

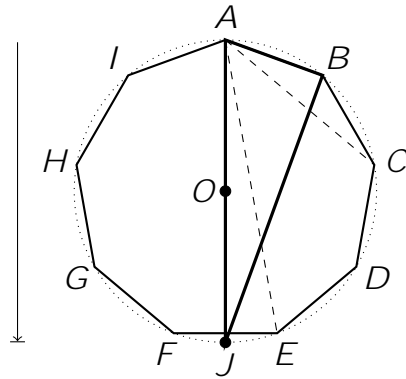
**MATHEMATICS ENRICHMENT CLUB.**  
**Problem Sheet 14 Solutions, September 10, 2019**

1. Suppose we take four socks and none of them match, then the next sock we draw must



## Senior Questions

1. Without loss of generality, we can inscribe the regular nonagon in a circle centered at  $O$  with diameter 1 unit. Let  $J$  be the point on the circle opposite  $A$ , so that  $AOJ$  is a diameter of the circle. Then  $AJ$  is one unit.



- i.  $b + c < d$ . Then each of six triples in which two numbers are from the set  $\{a; b; c\}$  and the third number is from the set  $\{d; e; g\}$  does not form a triangle.
- ii.  $c + d < e$ . Then each of six triples which includes  $e$  does not form a triangle.
- iii.  $b + d < e$  and  $a + b < d$ . Then each of six triples  $\{a; b; d\}$ ,  $\{a; b; e\}$ ,  $\{a; c; e\}$ ,  $\{a; d; e\}$ ,  $\{b; c; e\}$ ,  $\{b; d; e\}$  does not form a triangle.

Suppose that neither of above cases takes place, that is,  $b + c > d$ ,  $c + d > e$  and at least one of inequalities  $b + d > e$  and  $a + b > d$  holds. We shall show that this is impossible.

- iv. If  $b + c > d$ ,  $b + d > e$  then

$$a^2 + b^2 + c^2 + d^2 + e^2 < ab + bc + ce + (b + c)d + (b + d)e:$$

Contradiction.

- v. If  $c + d > e$ ,  $a + b > d$  then

$$a^2 + b^2 + c^2 + d^2 + e^2 < ab + bc + cd + (a + d)d + (c + d)e:$$

Contradiction.