cb a Uf_ ]Zbch gi Va ]HYX @cgg cZ&") i ci hcZ%\$i dUfr]WdUh]cb a Uf_ ]Zbch gi Va ]HYX	H\Yg]g'7c!cfX]bUhcf . Gi dYfj ]gcf
	%" @ hYfUhi fY`UbX Dfc[fYgg FYj]Yk	%\$	<b>%\$</b> 1	. Gi dYfj ]gcf#5ggYggcf'f) \$#) \$Ł
H\Yg]g <sup>·</sup> 6ł	%" I Dfc[fYgg I dXUh/		@cgg cZ&") ı ci hcZ%\$ı dUff]W]dUf]cb a Uf_ ]Zbch gi Va ]HYX	Gi dYfj ]gcf .
	%" GYa ]bUf DfYgYbHJhjcb &" Dfc[ fYgg F Ydcfh	+	%\$1	Gi dYfj ]gcf#5ggYggcf'f) \$#) \$Ł
11) \/1:7		%\$ 0/\$	)	Gi dYfj ]gcf
H\Yg]g <sup>·</sup> 7	%": ]bU`FYdcfh &" DUff]W]dUf]cb ' " DfYgYbHJh]cb cZk cf_	%\$ %\$	*\$I %\$I	Gi dYfj ]gcf#5ggYggcf'f) \$#) \$Ł Gi dYfk]gcf
		%\$	)	Gi dYfj ]gcf#5ggYggcf'f) \$#) \$Ł

BCH9.: cf'H\Yg]g'5ž6'a Uf\_'k]``'VY'97ž'UbX

#### 5bm`UhY`UggYgga Ybhg`k]```]bW'f'U')ı 'dYbU`hm'dYf'XUm'

K cf\_'gi Va ]hhYX``UhY`k ]h\ci hUb`Uddfcj YX`YI hYbg]cb`Vmh\Y`Wci fgY`WccfX]bUhcf`cf`XY`Y[ UhYX`Ui h\cf]hm]g gi VYWihc`U``UhY`dYbU`hmcZ)ı 'dYf`XUmcZh\Y`a Ul ]a i a 'a Uf\_'dcgg]V`Y`Zcf`h\UhUggYgga Ybh]hYa ž'dYf WU`YbXUf`XUm'

 $\label{thm:condition} $$HY^UHY^dYbU^m]g^Udd^]YX^dYf^WUYbXUf^XUmf]bWi X]b[^k YY_YbXg^UbX^di V^]W^c^]XUmg&h^Uhh^Y UggYgga Ybh]g^cj YfXi Y''^H\YfY^]g^bc^dfc!fUhU^cZh\Y^^UhY^dYbU^mZcf^gi Va ]gg]cbg^a UXY^dUfhk Umh\fci [^UXUm']$ 

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### DfYgYbHJh]cb

5```gi Va ]gg]cbg`UfY`YI dYWYX`kc`VY`bYUh`UbX`WYUf`mgYhci H'`Mci f`fYgi `hg`UfY`h\Y`d]bbUWY`cZU```mci f`\UfX k cf\_'UbX`g\ci `X`VY`hfYUh'X`k ]h\ Xi Y`fYgdYWf'`DfYgYbh]b[ `fYgi `hg`WYUf`m[ ]j Yg`h\Y`a Uf\_Yf`h\Y`VYgh'W\UbW' cZi bXYfgHJbX]b[ `mci f`a Yh\cX/Yj Yb`]Zh\Y`bi a Yf]WJ``fYgi `hg`UfY`]bWcffYWf

### A Uf\_]b[

A Uf\_]b[ '[i ]XY`]bYgʻZcfʻUgg][ ba Ybhgi Va ]gg]cbgʻk ]``VY`dfcj ]XYX`Uhh\Y`gUa Y`h]a Y`UgʻUgg][ ba YbhXYhU]`g hcʻUgg]ghk ]h\ a YYh]b[ 'UggYggUV`Y`fYei ]fYa Ybhg"'Gi Va ]gg]cbgʻk ]``VY`a Uf\_YX`UVWcfX]b[ 'hc'h\Y`a Uf\_]b[ [i ]XY`]bYgʻdfcj ]XYX"

