

## Solution Sheet 13, August 20, 2012

### Answers

1.  $2083\frac{1}{3}$  profit

2.  $\pi z z a$

3. Construct a triangle  $ABD$  with sides of length  $a, b, d$  such that  $\angle ADB = 90^\circ$ . Show that  $d = c$ .

4.

$$31^{24} < 32^{24} = 2^{5 \cdot 24} = 2^{120} = 2^{8 \cdot 15} = 256^{15} < 257^{15}$$

5. (a) Conjecture that  $S_n = 2^n(2n-1)(2n-3)\cdots(5)(3)(1)$  - that is,  $2^n$  times all the odd numbers from  $2n-1$  down to 1. Since every factor aside from  $2^n$  is odd, the power of 2 in the prime factorisation is  $n$ .

(b) First prove  $S_n = 2(2n-1)S_{n-1}$ . Use this to prove our conjecture.