

MATHEMATICS ENRICHMENT CLUB.<sup>1</sup>  
Solution Sheet 3, May 21, 2013

1. The dimensions of the brick are integers

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$$a_0 = c_0 \pmod{10}$$

$$a_1 = \frac{c_0}{10} + c_1 \pmod{10}$$

$$a_2 = \frac{c_1 + \frac{c_0}{10}}{10} + c_2 \pmod{10}$$

$$a_3 = \frac{c_2 + \frac{c_1 + \frac{c_0}{10}}{10}}{10} + c_3 \pmod{10}$$

Solving in order from  $b_0$  to  $b_3$  one finds  $b_0 = 3, b_1 = 5$  or  $0$ . Then if  $b_1 = 5$  we find no solution for  $b_3$ , so  $b_1 = 0$ . Then  $b_2 = 0$  or  $5$ , but this time if  $b_2 = 5$ , so

## Senior Questions

1. Solve using induction, or visit [http://en.wikipedia.org/wiki/Squared\\_triangular\\_number#Proofs](http://en.wikipedia.org/wiki/Squared_triangular_number#Proofs) for a cute geometrical representation.
2.  $1^2 + 2^2 + \dots + n^2 = \frac{1}{6}(2n^3 + 3n^2 + n)$ , so  $\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + \dots + n^2}{n^3} = \frac{1}{3}$ .
3. (I could be wrong) Choose one of 13 values for the triplet and one of 4 suits to exclude and there are  $13 \cdot 4$  possible triplets, then  $\binom{12}{5}$  combinations of the remaining suit are left. So there are  $13 \cdot 4 \cdot \binom{12}{5}$  possible hands.