5ggYgga Ybh'tUg_	K Y][ \h	8iY'8UHY	7 ci fgY @YUfb]b[ Ci HWca Yg 5ggYggYX
%"'H\Yg]g'Bca ]bUh]cb': cfa	B#5	' \$#\$) #&\$&&`\$\$\$`5A	
&"`DfcYWhD`Ub	B#5	%+#\$*#&\$&&`\$).\$\$`DA	%
' "'@]hYfUhi fY'FYj]Yk'UbX DfcYWhD`Ub#Dfc[fYgg	%\$ı	\$'#\$,#&\$&&`\$).\$\$`DA	%ž &ž )

## 5ggYgga Ybh% H\Yg]g Bca ]bUh]cb: cfa

**Gi Va ]gg]cb bch/g.** 'Gi Va ]hZcfa 'cb'A ccX`Y'UbX'UXX'mci f'bUa Y'hc'h\Y'XUhUVUgY **8i Y'XUh'**.'' \$#\$) #&\$&&\*\$-.\$\$\*5A

<uj Y`U`h\Yg]g`bca ]bUh]cb`Zcfa `g][ bYX`Vm`mci f`gi dYfj g]cf`UbX`fY[ ]ghYf`mci fgY`Zcb`A ccX`Y`]b`h\Y`H\Yg]g XUhUVUgY`UbX`i d`cUX`mci f`g][ bYX`Zcfa `hc`A ccX`Y"

#### 5ggYgga Ybh'Wf]hYf]U

Bc'AUf\_g'`cgh']Z\UbXYX']b'cb'hjaYž']Z`UhYž'U'`cgg'cZ&i 'Zfca'mci f'gi dYfj ]gcf'dUfhjWjdUhjcb'aUf\_"

#### 5ggYgga Ybh'& 'Dfc YWh'D`Ub

_	

# FYgci fWYg

# FYWca a YbXYX FYgci fWYg

• Ghufhik cf\_icbimci fihcd]WUgigccbiUgimci iWUb"iH\]gik ]```[ ]j Yimci idiYbhmicZh]a YihciUXXfYgg dfcViYa gih\Uhimci ia UmiYbWci bhYficbih\Yik Umi

.

Zmci f'dfcYWhXcYg'fYei ]fY'Uddfcj U'ž']b'h\Y'Z]fgh]bghUbW'žX]gW gg'h\]g'k ]h\'mci f'Gi dYfj ]gcf"

• Zmci '\Uj Y'U'ei Ygh]cb'-'Ug\_°

G@EI >[TdSdk i WTe[fW:\hdg.#k k k ":]VfUfm"i bgk "YXi "Ui #

? aaVW \hdg.##a ccX`Y"hY`h'i bgk "YXi "Ui #c[]b#bXYI "d\d

#### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

: YYXVUW\_cb^h\Y`Wci fgY`]g`[ Uh\YfYX`dYf]cX]WU``mi g]b[ `j Uf]ci g`a YUbgž]bWi X]b[ `h\Y`I BGK a m9I dYf]YbW'`dfcW'ggž]bZcfa U``X]gW gg]cb^]b`h\Y`Z]bU``WUgg`Zcf`h\Y`Wci fgYžUbX`h\Y`GW\cc`'g Gh XYbh#GhUZZa YYh]b[ g"'Mci f`ZYYXVUW\_`]g`hU\_Yb`gYf]ci g`mžUbX`Wcbh]bi U``]a dfcj Ya Ybhg`UfY`a UXY`hc`h\Y Wci fgY`VUgYXž]b`dUfhžcb`gi W\`ZYYXVUW\_"

## 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W`hifY`cZ`YUfb]b[`]bZcfaYX`Vm`UWJXYa]W]bhY[f]hm'`5``IBGK `ghiXYbhg`UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWJXYa]W]bhY[f]hm'D`U[]Uf]gaibXYfa]bYg`UWJXYa]W]bhY[f]hm`UbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{\WSeaXafZWbeS`VbSee[YfZW_aX\Sekagdai`ž$ 

