

MATHEMATICS ENRICHMENT CLUB.  
Solution Sheet 10, July 30, 2013

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1.

$$\begin{aligned}(x^{-1} + y^{-1})^{-1} &= \frac{1}{\frac{1}{x} + \frac{1}{y}} \\ &= \frac{1}{\frac{y+x}{xy}} \\ &= \frac{xy}{y+x}.\end{aligned}$$

2. For a number to be a cube its prime factorisation must contain only cubes. The prime factorisation of  $60 = 2^2 \cdot 3 \cdot 5$  so  $2^3 \cdot 3^2 \cdot 5^2 = 450$ .
3. Using long division we can see that 12 950 264 876 is divisible by 3 but that  $4\,650\,088\,292 = \frac{12\,950\,264\,876}{3}$  is not. Thus the prime factorisation of 12 950 264 876 contains a 3 which is not squared, so cannot be a square.
4. There is a multiplier such that the angles of our triangle are 2, 3 and 4. Using the angle sum  $2 + 3 + 4 = 180 \Rightarrow = 20$ . So the angles are 40, 60 and 80.
5. Let the median from C to AB meet AB at D. Since DC has length  $\frac{1}{2}AB$

Now we divide  $2^{20} + 1$  by  $2^{10}$

2. Using various angle expansions obtain

$$\begin{aligned}
 \cos(n+2) &= \cos(n+1)\cos\theta - \sin(n+1)\sin\theta \\
 &= \cos(n+1)\cos\theta - \sin(n)\cos\theta\sin\theta - \cos(n)\sin\theta\cos\theta \\
 &= \cos(n+1)\cos\theta - \sin(n)\sin\theta\cos\theta - \cos(n)\sin\theta\cos\theta \\
 &= \cos(n+1)\cos\theta - \frac{1}{2}\sin(n)\sin(2\theta) - \cos(n)\sin^2\theta :
 \end{aligned}$$

Now note

$$\begin{aligned}
 \sin(n)\sin(2\theta) &= \cos(n)\cos(2\theta) - \cos(n+2\theta) \\
 &= \cos(n)(2\cos^2\theta - 1) - \cos(n+2\theta) :
 \end{aligned}$$

So

$$\cos(n+2) = \cos(n)\cos(2\theta) - \frac{1}{2}\sin(n)\sin(2\theta) - \cos(n)\sin^2\theta :$$