MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 10 Solutions, August 13, 2019

1. Carla wins if either or both players rolls a 5 or 6. Summing the probability of each case to occur gives

$$2 \qquad \frac{2}{6} \quad \frac{4}{6} \quad + \frac{2}{6} \quad \frac{2}{6} = \frac{5}{9};$$

   Carla is more likely to win.

2. First note that $2^{10} = 1024$, so that the distinct numbers we are adding can have at most a power of 9 on 2. Now adding the ten possible distinct powers of 2, gives $2^0 + 2^1 + \ldots + 2^9 = 2^{10} \quad 1 = 1023$, and we have to delete a number from the summation

Now $x \neq y$, so we can cancel the common factor of $\sqrt{x} - \sqrt{y}$ to obtain

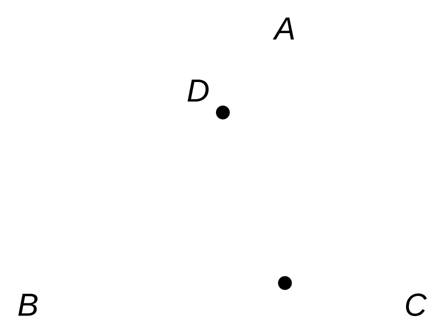
$$\sqrt{x} + \sqrt{y} = 1.$$

Squaring both sides and rearranging, we have

$$\begin{aligned} x + 2\sqrt{xy} + y &= 1 \\ x + y &= 1 - 2\sqrt{xy} \end{aligned}$$

Now the LHS is what we are trying to maximise. If we look at the RHS, we can see that this is maximised if either $x$ or $y$ is zero and the other number must then be one. So the maximum value is one.

5. Let $\angle BAC =$ and $\angle ABC =$ .



## Senior Questions

1. Since we are dividing by $x^2 - 1$, the remainder is a polynomial of $x$ of at most degree 1; that is the remainder takes the form $ax + b$, for some constants $a$ and $b$.

   To find $a$ and $b$, write

$$\begin{aligned} x^{2019} &= Q(x)(x^2 - 1) + ax + b \\ &= Q(x)(x - 1)(x + 1) + ax + b; \end{aligned}$$

   where $Q(x)$ is a polynomial of $x$. Then by putting $x = 1$ and $x = -1$ into the last line of the above equation, we have $a + b = 1^{2019} = 1$ and $-a + b = -1^{2019} = -1$. Solving these simultaneously, we arrive at $a = 1$ and $b = 0$.

2. By the sine rule,

$$\begin{aligned} \frac{\sin}{4} &= \frac{\sin 2}{6} \\ 3\sin &= 2\sin 2 \\ 3\sin &= 4\sin\ \cos ; \end{aligned}$$

   so

$$\sin = \frac{4}{3}$$