Mci 'UfY'U'gc'fYa ]bXYX'h\UhWUfYZ ``h]a Y'a UbU[ Ya Ybh]g'Ub']a dcfhUbhdUfhcZgh XmUbX'cbY'cZh\Y ]XYbh]Z]YX'WUi gYg'cZd`U[ ]Uf]ga ']g'dccf'h]a Y'a UbU[ Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b[ 'UbX'h\Y'dfcdYf'fYZYfYbW]b[ 'cZgci fW/g']b'dfYdUf]b[ 'U```UggYgga Ybh'hUg\_g''

Zd`U[]Uf]ga `]gʻZci bX`]bʻmci fʻk cf\_ʻk \Ybʻmci ʻUfY`]bʻZ]fghmYUfžmci fʻ`YWi fYfʻk]``cZYfʻmci ʻUgg]ghUbWYʻhc
]a dfcj Yʻmci fʻUWUXYa ]Wg\_]``g"'H\Yma UmiUg\_ʻmci 'hcʻ`cc\_ʻUhgca Yʻcb`]bY fYgci fWYgž'UhHYbX`h\Yʻ@YUfb]b[
7 YbhfYžcfʻgca Yh]a YgʻfYgi Va ]hmci fʻk cf\_ʻk]h\ʻh\YʻdfcV`Ya `Z|I YX"'<ck Yj Yfʻa cfYʻgYf]ci gʻ]bghUbWYgʻ]bʻZ]fgh
mYUfžgi W\ʻUgʻghYU`]b[ 'Ubch\Yfʻghi XYbhgʻk cf\_ʻcfʻdUm]b[ 'gca YcbY'hc'Xc'mci fʻk cf\_ža Um'VY`]bj Ygh][ UhYXfYʻgYf]ci gʻ]

 $5 ```ghi XYbhg`UfY`YI dYWhYX`hc`fYUX`UbX`VY`ZJa ]`]Uf`k ]h\`I BGK `; i ]XY`]bYg`UbX`Dc`]WYg''` ± b`dUfh]W``Ufž ghi XYbhg`g\ci `X`VY`ZJa ]`]Uf`k ]h\`h\Y`Zc``ck ]b[ .$ 

**BHYDXUDW**.....Œ

OH BGK '9a U] '5XXfYgg

ŒdYWU`'7cbalXYfUhcb

⊞l Ua q

(Enddfci YX'7 U'W 'Uhcfa

(Bei huv'y @Yufb]b[ GYfi ]WYa

.

## 8]gWU]a Yf

 $\label{thm:continuous} FZ [e UagdeWagf' \ WeWfe agf VWeUd[bf[a` aXU'SeeWe Sf fZWVSfWfZW5 agdeWA gf' W[e bgT' [eZW\ZFZW `SfgdWaXU'SeeWe _ Sk UZS` YWVgd `Y fZWF Wd _ SXfWd fZW5 agdeWA gf' W[e bgT' [eZ\W\Z ? aaV' WeZag' V TW Ua` eg fW Xad fZWgb fa VSfWU'See VWeUd[bf[a` e\Z ;XfZ\WdW[e S` k [` Ua` e[efW Uk [` fZWV\WeUd[bf[a` aX SUf[h[f[\We T\Wf | WW fZ\WG` [h\Wde[fk f[_ \WfST'\WS` V fZ\W5 agdeWA gf' \W/Se gbVSf\W [` ? aaV'\Wfi fZ\W V\WeUd[bf[a` [` fZ\W5 agdeWA gf' \W? aaV'\WSbb' [\We\Z ] \]$ 

#### 7F = 7CG

7F=7CG'Dfcj]XYf'7cXY.'\$\$\$-,;

