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MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 5, June 10, 2019[1]

1. You can think of an arithmetic sequence as a set of stairs, where the size of the\jump" from one step to the next is the same in every case.

into 23 chairs, but we don't care how the 12 or 11 people are arranged within the group, so we remove 12! and 11!.

Next we consider how many ways 11 and 12 people can be arranged at a round table. For the group of 12 people, there are 12! ways in which they can be arranged into 12 chairs. However because the table is round, we can rotate the table and get the same arrangement; the number of rotations is 12. Hence we conclude that there is 12!=12 = 11! ways the 12 person group can be arranged. For the group of 11 people, there are 10! ways to arrange them following a similar argument as the 12 group case. Thus we conclude the total number of ways Bernard can do this is

$$\frac{23!}{12! \quad 11!} \quad 11! \quad 10! = \frac{23!}{12 \quad 11}:$$

3. Suppose that n

5. We can always pair up the divisors of a number x in such a way that the product of the pair is equal to the number itself. For example the divisors of 4 are $1, 2$ and 4, which can be paired up into $\{1, 4\}$ and $\{2\}$. In particular, note that if x has an odd number of divisors, then x is a perfect square.

   Suppose we have a number x with an odd number of even divisors and even number of odd divisors, then the total number of divisors of x must be odd. Hence x is a perfect square.

   If x is odd, then is has no even divisors. Since zero is an even number, this means that x has an even number of even divisors, contrary to our assumption. Thus x is even.

   We can write x in terms of its prime factors as

$$x = p_1^{e_1} 4$$

(b) Using part (a), we have $a^n \quad 1 = (a \quad 1)(a^n + a^{n \quad 1} + \ldots + a + 1)$. Since $a^n \quad 1$ is

(c) The first 5 Lucas numbers are

$$
\begin{aligned}
L_0 &= 2 \\
L_1 &= 1 \\
L_2 &= 3 \\
L_3 &= 4 \\
L_4 &= 7
\end{aligned}
$$

You should obtain the formula

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n.$$