

MATHEMATICS ENRICHMENT CLUB.¹
Problem Sheet 5, May 28, 2012

1. Two classes of 20 and 30 students average 66% and 56% respectively on an examination. What is the average for all the students on the exam?
2. A mathematics test has 5 questions on each of which people can score 0,1,2 or 3 marks.

Senior Questions.

1. Let $f(x) = \left(1 + \frac{1}{x}\right)^x$.

(a) Prove that $\frac{f'(x)}{f(x)} = \log\left(1 + \frac{1}{x}\right) - \frac{1}{1+x}$:

(b) By considering the area under the curve $y = \frac{1}{t}$ for t from 1 to $1 + \frac{1}{x}$, show that $\log\left(1 + \frac{1}{x}\right) > \frac{1}{1+x}$ and deduce that $f(x)$ is increasing.

2. Suppose $a > b > 0$. Find $\lim_{n \rightarrow \infty} (a^n + b^n)^{\frac{1}{n}}$.

3. By considering $\cos(A + B) + \sin(A - B) = 0$ find the general solution (for θ) of $\cos n\theta + \sin m\theta = 0$.