#### 5W\_bck `YX[ Ya YbhcZ7ci bffm

 $K Y : UW\underline{b}ck "YX[Y : h, Y : 6YXY[U : dYcd"Y : k \c : UfY : h, Y : hfUX]h\underline{b}cbU "W ghcX]Ubg"cZh, Y : `UbXg"cb : k \] WX : I BGK ? Ybg]b[hcb : WUa di q:]q: "cWUhYX"$ 

# 5ddYbX]I . '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa =bhybXYX=@YUfb]b[ 'Ci hWca Yg	
?bck `YX[Y`UbX'g_]```VUgY	
D9%"%7ca dfY\Ybg]j Yž'h\YcfmVUgYX'i bXYfghUbX]b[ 'cZh\Y'i bXYfd]bb]b[ 'bUhi fU`'UbX d\mg]\WJ`'g\WJYb\Wg'UbX'h\Y'Yb[ ]bYYf]b[ 'Z bXUa YbhJ`g'Udd`]\WJV`Y'hc'h\Y'Yb[ ]bYYf]b[ 'X]g\WJd`]bY	
D9%"&"7cbWYdh U``i bXYfgHJbX]b['cZh\Y'a Uh\Ya Uh]Wgžbi a Yf]WJ``UbU`mg]gžgHJh]gh]Wgž'UbX Wca di hYf`UbX`]bZcfa Uh]cb'gW]YbWYg'k\]W\'i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" '=b!XYdh\'i bXYfghUbX]b['cZgdYW]U`]ghVcX]Yg'cZ_bck `YX[Y'k]h\]b'h\Y'Yb[]bYYf]b[X]gW]d`]bY	
D9%'( '8]gWfba YbhcZ_bck `YX[ Y 'XYj Y`cda YbhUbX'fYgYUfWX 'X]fYWf]cbg'k ]h\]b'h\Y Yb[ ]bYYf]b[ 'X]gW]d`]bY	
D9%) '? bck 'YX[Y'cZYb[]bYYf]b[ 'XYg][b'dfUWjWY'UbX'WcbhYl hi U`'ZJWcfg']a dUWjb[ 'h\Y Yb[]bYYf]b[ 'X]gWjd`]bY	
D9%"* I bXYfgHJbX]b[ 'cZh\Y'gWcdYž'df]bW]d`Ygž'bcfa gž'UWVci bHJV]`]h]Yg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[ ]bYYf]b[ 'dfUWfJWY']b'h\Y'gdYW]ZJWX]gW]d`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUh]cb`cZYgHUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j ]b[	
D9&"&": `i YbhiUdd`]WUh]cb'cZYb[]bYYf]b['hYVXb]ei Ygžhcc`g'UbX'fYgci fWYg	
D9&" '5dd`]WUh]cb`cZgmghYa Uh]WYb[]bYYf]b[`gmbh\Yg]g`UbX`XYg][b`dfcWYggYg	
D9&"('5dd`]WUh]cb'cZgmghYa Uh]WUddfcUWXYg'hc'h\Y'WcbXi WhUbX'a UbU[Ya YbhcZYb[]bYYf]b[dfcYWng	
DfcZYgg]cbU`'UbX'dYfgcbU`'Ultf]Vi hYg	
D9' "%'9h\]WU`'WcbXi Wh'UbX'dfcZYgg]cbU`'UWWci bHJV]`]hm	
D9' "&'9ZYWAj Y'cfU`'UbX'k f]MYb'Wcaaib]WUAjcb']b'dfcZYgg]cbU`'UbX'`Um'XcaU]bg	
D9' "' '7 fYUhjj Yž']bbcj Uhjj Y'UbX'dfc!UWhj Y'XYa YUbci f	
D9' "( 'DfcZ/gg]cbU`i gY'UbX'a UbU[ Ya YbhcZ]bZcfa Uh]cb	
D9' ") 'C fXYf'mia UbU[ Ya YbhcZgY`Zi'UbX`dfcZygg]cbU``WcbXi Wh	
D9' "* '9ZZYWNjj Y'NYUa 'a Ya VYfg\]d'UbX'NYUa 'YUXYfg\]d	