MATHEMATICS ENRICHMENT CLUB.

Solution Sheet 17, September 17, 2013

1. There are 5 odd integer digits to choose from. Once one is chosen for the rst digit, only 4 remain, then 3. So there are 5 4 3 = 60 3 digit numbers with distinct odd digits.

2.

5. (a) $a_6 = 6 + 5 + 4 + 3 + 2 = 20$

   (b) a_n is simply the sum of integers from 2 to n, which is an arithmetic series, so $a_n = \frac{n-1}{2}(n+2)$.

   (c) $b_6 = 6^2 + 5^2 + 4^2 + 3^2 + 2^2 = 90$

   (d) Some may recognize that b_n is the nth square pyramidal number (`en.wikipedia.org/wiki/Square_pyramidal_number`) minus 1. The formula for the nth square pyramidal number is $\frac{n}{6}(n+1)(2n+1)$, so $b_n = \frac{n}{6}(n+1)(2n+1) - 1$.

6. (a) $ABCB_1$ is a parallelogram since BC is parallel to AB_1 and CB_1 is parallel to AB. Similarly CBC_1A is a parallelogram. So now we know that A is the midpoint of B_1C_1.

   Now $\angle B_1AC = \angle ACB$ because they are alternate. If D is the point at which the altitude from A meets BC then $\angle DAC = 90^\circ - \angle ACD = 90^\circ - \angle ACB$ so $\angle DAC + \angle B_1AC = 90^\circ$, and AD is the perpendicular bisector of B_1C_1.

   (b) Since all the altitudes are also perpendicular bisectors of a triangle, and perpendicular bisectors of a triangle are concurrent, these altitudes are also.

7. P must be on the opposite side of the chord AB from O otherwise the angle with be zero. Instead, let the angle at P be θ, then the angle at O is $180^\circ - 2\theta$. Setting these equal gives $\theta = 180^\circ/3 = 60^\circ$.